13th lecture

ELECTROWEAK INTERACTIONS

— part 1 —

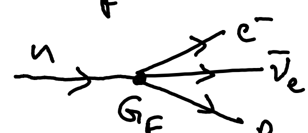
Fermi theory & weak currents

- $\mathcal{L}_{\text{Fermi}} = G_F \, \bar{p} \gamma_\mu n \cdot \bar{e} \gamma^\mu \nu_e$ for neutron decay (1934)

p, n, e, ν_e = Dirac spinors for those particles

$[\text{spinor}] = M^{3/2} \rightsquigarrow [G_F] = M^{-2}$ $G_F \approx 1.2 \times 10^{-5}\ \text{GeV}^{-2}$

tree amplitude for n decay



"4-fermion"

$$\mathcal{M}_{n \to p e \bar{\nu}_e} = G_F \, \bar{u}_p(p_p) \gamma_\mu u_n(p_n) \cdot \bar{u}_e(p_e) \gamma^\mu u_{\nu_e}(p_{\nu_e})$$

$[u] = M^{1/2} \rightsquigarrow [\mathcal{M}] = M^0$ ✓ works well at low energies but not quite correct $\longrightarrow$ parity-breaking is absent here!

- true effective (= low-energy) Lagrangian is

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \cos\theta_c \cdot \underbrace{\bar{p}\gamma_\mu(1-g_A\gamma^5)n}_{j_\mu^{+(N)}} \cdot \underbrace{\bar{e}\gamma^\mu(1-\gamma^5)\nu_e}_{j^{\mu-(e)}} + \text{h.c.}$$

where $g_A \approx 1.26$, θ_c = "Cabbibo angle", $\sqrt{2}$ is matching factor

- $j_\mu^{-(e)}$ = "weak charged lepton current"
  ↳ corresponding $\hat{H}$ creates e^- & destroys e^+ in quantum state
  - in analogy to the electromagnetic neutral current $\bar{e}\gamma_\mu e$
  - describes transitions $\nu_e \to e$, or creation of $\bar{\nu}_e$ & e^- from $|0\rangle$
  - is not a pure vector, but combination of vector & axial vector
    ↳ "V-A"
- $j_\mu^{+(N)}$ = "weak charged nucleon current"
  - not exactly V-A, because p & n are not fundamental
  - electron charge = − proton charge $\Leftrightarrow$ V-coefficient exactly $g_V = 1$
  - Strong interactions distort axial charge $\rightsquigarrow$ $g_A \neq 1$ (computable in low-energy QCD)

- fundamental fields are the quark fields $\rightsquigarrow$

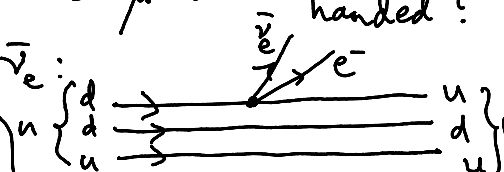
$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ j_\mu^{-(e)} j^{\mu+(q)} + j_\mu^{+(e)} j^{\mu-(q)} \right\} \quad \text{with}$$

$$j_\mu^{-(e)} = \bar{e}\gamma_\mu(1-\gamma^5)\nu_e \quad , \quad j_\mu^{+(e)} = \bar{\nu}_e \gamma_\mu (1-\gamma^5) e \quad \text{(check!)}$$

$$j_\mu^{+(q)} = \bar{u}\gamma_\mu(1-\gamma^5) d \quad , \quad j_\mu^{-(q)} = \bar{d}\gamma_\mu(1-\gamma^5) u$$

"weak charged quark currents"

- quark currents are V–A $\rightsquigarrow$
- $j_\mu^{-(e)} = 2\bar{e}_L \gamma_\mu \nu_{eL} \quad , \quad j_\mu^{-(q)} = 2\bar{d}_L \gamma_\mu u_L$ only left-handed!
- neutron decay caused by $d \to u e^- \bar{\nu}_e$:



