12th lecture

QUANTUM CHROMODYNAMICS

– part 2 –

Light mesons & baryons

- even before 1974, most theorists would have bet on quarks
reason: many light hadron properties explained by the "constituent quark model"
→ light hadrons are bound states of light quarks
- quark masses: $m_u \approx 3$ MeV, $m_d \approx 6$ MeV, $m_s \approx 150$ MeV
are so-called "current masses" (in QCD Lagrangian)
relevant at $E \gg \Lambda_{QCD}$, in perturbative regime
but at $E \lesssim \Lambda_{QCD}$ each quark is dressed by a gluon cloud
$m_{cloud} \simeq 300$ MeV $\Rightarrow$ constituent masses $m_q + m_{cloud} \gg m_q$

- approximate constituent mass degeneracy for u & d

$\rightsquigarrow$ approximate SU(2) "isotopic symmetry"

- key example $m_{p=uud} \approx m_{n=udd}$ ($\Delta m \approx 1.3$ MeV < current masses, Coulomb energy)

arrange $|p\rangle$ & $|n\rangle$ in 2-component $|N\rangle = \begin{pmatrix} |p\rangle \\ |n\rangle \end{pmatrix} \in \mathbb{C}^2 \times \mathcal{H}$

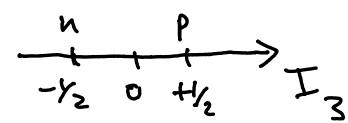
"isospin" symmetry rotates $|p\rangle \leftrightarrow |n\rangle$: $|N\rangle \mapsto U|N\rangle$

$U \in SU(2)$

$\vec{I} = (I_1, I_2, I_3)$ $\quad [I_i, I_j] = i\varepsilon_{ijk} I_k$

$I_3|p\rangle = +\frac{1}{2}|p\rangle$, $I_3|n\rangle = -\frac{1}{2}|n\rangle$, $I_+|n\rangle \sim |p\rangle$, $I_-|p\rangle \sim |n\rangle$

$\rightsquigarrow$ p & n form an iso(topic) doublet



$\rightsquigarrow$ $SU_I(2)$ in analogy with ordinary spin:

particle states form irreps labelled by isospin i

$\vec{I}^2 |\ \rangle = i(i+1)|\ \rangle$, $i = 0, \frac{1}{2}, 1, \dots$, basis $\{|i_3\rangle, i_3 = -i, \dots, +i\}$

the $2i+1$ basis states form an "isomultiplet"

- other examples

isotriplet Σ $(i=1)$: $|\Sigma^+\rangle = |uus\rangle$

isosinglet Λ $(i=0)$: $|\Sigma^0\rangle = |uds\rangle$

$|\Lambda\rangle = |uds\rangle$ | $|\Sigma^-\rangle = |dds\rangle$

isodoublet Ξ $(i=\frac{1}{2})$: $|\Xi^0\rangle = |uss\rangle$, $|\Xi^-\rangle = |dss\rangle$

- strange constituent mass not far from u/d $\rightsquigarrow$ broken larger symmetry

$\rightsquigarrow$ enhance isospin $SU(2)$ to approximate flavor $SU(3)$

! do never confuse global flavor $SU_F(3)$ with color $SU_C(3)$!

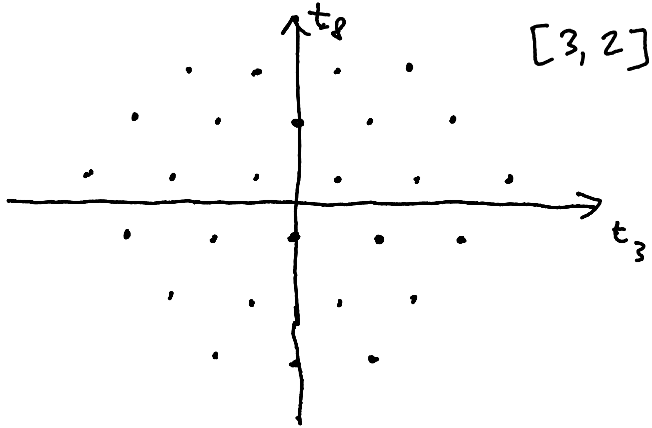
- flavor $SU_F(3)$ $\rightsquigarrow$ particles grouped in $SU_F(3)$ multiplets (irreps)

