On the possibility of a non-zero graviton mass

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FAQs

- What are the experimental upper bounds on the graviton mass?
- What are they based on - experimentally and theoretically?
- What about a linear theory of massive gravitons ?
- What about a non-linear theory of massive gravitons ?

Mass and Length Units

- The Compton wavelength of a mass M is given by

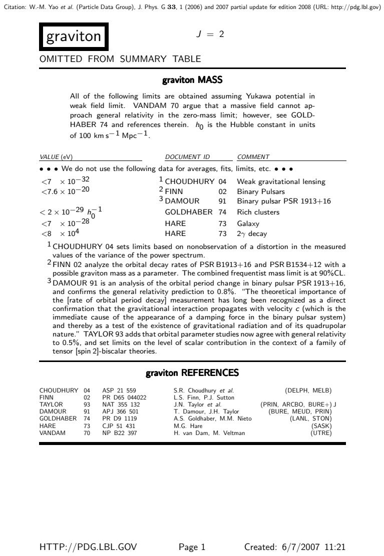
$$\lambda_M = \frac{h}{cM} \approx \frac{10^{-6}\,\mathrm{m}}{M[\mathrm{eV}]}\,.$$

- In astrophysical or cosmological length-units of light years and cosmological-horizon distance respectively,

$$\mathrm{ly} \approx 10^{16}\,\mathrm{m} \qquad \mathrm{d_H} := c/H \approx 10^{10}\,\mathrm{ly}$$

one has

$$\lambda_M = \frac{10^{-22}\,\mathrm{ly}}{M[\mathrm{eV}]} = \frac{10^{-32}\,\mathrm{d_H}}{M[\mathrm{eV}]}\,.$$



Citation: W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006) and 2007 partial update for edition 2008 (URL: http://pdg.lbl.gov)

graviton $J = 2$

OMITTED FROM SUMMARY TABLE

graviton MASS

All of the following limits are obtained assuming Yukawa potential in weak field limit. VANDAM 70 argue that a massive field cannot approach general relativity in the zero-mass limit; however, see GOLDHABER 74 and references therein. h_0 is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}.

VALUE (eV)	*DOCUMENT ID*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •		
$<7 \times 10^{-32}$	[1] CHOUDHURY 04	Weak gravitational lensing
$<7.6 \times 10^{-20}$	[2] FINN 02	Binary Pulsars
	[3] DAMOUR 91	Binary pulsar PSR 1913+16
$< 2 \times 10^{-29}\ h_0^{-1}$	GOLDHABER 74	Rich clusters
$<7 \times 10^{-28}$	HARE 73	Galaxy
$<8 \times 10^{4}$	HARE 73	2γ decay

[1] CHOUDHURY 04 sets limits based on nonobservation of a distortion in the measured values of the variance of the power spectrum.

[2] FINN 02 analyze the orbital decay rates of PSR B1913+16 and PSR B1534+12 with a possible graviton mass as a parameter. The combined frequentist mass limit is at 90%CL.

[3] DAMOUR 91 is an analysis of the orbital period change in binary pulsar PSR 1913+16, and confirms the general relativity prediction to 0.8%. "The theoretical importance of the [rate of orbital period decay] measurement has long been recognized as a direct confirmation that the gravitational interaction propagates with velocity c (which is the immediate cause of the appearance of a damping force in the binary pulsar system) and thereby as a test of the existence of gravitational radiation and of its quadrupolar nature." TAYLOR 93 adds that orbital parameter studies now agree with general relativity to 0.5%, and set limits on the level of scalar contribution in the context of a family of tensor [spin 2]-biscalar theories.

graviton REFERENCES

CHOUDHURY	04	ASP 21 559	S.R. Choudhury *et al.*	(DELPH, MELB)
FINN	02	PR D65 044022	L.S. Finn, P.J. Sutton	
TAYLOR	93	NAT 355 132	J.N. Taylor *et al.*	(PRIN, ARCBO, BURE+) J
DAMOUR	91	APJ 366 501	T. Damour, J.H. Taylor	(BURE, MEUD, PRIN)
GOLDHABER	74	PR D9 1119	A.S. Goldhaber, M.M. Nieto	(LANL, STON)
HARE	73	CJP 51 431	M.G. Hare	(SASK)
VANDAM	70	NP B22 397	H. van Dam, M. Veltman	(UTRE)