- quantum $\hat{H}$ contains $\hat{V}$ from $V = -\mathcal{L}_{eff}$

amplitude for n decay given (in leading order) by matrix element

$$M = \langle p e^- \bar{\nu}_e | \hat{V} | n \rangle = \frac{G_F}{\sqrt{2}} \langle p | j_\mu^{+(q)} | n \rangle \cdot \langle e^- \bar{\nu}_e | j^{\mu-(e)} | vac \rangle$$

$$= \frac{G_F}{\sqrt{2}} \bar{u}_p(p_p)\gamma_\mu(1-g_A\gamma^5) u_n(p_n) \cdot \bar{u}_e(p_e)\gamma^\mu(1-\gamma^5) u_{\nu_e}(p_{\nu_e})$$

- origin of factor $\cos\theta_c$:

weak interactions between different quark & lepton generations!

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \quad \& \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

e.g. $\Lambda \longrightarrow p e^- \bar{\nu}_e$ via $s \to u e^- \bar{\nu}_e$ $\rightsquigarrow$ need current $\sim \bar{u}\gamma_\mu s$

for simplicity consider only two generations:

$$j_\mu^{-(q)} = \bar{d}\gamma_\mu(1-\gamma^5)u + \bar{s}\gamma_\mu(1-\gamma^5)c \quad \text{is wrong, rather}$$

$$\begin{aligned} j_\mu^{-(q)} &= \bar{d}\gamma_\mu(1-\gamma^5)(u\cdot\cos\theta_c - c\cdot\sin\theta_c) \\ &\quad + \bar{s}\gamma_\mu(1-\gamma^5)(u\cdot\sin\theta_c + c\cdot\cos\theta_c) \\ &= \begin{pmatrix} d \\ s \end{pmatrix}^T \gamma_\mu(1-\gamma^5) \begin{pmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix} = \begin{pmatrix} d \\ s \end{pmatrix}^T \gamma_\mu(1-\gamma^5) \begin{pmatrix} u' \\ c' \end{pmatrix} \end{aligned}$$

$\rightsquigarrow$ "generation mixing", Cabbibo angle $\theta_c \approx 13^\circ$ ($\cos\theta_c \approx 0.97$)

$\longrightarrow$ strange baryon decay with $e^-\bar{\nu}_e$ emission suppressed by $\sin\theta_c$

neutron beta decay almost unaffected ($\sim\cos\theta_c$)

- 3-generation mixing:

$$\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} = \underset{\substack{\uparrow \\ 3\times 3 \text{ matrix}}}{\text{CKM}} \cdot \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$ "Cabbibo-Kobayashi-Maskawa"

3x3 matrix ⟶ 3 angles + 1 phase

⇝ matrix is complex ($\approx 10^{-3}$) ⟶ CP violation!

$$\begin{pmatrix} \nu_e' \\ \nu_\mu' \\ \nu_\tau' \end{pmatrix} = \text{PMNS} \cdot \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$ "Pontecorvo-Maki-Nakagawa-Sakata"

↳ neutrino oscillations! also 4 parameters

- full modernized low-energy Fermi Lagrangian:

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} j_\mu^{+} j^{\mu -}$$ with $j_\mu^{\pm} = j_\mu^{\pm(\ell)} + j_\mu^{\pm(q)}$ "current-current"

lepton & quark currents including generation mixing

So far looked only at $j^{(\ell)} \cdot j^{(q)}$ "semi-leptonic process"

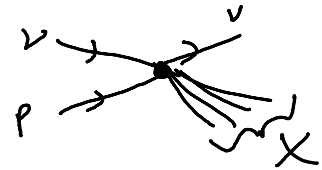
but $\mathcal{L}_{eff}$ contains also $j^{(\ell)} \cdot j^{(\ell)}$ "purely leptonic" (e.g. $\nu e \to \nu e$)

and also $j^{(q)} \cdot j^{(q)}$ "non-leptonic" (e.g. $\Lambda \to p\pi^-$)

[⇝ small parity violating effects in hadronic processes]