recall Lie algebra basis $\{T_1, \dots, T_8\}$ $\to$ T_3, T_8 diagonal; $\to$ 3 pairs of ladders

an $SU(3)$ irrep has basis $\{|t_3, t_8\rangle\}$ $\leftarrow$ simultaneous eigenvalues of T_3 & T_8

plot pairs (t_3, t_8) $\to$ weight space

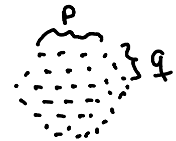
- an example



[3, 2]

ladders

$\leftrightarrow \quad \nwarrow\!\searrow \quad \nearrow\!\swarrow$

shape 

$\Longleftrightarrow$ irrep [p, q]

degeneracies:
increase from outside to inside
remain constant once triangular

rescale: $i_3 = t_3$, $y = \frac{2}{\sqrt{3}} t_8$ $\rightsquigarrow$ $\underset{1}{\leftrightarrow}$ $\updownarrow 1$

"isospin" "hypercharge"

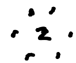
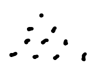
& $y = b + s$

baryon number "strangeness" (#$\bar{s}$ − #s)

baryon number: +1 for baryons
−1 for antibaryons
0 for mesons

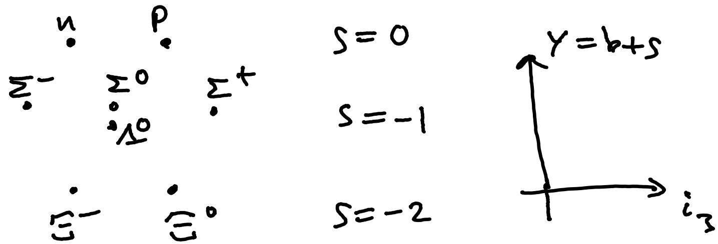
strangeness: historical notion for long-lived hadrons, strongly cons'd

- simplest representations

$[0,0]$		singlet	1
$[1,0]$		triplet (fundamental)	3
$[0,1]$		antitriplet (anti-fund'l)	$\bar{3}$
$[1,1]$		octet (adjoint)	8
$[2,0]$		sextet	6
$[0,2]$		antisextet	$\bar{6}$
$[3,0]$		decuplet	10
$[0,3]$		antidecuplet	$\overline{10}$

some irreps occur in hadron spectrum
but some do not why?

- baryon octet (spin ½ : $|\uparrow\uparrow\downarrow\rangle$, $|\uparrow\downarrow\downarrow\rangle$)



$|udd\rangle$	$|uud\rangle$		$i=\frac{1}{2}$
$|dds\rangle$	$|uds\rangle$	$|uus\rangle$	$i=1$
$|dss\rangle$	$|uss\rangle$		$i=\frac{1}{2}$

lifetimes: p stable, $n \to p$ via $d \to u e^- \bar{\nu}_e$

$\Sigma, \Lambda \to n$ or p via $s \to u e^- \bar{\nu}_e$

$\Xi \to \Sigma, \Lambda \to n, p$ via "cascade"

$\Big\}$ $\tau \approx 10^{-10}$ s decay weakly

observe: electric charge $q = i_3 + \frac{1}{2} Y$ Gell-Mann – Nishijima

Δq ↗

masses: very close inside same isomultiplet

$m_\Xi > m_{\Sigma,\Lambda} > m_N$ but differences both ≈ 190 MeV

equality of $m_{2s} - m_{1s} = m_{1s} - m_{0s}$: Gell-Mann – Okubo (GMO)