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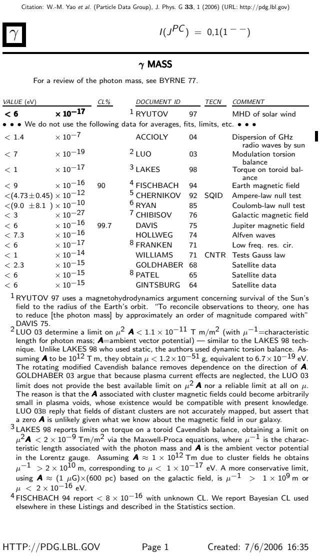

Citation: W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006) (URL: http://pdg.lbl.gov)

γ $I(J^{PC}) = 0,1(1^{--})$

γ MASS

For a review of the photon mass, see BYRNE 77.

VALUE (eV)	*CL%*	*DOCUMENT ID*	*TECN*	*COMMENT*
$\mathbf{< 6 \times 10^{-17}}$		[1] RYUTOV 97		MHD of solar wind
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$< 1.4 \times 10^{-7}$		ACCIOLY 04		Dispersion of GHz radio waves by sun
$< 7 \times 10^{-19}$		[2] LUO 03		Modulation torsion balance
$< 1 \times 10^{-17}$		[3] LAKES 98		Torque on toroid balance
$< 9 \times 10^{-16}$	90	[4] FISCHBACH 94		Earth magnetic field
$<(4.73\pm0.45) \times 10^{-12}$		[5] CHERNIKOV 92	SQID	Ampere-law null test
$<(9.0 \pm 8.1) \times 10^{-10}$		[6] RYAN 85		Coulomb-law null test
$< 3 \times 10^{-27}$		[7] CHIBISOV 76		Galactic magnetic field
$< 6 \times 10^{-16}$	99.7	DAVIS 75		Jupiter magnetic field
$< 7.3 \times 10^{-16}$		HOLLWEG 74		Alfven waves
$< 6 \times 10^{-17}$		[8] FRANKEN 71		Low freq. res. cir.
$< 1 \times 10^{-14}$		WILLIAMS 71	CNTR	Tests Gauss law
$< 2.3 \times 10^{-15}$		GOLDHABER 68		Satellite data
$< 6 \times 10^{-15}$		[8] PATEL 65		Satellite data
$< 6 \times 10^{-15}$		GINTSBURG 64		Satellite data

[1] RYUTOV 97 uses a magnetohydrodynamics argument concerning survival of the Sun's field to the radius of the Earth's orbit. "To reconcile observations to theory, one has to reduce [the photon mass] by approximately an order of magnitude compared with" DAVIS 75.

[2] LUO 03 determine a limit on $\mu^2 \boldsymbol{A} < 1.1 \times 10^{-11}$ T m/m^2 (with μ^{-1}=characteristic length for photon mass; $\boldsymbol{A}$=ambient vector potential) — similar to the LAKES 98 technique. Unlike LAKES 98 who used static, the authors used dynamic torsion balance. Assuming $\boldsymbol{A}$ to be 10^{12} T m, they obtain $\mu < 1.2 \times 10^{-51}$ g, equivalent to 6.7×10^{-19} eV. The rotating modified Cavendish balance removes dependence on the direction of $\boldsymbol{A}$. GOLDHABER 03 argue that because plasma current effects are neglected, the LUO 03 limit does not provide the best available limit on $\mu^2 \boldsymbol{A}$ nor a reliable limit at all on μ. The reason is that the $\boldsymbol{A}$ associated with cluster magnetic fields could become arbitrarily small in plasma voids, whose existence would be compatible with present knowledge. LUO 03B reply that fields of distant clusters are not accurately mapped, but assert that a zero $\boldsymbol{A}$ is unlikely given what we know about the magnetic field in our galaxy.

[3] LAKES 98 reports limits on torque on a toroid Cavendish balance, obtaining a limit on $\mu^2 \boldsymbol{A} < 2 \times 10^{-9}$ Tm/m^2 via the Maxwell-Proca equations, where μ^{-1} is the characteristic length associated with the photon mass and $\boldsymbol{A}$ is the ambient vector potential in the Lorentz gauge. Assuming $\boldsymbol{A} \approx 1 \times 10^{12}$ Tm due to cluster fields he obtains $\mu^{-1} > 2 \times 10^{10}$ m, corresponding to $\mu < 1 \times 10^{-17}$ eV. A more conservative limit, using $\boldsymbol{A} \approx (1\ \mu\text{G}) \times (600\ \text{pc})$ based on the galactic field, is $\mu^{-1} > 1 \times 10^{9}$ m or $\mu < 2 \times 10^{-16}$ eV.