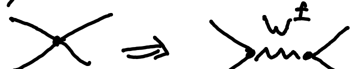
- still something missing: "weak neutral currents"

ex.: $\nu p \longrightarrow \nu X$
$\uparrow$ other stuff

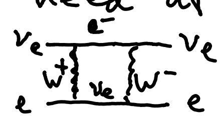


needs $j_\mu^{(\nu)} = \bar{\nu}\gamma_\mu(1-\gamma^5)\nu$

in addition, electron neutral current picks up a weak contribution

- this zoo of currents begs underlying explanation
  - Fermi theory not renormalizable* $\rightsquigarrow$ "high-energy completion"?
  - resolution 

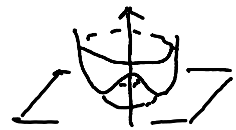
? but massive vector bosons as mediators are not gauge-invariant
  - way out: W bosons get mass via "Higgs mechanism"
  - weak interactions entangled with electromagnetism
  - need at least four mediators: $W^\pm, Z^0, \gamma \rightsquigarrow 4$ gauge fields

* 

W propagators do <u>not</u> fall off at high momenta
Fermi propagators $\sim 1/p$ $\Big\} \Rightarrow \int d^4p\, p^{-2}$ div.

Higgs mechanism

- Abelian case



– recall spontaneous symmetry breaking of global $U(1)$ in complex scalar field theory with Mexican hat potential

– now assume our scalar particles carry electric charge $\rightsquigarrow$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D^\mu \phi)^* (D_\mu \phi) - \frac{\lambda}{4} (\phi^* \phi - v^2)^2 \, , \quad D_\mu = \partial_\mu + ieA_\mu$$

now the $U(1)$ invariance is local (is a redundancy):

$$\phi(x) \longmapsto e^{-ie\chi(x)} \phi(x) \quad \& \quad A_\mu(x) \longmapsto A_\mu(x) + \partial_\mu \chi(x)$$

– no longer a vacuum manifold S^1 of ∞ many vacua labelled by $\arg(\phi)$, because changing the phase of $\phi(x)$ is a gauge transformation (a redundancy) $\rightsquigarrow$ one vacuum

$\rightsquigarrow$ no longer expect a Goldstone boson, and it is absent:

$$\mathcal{L}_{vac} \equiv \mathcal{L}(\phi_{vac} = v e^{i\alpha(x)/\sqrt{2}}) = \frac{v^2}{2} (\partial_\mu \alpha + \sqrt{2} e A_\mu)^2$$

$\hookrightarrow V \equiv 0$

apply gauge transfm with $\chi = -\alpha / e\sqrt{2}$

$\rightsquigarrow \alpha(x)$ removed (unphysical)

– choose a gauge where $\phi(x)$ is real

$$\phi(x) = e^{i\alpha(x)/\sqrt{2}}\left(v + H(x)/\sqrt{2}\right) \xrightarrow[\alpha=0]{\text{gauge}} v + H(x)/\sqrt{2} \in \mathbb{R}$$

radial fluctuations

$$\rightsquigarrow \mathcal{L}\left(v + H(x)/\sqrt{2}\right) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + e^2v^2 A_\mu A^\mu + \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - \frac{\lambda v^2}{2}H^2$$

$$+ \mathcal{O}(H^3, H^4) \text{ interactions}$$

from $|D_\mu \phi|^2 \to |ieA_\mu v|^2 \to \boxed{\text{photon mass } m_A^2 = 2e^2v^2}$

$\boxed{H \text{ mass } m_H^2 = \lambda v^2}$

H is called the "Higgs boson" (gauge-invariant part of ϕ)

gauge invariance lost? NO! need $\mathcal{O}(H^3, H^4)$ interactions and must "ungauge" (put α back) to see full gauge invariance ✓

– count degrees of freedom (#):

without sym. breaking ($\lambda = 0$): $\phi \in \mathbb{C}$, A_μ massless $\rightsquigarrow$ # $= 2 + 2$

with symmetry breaking ($\lambda \neq 0$): $H \in \mathbb{R}$, A_μ massive $\rightsquigarrow$ # $= 1 + 3$

$\rightsquigarrow$ gauge field A_μ has swallowed the Goldstone boson $\alpha \to$ massive