- baryon decuplet (spin 3/2 : $|\uparrow\uparrow\uparrow\rangle, |\uparrow\uparrow\downarrow\rangle, |\uparrow\downarrow\downarrow\rangle, |\downarrow\downarrow\downarrow\rangle$)

$S=0$ $\quad \Delta^- \quad \Delta^0 \quad \Delta^+ \quad \Delta^{++} \longrightarrow N\pi$ decay $\qquad |ddd\rangle \quad |udd\rangle \quad |uud\rangle \quad |uuu\rangle$

$S=-1$ $\quad \Sigma^{*-} \quad \Sigma^{*0} \quad \Sigma^{*+}$ $\qquad |dds\rangle \quad |uds\rangle \quad |uus\rangle$

$S=-2$ $\quad \Xi^{*-} \quad \Xi^{*0}$ $\qquad |dss\rangle \quad |uss\rangle$

excited versions of Σ, Ξ } lifetimes $\sim 10^{-23}$ s

$S=-3$ $\quad \Omega^- \longrightarrow$ must decay weakly: $\longrightarrow \tau \approx 10^{-10}$ s ! $\qquad |sss\rangle$

$\Omega^- \to \Xi \to \Sigma \to N$

GMO works well

Ω^- predicted 1961, observed in 1964

- meson octet (spin 0, pseudoscalar)

$K^0 \quad K^+$ $\qquad S=1$ $\qquad |d\bar{s}\rangle \quad |u\bar{s}\rangle$

$\pi^- \quad \pi^0 \quad \pi^+$, η $\qquad S=0$ $\qquad |d\bar{u}\rangle \quad \frac{1}{\sqrt{2}}(|u\bar{u}\rangle - |d\bar{d}\rangle) \quad |u\bar{d}\rangle$

$\frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)$

$K^- \quad \bar{K}^0$ $\qquad S=-1$ $\qquad |s\bar{u}\rangle \quad |s\bar{d}\rangle$

GMO does not work so well because of "spontaneous breaking of chiral symmetry" $\longrightarrow$ later

- why only certain $SU_F(3)$ reps appear?
  - all irreps can be constructed by iterated tensor products of basic irreps 3 & $\bar{3}$ ($[10]$ & $[01]$) plus symmetry projection
  - physical interpretation: baryons are bound states of objects transforming in basic reps:

    $[10]$ = "quark triplet" $\quad d \; u \; s \quad$ with spin $\frac{1}{2}$ & $b = \frac{1}{3}$

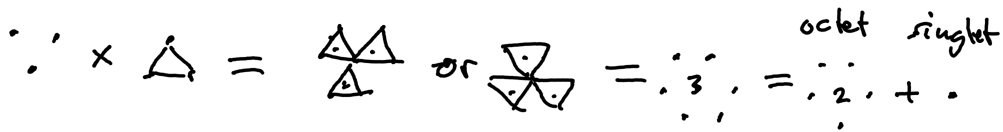
    $[01]$ = "antiquark triplet" $\quad \bar{s} \; \bar{u} \; \bar{d} \quad$ with spin $\frac{1}{2}$ & $b = -\frac{1}{3}$

    $q_u = +\frac{2}{3}$

    $q_d = q_s = -\frac{1}{3}$

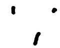
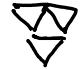
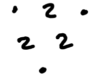
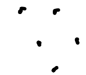
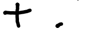
    but these states are forbidden by confinement!
    (only color-neutral asymptotic states exist)
  - tensor products

    $3 \times \bar{3}$:



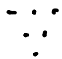
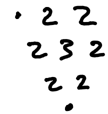
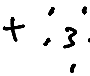
$$= 8 + 1$$

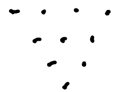
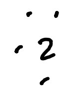
$\rightsquigarrow$ $q\bar{q} \to$ meson octet + singlet (η')

3×3 :  $\times$  $=$  $=$  $=$  $+$  $= 6 + \bar{3}$

(not allowed by confinement)