[4] FISCHBACH 94 report $< 8 \times 10^{-16}$ with unknown CL. We report Bayesian CL used elsewhere in these Listings and described in the Statistics section.

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Naive Theory

- A naive generalisation of Newtonian gravity to include a graviton mass m and a cosmological constant Λ is given by

$$\Delta\phi - m^2\,\phi + \Lambda = 4\pi G\,\rho\,. \tag{1}$$

- For a point mass $\rho(\vec{x}) = M\,\delta^{(3)}(\vec{x})$ the solution is

$$\phi(r) = -\frac{\Lambda}{m^2}\left\{\frac{\sinh(mr)}{mr} - 1\right\} - \frac{GM}{r}\exp(-mr)\,.$$

- Has smooth limits $m \to 0$ and $\Lambda \to 0$ to the corresponding solutions of (1).

Questions

Units for mass and length

Naive theory & upper bounds

Linear theory

Inclusion of Λ

Non-linear theory

Alternative approaches

The End

Upper bounds

- For $\Lambda = 0$ have Yukawa-type potential. Checking Kepler's 3rd law within solar system gives upper bound (C. Will 1998)

$$\lambda_m > 10^{12}\,\mathrm{km} \approx 0.1\,\mathrm{ly}\,. \tag{2}$$

- Stability of groups of galaxies well above diameters of $2 \cdot 10^6\,\mathrm{ly}$ yields (Goldhaber & Nieto, 1974)

$$\lambda_m > 10^7\,\mathrm{ly} \approx 10^{-3}\,\mathrm{d_H} \quad \Leftrightarrow \quad m < 10^{-29}\,\mathrm{eV}\,.$$

Caution: This estimate neglects dark-matter/energy problems

Upper bounds - contd. I

- In leading-order approximation, light rays in space behave as in a medium of diffractive index

$$n(\vec{x}) = 1 - 2\,\phi(\vec{x})/c^2\,.$$

- Assuming this to hold in presence of graviton mass, a Yukawa-type suppression of potential $\phi(\vec{x})$ will influence deflection angles and hence convergences (derivative w.r.t. initial angle).
- Using a well studied cluster of stars at redshift $z = 1.1$ (Waerbeke 2001) one can derive the bound (Choudhury 2004)

$$\lambda_m > 3 \cdot 10^{-2}\,\mathrm{d_H} \quad \Leftrightarrow \quad m < 3 \cdot 10^{-31}\,\mathrm{eV}\,.$$

Upper bounds - contd. II

- In Minkowski space, a massive graviton would give rise to a dispersion relation

  $$E^2 = (cp)^2 + (mc^2)^2 \,,$$

  which in terms of the group velocity, $v_g := dE/dk$, of gravity waves gives

  $$\frac{v_g^2}{c^2} = 1 - \frac{mc^2}{E^2} \,.$$

- Estimated upper bounds on observable distortions of phasing and arrival times of gravitational waves from compact inspiral systems of stellar- to massive-BH masses ($10^7\, m_\odot$), corresponding to frequencies of 100 to 10^{-3} Hz, are as follows (C. Will 1998):

  $$\lambda_m > 0.5 \,/\, 7 \cdot 10^3\,\mathrm{ly} \quad \text{for Ligo-Virgo / Lisa}\,.$$

How can this be put within a consistent theoretical framework ?

The argument of ZvDV

- Consider a linear mass-m spin-2 theory of gravity in a Poincaré invariant context. The free momentum-space propagators are:

$$P^{m}_{\mu\nu\,\alpha\beta} = \frac{\frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}\right) - \Theta\,\eta_{\mu\nu}\eta_{\alpha\beta}}{p^2 - m^2}\,, \tag{3}$$

where

$$\Theta = \begin{cases} 1/3 & \text{for } m > 0\,, \\ 1/2 & \text{for } m = 0\,. \end{cases}$$

- This leads to one-graviton interaction

$$\kappa_m\,\frac{T^{\mu\nu}t_{\mu\nu} - \Theta\,T^{\mu}_{\mu}t^{\nu}_{\nu}}{p^2 - m^2}\,.$$

- Applied to $T = t =$ pressureless dust one obtains Newtonian limit iff

$$\kappa^2_{m>0} = \tfrac{3}{4}\kappa^2_0 = 12\pi\,G\,.$$

The argument of ZvDV - contd.