$3 \times 3 \times 3 = (6 + \bar{3}) \times 3 = 6 \times 3 + \bar{3} \times 3$

$=$  $\times$  $+$  $\times$  $=$  $+$  $=$  $+$ 

$=$  $+$  $+$  $+$ $= 10 + 8 + 8 + 1$

(allowed by confinement because $\exists$ $SU_c(3)$ singlet here)

$\Rightarrow$ $qqq \rightarrow$ baryon $<$ decuplet / octet

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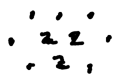
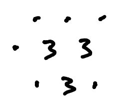
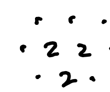
answers to polling questions

1) $[2,1]$  12 dots = weights

2) $\dim([2,1]) = 15$

3)  $\times$ $=$  $=$  $=$  $+$  $= 15 + 3$ $= [2,1] + [1,0]$

Chiral symmetry in QCD

- consider a world in which $m_u = m_d = 0$

$\rightsquigarrow \quad \mathcal{L}^0_{u,d} = i\bar{q}\gamma^\mu(\partial_\mu + g\hat{A}_\mu)q$ with $q = \begin{pmatrix} u \\ d \end{pmatrix}$

global flavor symmetry: $q \mapsto S_V q$, $S_V \in U_V(2)$ 2×2 unitary

is "vector-like": Noether currents

$$(j^\mu_V)^i{}_j = \bar{q}^i \gamma^\mu q_j \qquad i,j = 1,2$$

are Lorentz vectors, can be grouped in SU(2) irreps: ← "iso"

$\rightsquigarrow$ isosinglet $\bar{q}\gamma_\mu \mathbb{1} q$ & isotriplet $\bar{q}\gamma_\mu\sigma^a q$ $\quad a = 1,2,3$

$\hookrightarrow U_V(1)$ $\qquad\qquad$ $\hookrightarrow SU_V(2)$

- but there is a larger symmetry:

split into left- & right-handed quark fields:

$q_{L,R} = \frac{1}{2}(\mathbb{1} \mp \gamma^5)q \quad \rightsquigarrow \quad \bar{q}_{L,R} = \bar{q}\,\frac{1}{2}(\mathbb{1} \pm \gamma^5)$

$\rightsquigarrow$ $\mathcal{L}^0_{u,d} = i\bar{q}_L \gamma^\mu (\partial_\mu + g\hat{A}_\mu) q_L + i\bar{q}_R \gamma^\mu (\partial_\mu + g\hat{A}_\mu) q_R$

is invariant under <u>separate</u> $q_L \mapsto S_L q_L$, $q_R \mapsto S_R q_R$

$\rightsquigarrow$ full symmetry is $U_L(2) \times U_R(2)$!

- turn on quark masses but keep $m_u = m_d$

$\rightsquigarrow$ mass term $\mathcal{L}_m = m\bar{q}q = m(\bar{q}_L q_R + \bar{q}_R q_L)$

is invariant only if $S_L = S_R$ $\rightsquigarrow$ $U_V(2) \subset U_L(2) \times U_R(2)$

- but for $m=0$ one also has $S_L = S_R^\dagger$

$\rightsquigarrow$ "chiral" or "axial" transformations $\rightsquigarrow$ other subgroup, $U_A(2)$

its Noether currents $j^\mu_A = \{\underbrace{\bar{q}\gamma^\mu\gamma^5 q}_{U_A(1)}, \underbrace{\bar{q}\gamma^\mu\gamma^5\sigma^a q}_{SU_A(2)}\}$

however, quantum effects break $U_A(1)$ ("anomalous")

$\rightsquigarrow$ must impose constraint $\det(S_L S_R^\dagger) = 1$ $\rightsquigarrow$ project onto $SU_A(2)$

- in massless 2-flavor QCD the global symmetry is

$$U_V(2) \times SU_A(2)$$

However: $SU_A(2)$ gets spontaneously broken

What does that mean?

Spontaneous symmetry breaking

a concept also essential for electroweak part!

- toy model I: real scalar field

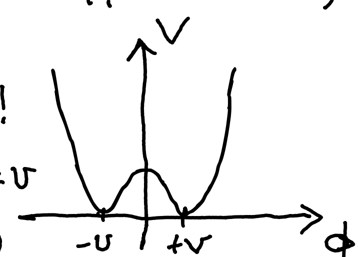
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) \quad , \quad V = \frac{\lambda}{4!}(\phi^2 - v^2)^2$$

discrete $\mathbb{Z}_2$ symmetry $\phi \mapsto -\phi$

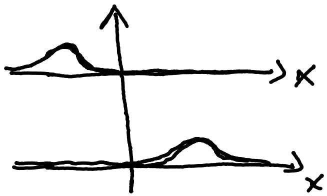
but symmetric point ($\phi = 0$) is not minimum!

instead: <u>two</u> non-symmetric minima $\phi = \pm v$

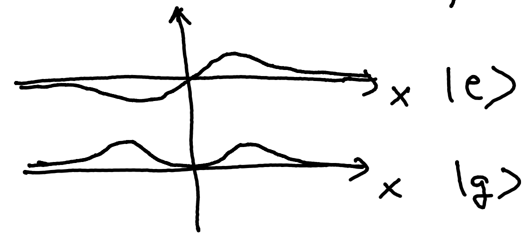
$\rightsquigarrow$ vacuum expectation value $\langle \phi \rangle = \pm v \neq 0$



- remark: in quantum mechanics the two wave functions ψ



combine via tunneling



QFT: ∞ degrees of freedom $\rightsquigarrow$ ∞ tunneling barrier

$\rightsquigarrow$ <u>two</u> degenerate ground states, related by symmetry

- spontaneous symmetry breaking:

– $\mathcal{L}$, H enjoy a symmetry $\rightsquigarrow$ eq. of motion are invariant

– ground state is <u>not</u> $\rightsquigarrow$ collection of degenerate vacua related by symmetry transformations

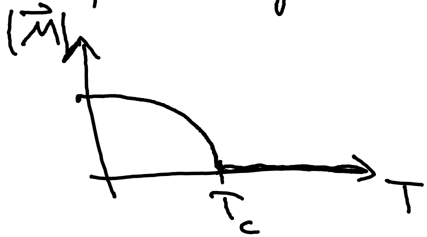
- examples

– $\ddot{x} + \omega^2 x = 0$ sym.: $x(t) \xmapsto{T_a} x_a(t) = x(t-a)$ transl. inv.

solutions $x_{\alpha,\beta}(t) = \alpha \cos(\omega t + \beta) \xmapsto{T_a} \alpha \cos(\omega(t-a)+\beta) = x_{\alpha,\beta-\omega a}(t)$

continuous family of nonsym. sol'us, only $x_0 \equiv 0$ is invariant

- parity symmetry QED
  - $\rightsquigarrow$ chiral moleculer, on Earth only one variant prevalent
- ferromagnet: eqs. are rotationally symmetric $\rightsquigarrow SO(3)$



$T > T_c$: $\vec{M} = 0$ $\quad$ $SO(3)$ symmetric

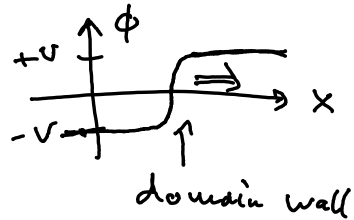
$T < T_c$: $\vec{M} \neq 0$ $\quad$ breaks $SO(3) \to SO(2)$
selects a direction