- Applied to the interaction of pressureless dust ($T^{\mu\nu}$) with light (traceless $t_{\mu\nu}$) the Θ dependence drops out but difference of $3/4$ in identification of κ remains.
- This leads to a finite difference of $m \to 0$ limit of some observables, like light deflection:

$$\lim_{m\to 0} \Delta^{m} = \tfrac{3}{4}\Delta^{\text{Einstein}} \, .$$

- Does this mean that current observations on light deflection strictly rule out $m > 0$?
- A more detailed analysis shows that the $m \to 0$ limit is precarious in several respects.

Analysis of ZvDV

- The classical field equations for a Poincaré invariant mass-m spin-2 field $h_{\mu\nu}$ outside sources are

$$(\Box + m^2)h_{\mu\nu} = 0\,, \quad \partial^\mu h_{\mu\nu} = 0\,, \quad h^\mu_\mu = 0\,.$$

- Their unique, one-parameter family of static and spherically symmetric solutions is given by

$$h_{\mu\nu}(r) = -\frac{b}{2}\begin{pmatrix} 2 & \vec{0}^\top \\ \vec{0} & (\delta_{ij} - m^{-2}\partial_i\partial_j) \end{pmatrix} \frac{\exp(-mr)}{r}\,.$$

- The term $\propto m^{-2}$ diverges for $m \to 0$. On the other hand, it is $\propto \partial_i\partial_j f$ which becomes a gauge transformation at $m = 0$. Which tendency wins on observables ?

Analysis of ZvDV - contd. I

- It can be shown that the most general coupling to matter is given by $\kappa h_{\mu\nu} T^{\mu\nu}$, so that full set of field equations with sources are

$$\begin{aligned}
(\Box + m^2) h_{\mu\nu} &= -\kappa \Pi^{m}_{\mu\nu\alpha\beta} T^{\alpha\beta} \\
\partial^\mu h_{\mu\nu} &= -\frac{\kappa}{m^2} \partial^\mu \left(T_{\mu\nu} - \tfrac{1}{3} \eta_{\mu\nu} \Sigma_{\alpha\beta} T^{\alpha\beta} \right) \qquad (4) \\
h^\mu_\mu &= \frac{\kappa}{3m^2} \Sigma_{\alpha\beta} T^{\alpha\beta}
\end{aligned}$$

where

$$\begin{aligned}
\Pi^{m}_{\mu\nu\alpha\beta} &= \tfrac{1}{2} \left(\pi^{m}_{\mu\alpha} \pi^{m}_{\nu\beta} + \pi^{m}_{\mu\beta} \pi^{m}_{\nu\alpha} - \tfrac{1}{3} \pi^{m}_{\mu\nu} \pi^{m}_{\alpha\beta} \right) \\
\pi^{m}_{\mu\nu} &= -\left(m^{-2} \partial_\mu \partial_\nu + \eta_{\mu\nu} \right) \\
\Sigma_{\mu\nu} &= \eta_{\mu\nu} - 2 m^{-2} \partial_\mu \partial_\nu \, .
\end{aligned}$$

- The 2nd and 3rd equation in (4) are constraints which are preserved under the evolution given by the first equation.

Analysis of ZvDV - contd. II

- From the coupling to a Maxwell field one shows that light rays are lightlike geodesics in the metric

  $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \, .$$

  This allows to determine the deflection angle as a function of m. It turns out to have a finite limit as $m \to 0$, given indeed by 3/4 of Einstein's value.
- Note that electromagnetic fields and gravitational fields propagate on different characteristics. This may be conjectured to be an artifact of the linear approximation.
- We also see that $|h_{\mu\nu}|$ is unbounded as $m \to 0$. That is, the linear approximation is not uniform in m.
- As $m \to 0$ the 5 degrees of freedom for the massive field turn into $2+2+1$ for a massless tensor, vector, and scalar field. The reason for the factor 3/4 is that the scalar still couples to the source T^μ_μ, whereas the vector decouples from conserved $T^{\mu\nu}$.