- phase transitions

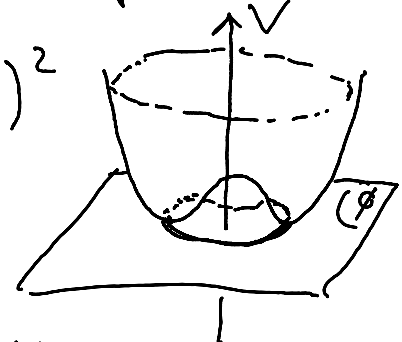
order parameter $\langle \phi \rangle$ $\begin{cases} = 0 & \text{unbroken phase} \\ \neq 0 & \text{broken phase} \end{cases}$ $\quad$ transition mostly second order

first-order transition come with domain walls $\leftrightarrow$ latent heat
expanding bubbles of "true vacuum" into "false vacuum"

can get hit
by domain wall



- toy model II: complex scalar field



$$\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) - \frac{\lambda}{4}(\phi^*\phi - v)^2$$

continuous U(1) symmetry

$$\phi \longmapsto e^{i\chi}\phi$$

continuous family of minima: $|\phi| = v$, phase arbitrary

↪ "vacuum manifold" $= S^1 = \{ v e^{i\alpha/\sqrt{2}} \mid \alpha \in [0, \sqrt{8}\pi] \}$

U(1) symmetry spontaneously broken to $\mathbb{1}$

$\alpha =$ order parameter

- imagine α has slow x dependence: $\phi_{vac}(x) = v e^{i\alpha(x)/\sqrt{2}}$

$\rightsquigarrow \mathcal{L}_{eff} = \mathcal{L}(\phi_{vac}) = \frac{v^2}{2}(\partial_\mu \alpha)(\partial^\mu \alpha)$ inv. under $\underbrace{\alpha \to \alpha + \chi}_{U(1)}$

"walk in vacuum mfd" ⇔ low-energy spectrum ⇔ massless free scalar

this soft mode is called "Goldstone boson"

(fluctuation of order parameter)

- Goldstone theorem

Lagrangian with global symmetry group G spontaneously broken to a subgroup $H \subset G$.

Then spectrum includes $\dim G - \dim H$ massless particles. Their interaction strength depends on E, decouple for $E \to 0$

Vacuum manifold $=$ coset G/H, parametrized by the Goldstone bosons
$(V=0)$

Quark condensate

back to QCD: axial $SU_A(2)$ is broken spontaneously

where are the 3 Goldstone bosons? which order parameter?

consider composite $\mathcal{O}^{ij} = q_L^i \bar{q}_R^j$ $\quad i,j = 1,2$ or u,d

the order parameter is VEV $\Sigma^{ij} = \langle \mathcal{O}^{ij} \rangle$ complex 2×2

$\rightarrow$ 8 real parameters $\Leftrightarrow$ dim of $U_L(2) \times U_R(2)$ or $U_V(2) \times U_A(2) = G$

in QCD only $G = U_V(2) \times SU_A(2)$ broken by

$\Sigma^{ij} = \Sigma \cdot \Omega^{ij}$ with Ω^{ij} unitary

$U_V(1) \leftrightarrow$ size (for Σ), orientation (for Ω^{ij})

still invariant under $SU_V(2)$ subgroup

broken to $U_V(2)$

$\rightsquigarrow \mathcal{M}_{vac} = SU_A(2) = S^3$

size $\Sigma \approx (250\ \text{MeV})^3$ "quark condensate" ($250\ \text{MeV} \sim \Lambda_{QCD}$)

$\rightarrow$ chiral sym. is broken $\Rightarrow$ 3 Goldstone bosons are the pions (π^-, π^0, π^+)!

it is a breaking of approximate symmetry $\rightarrow m_\pi \neq 0$

start with massless QCD & treat $m_u \approx m_d$ as perturbations

$\rightarrow \pi$ are "pseudo-Goldstones", have small mass

picture: tilted champagne bottle

$\exists$ formula $m_\pi^2 \sim (m_u + m_d) \cdot \Sigma \rightarrow 0$ in chiral limit $m_q \rightarrow 0$

- proton mass? Ioffe formula $m_p \approx \sqrt[3]{4\pi^2 \Sigma} \Leftarrow$ chiral sym. breaking gives hadrons mass!