Analysis of ZvDV - contd. III

- In order to get the right (Einstein) limit as $m \to 0$ one may start from a massive scalar-tensor theory (Will, Visser), where the additional scalar just cancels the emerging one as $m \to 0$. However, this scalar must be a ghost (negative kinetic part).
- One way to do this is to consider Non-Pauli-Fierz mass terms:

$$-\frac{m^2}{4}\left(h_{\mu\nu}h^{\mu\nu} - \alpha(h^\mu_\mu)^2\right).$$

- Working out the field equations for $\alpha \neq 1$ yields propagation for a massive spin-2 field of mass m and a massive spin-0 field, given by $(1-\alpha)h^\mu_\mu$, of mass

$$\bar{m} = m\sqrt{(4\alpha - 1)/2(1-\alpha))} \qquad \text{for} \quad \tfrac{1}{4} \leq \alpha < 1$$

Analysis of ZvDV - contd. IV

- The formal solution of the field equations is

$$\begin{aligned} h_{\mu\nu} = &- \kappa(\Box + m^2)^{-1}\left(T_{\mu\nu} - \tfrac{1}{3}\eta_{\mu\nu}T\right) \\ &+ \eta_{\mu\nu}(\kappa/6)(\Box + \bar{m}^2)^{-1}T \\ &+ \left\{(\Box + m^2)^{-1} - (\Box + \bar{m}^2)^{-1}\right\}\partial_\mu\partial_\nu T\,. \end{aligned}$$

- For $\alpha \neq 1$ this has a limit as $m \to 0$:

$$h_{\mu\nu} = -\kappa\,\Box^{-1}\left(T_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}T\right) + \frac{\kappa}{2}\,\frac{2\alpha - 1}{1 - \alpha}\,\partial_\mu\partial_\nu\Box^{-1}\Box^{-1}T\,.$$

- The first terms is the same as in linearised GR, the second has the form of a gauge transformation. (Visser and Will consider the case $\alpha = 1/2$.)

Inclusion of Λ

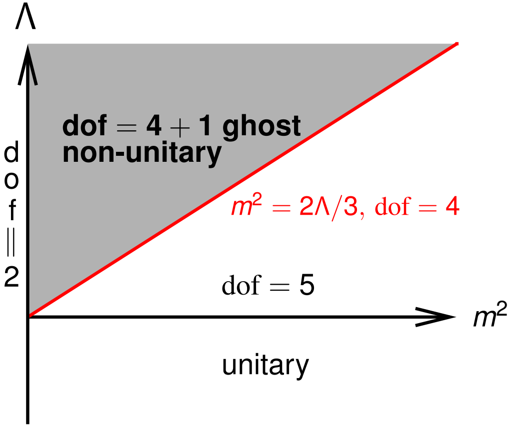
- Consider an $(A)dS_4$ background instead of Minkowski space, corresponding to a (negative) positive cosmological constant Λ.
- Due to maximal symmetry, the propagators can be explicitly computed (Naqvi 1999, Kogan *et al.* 2000). Their short-distance behaviour in case of a Pauli-Fierz mass term is as in (3), with

$$\Theta = \frac{1-(m^2/\Lambda)}{2-3(m^2/\Lambda)}$$

so that

$$\Theta \to \begin{cases} 1/2 & \text{for } m^2/\Lambda \to 0 \quad \text{(Einstein limit)} \\ 1/3 & \text{for } m^2/\Lambda \to \infty \quad \text{(ZvDV limit)} \end{cases}$$

- Limit $(m, \Lambda) \to (0, 0)$ is direction dependent.
- What goes on at $m^2/\Lambda = 2/3$?



In general, for integer/half-integer spins $s \geq 3/2$ one has upper/lower unitarity bounds (Deser & Waldron 2001)

$$\Lambda \leq 3m^2/(s-1)^2 \quad \text{bzw.} \quad \Lambda \geq -3m^2/(s-1/2)^2\,.$$

Non-linear theory

- Mass terms only exist if besides g there also exists a background metric f, where $h := g - f$:

  $$I_m = -\frac{M_{\mathrm{Pl}}^2\, m^2}{2} \int dx^4\, K[f,g]^{\mu\nu\,\alpha\beta}\, h_{\mu\nu} h_{\alpha\beta}\,.$$

  e.g. of Pauli-Fierz-type (not unique!)

  $$K[f,g]^{\mu\nu\,\alpha\beta} = \sqrt{-f}\,\left(f^{\mu\alpha} f^{\nu\beta} - f^{\mu\nu} f^{\alpha\beta}\right).$$

- 6 instead of only 5 field degrees of freedom: h^μ_μ is ghost (Boulware & Deser 1970). Leads to instabilities of Minkowski space on arbitrarily short timescales (Gabadadze & Gruzinov 2005).
- Superluminal propagation of gravitational waves on $k = 0$ FRW background (Rubakov 2008).

Non-linear theory - contd.

- A weak-field and low-mass approximation exists only locally (Vainshtein 1972, Carrera & D.G. 2001, Damour *et al.* 2003)

$$r_g \ll r \ll \left(m^{-4} \cdot r_g\right)^{1/5}.$$

- Non-trivial asymptotically flat solutions are conjectured to not exist (Carrera & D.G. 2001, Damour *et al.* 2003). Schwarzschild-DeSitter is, however, solution (Salam & Strathdee 1977).
- One-loop corrections to propagator are of the form (Aubert 2004)

$$\frac{1}{p^2 - m^2}\left\{1 + \frac{p^{10}\log(p^2)}{4320\pi M_{\mathrm{Pl}}^2 m^8} + \frac{P(p)}{M_{\mathrm{Pl}}^2 m^8} + O(m^{-6})\right\}.$$

  The strong-coupling scale is hence given by

$$\Lambda = \left(M_{\mathrm{Pl}} \cdot m^4\right)^{1/5}.$$

  For $m \approx 10^{-29}\mathrm{eV}$ this corresponds to length $10\,\mathrm{AU}$!

Non-Lorentz-invariant mass terms

- Writing $h_{\mu\nu} = (h_{00}, h_{0i}, h_{ij})$, the most general E_3-invariant mass-term is given by

$$L_m = (M_{\rm Pl}^2/2)\left(m_0^2\, h_{00}^2 + 2m_1^2\, h_{0i}^2 - m_2^2\, h_{ij}^2 + m_3^2\, h^2 - m_4^2\, h_{00}\, h\right) .$$

- $-m^2(h^2_{\mu\nu} h_{\mu\nu} - \alpha(h^\mu_\mu)^2)$ corresponds to

$$m_0^2 = (1-\alpha)m^2\,, \quad m_1^2 = m_2^2 = m^2 \quad m_3^2 = m_4^2 = \alpha m^2\,.$$

- Absence of ghosts and smooth ZvDV limit is guaranteed if (Rubakov 2004)

$$m_1^2 > m_4^2 > 0\,, \quad m_2^2 > m_3^2\,, \quad 4m_2^2 > m_4^2\,.$$

- The scale of strong coupling is now given by $\sqrt{mM_{\rm Pl}}$ (Rubakov 2004 → Arkani-Hamed 2003), corresponding to $3\,\mu\mathrm{m}$ for $m = 10^{-29}\,\mathrm{eV}$.

Induced gravity

- The DGP action is

$$S = -M_{\rm Pl}^4 \left\{ \int d^4x \sqrt{-g_4} R_4 + L^{-1} \int d^5x \sqrt{-g_5} R_5 \right\} .$$

- Consider 4-dim spherically-symmetric solution

$$g_5 = e^{\nu} dt^2 - e^{\lambda} dr^2 - e^{\mu} (d\chi^2 + sin^2\chi \, d^2\theta + sin^2\chi sin^2\theta \, d^2\varphi)$$

  with brane at $x^4 = r \cos \chi = 0$.

- For large L there is a leading-order correction of the Schwarzschild metric on the brane (Gruzinov 2002):

$$\frac{\delta\nu}{\nu} = -2\sqrt{2} \left(\frac{r}{L}\right) \left(\frac{r}{r_g}\right)^{1/2} , \quad \text{comp.} \quad \frac{\delta\nu}{\nu}\bigg|_{\rm Yukawa} = -mr \, .$$

- For the same data as in (2) one obtains from solar-system planetary motion (compare bound (2))

$$L > 3 \cdot 10^{-2} \, \mathrm{d_H}$$

Summary and Comments

- There seems to be no way to speak about massive gravitons within traditional framework (SR & GR in 3+1 dimensions).
- Possibilities for infrared modifications exist in higher-dimensional models (3+1 covariant) and possibly in 3+1 dimensions if Lorentz invariance is given up. In the first case there are potentially interesting phenomenological consequences, e.g. graviton-oscillations (Barvinsky et al. 2003).
- Strong coupling may set in well above the Planck energy, possibly causing anomalies in gravitational law of attraction at sub-$\mu\mathrm{m}$ scales.
- Caution: Conversely it does not follow that deviations from r^{-2} force-law at sub–$\mu\mathrm{m}$ scales and/or violations of equivalence principle are necessarily signatures of Quantum Gravity.
- Only through theory we can interpret observations, but not anything that can be written down constitutes a theory!