– Higgs effect was discovered by

- Ginzburg & Landau $\to$ superconductivity (nonrelativistic)
- Higgs • Brout & Englert • Guralnik, Hagen, Kibble ~ 1964

Higgs observed 2012
Nobel prize 2013

- Non-Abelian case

- $\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\left\{\hat{G}_{\mu\nu}\hat{G}^{\mu\nu}\right\} + (D^{\mu}\phi)^{*}(D_{\mu}\phi) - \frac{\lambda}{4}(\phi^{*}\phi - v^{2})^{2}$

  where $\phi = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \in \mathbb{C}^2$ is an $SU(2)$ doublet $(i=1,2)$

  $$D_{\mu}\phi = (\partial_{\mu} + g\hat{W}_{\mu})\phi \quad , \qquad \phi^{*}\phi \equiv \phi^{*i}\phi_i$$

  and $\hat{W}_{\mu} = W_{\mu}^{a}t_a$ is $SU(2)$ gauge field with fieldstrength

  $$\hat{G}_{\mu\nu} = \frac{1}{g}[D_{\mu}, D_{\nu}] = \partial_{\mu}\hat{W}_{\nu} - \partial_{\nu}\hat{W}_{\mu} + g[\hat{W}_{\mu}, \hat{W}_{\nu}]$$

- scalar field acquires a VEV $\rightsquigarrow \mathcal{M}_{vac} = S^3$

  $\langle\phi\rangle = U\begin{pmatrix}0\\ v\end{pmatrix}$ with $U \in SU(2)$ breaks $SU(2) \to \mathbb{1}$

  $\phi \in \mathbb{C}^2 \simeq \mathbb{R}^4$, $\langle\phi\rangle$ selects a direction, $SU(2) \simeq S^3$

  Goldstone's theorem would predict 3 massless bosons

  but gauge redundancy identifies all of $S^3 \rightsquigarrow$ choose gauge $U = \mathbb{1}$

  $|D_{\mu}\phi|^2 \longrightarrow (0\ v)\, g^2 \hat{W}_{\mu}^{\dagger}\hat{W}^{\mu}\begin{pmatrix}0\\ v\end{pmatrix} \underset{t_a = -\frac{i}{2}\sigma_a}{\overset{\uparrow}{=}} \frac{1}{4}g^2 v^2 W_{\mu}^{a}W^{\mu a} \rightsquigarrow m_W^2 = \frac{g^2 v^2}{2}$

  "unitary gauge" $\phi(x) = \begin{pmatrix}0\\ v + H(x)/\sqrt{2}\end{pmatrix} \longrightarrow m_H^2 = \lambda v^2$

- counting degrees of freedom:

unbroken: $(3\times 2)_W + 4_\phi = 10$

broken: $(3\times 3)_W + 1_\phi = 10$ ✓

Standard Model: gauge & Higgs sectors

need four gauge bosons: $W^\pm, Z^0, \gamma$ is not A_μ

- introduce $SU(2)$ gauge fields $\hat{W}_\mu$ plus $U(1)$ gauge field B_μ

$$\mathcal{L}^{SM}_{bos} = -\frac{1}{2}\operatorname{tr}\{\hat{G}_{\mu\nu}\hat{G}^{\mu\nu}\} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - \frac{\lambda}{4}(\phi^*\phi - v^2)^2$$

$\phi \in \mathbb{C}^2$ $SU(2)$ doublet, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $\hat{G}_{\mu\nu} = \partial_\mu \hat{W}_\nu - \partial_\nu \hat{W}_\mu + g[\hat{W}_\mu, \hat{W}_\nu]$

- new: ϕ is also charged w.r.t. $U(1)$ $\rightsquigarrow$ couples to B_μ in cov. der.:

$$D_\mu\phi = \left(\partial_\mu + g\hat{W}_\mu + \frac{i}{2}g' B_\mu\right)\phi \rightsquigarrow \text{independent charges} \begin{cases} g & SU(2) \\ g'/2 & U(1) \end{cases}$$

- potential minimum $\phi^*\phi = v^2 \rightsquigarrow$ scalar VEV $\langle\phi\rangle = U\begin{pmatrix}0\\v\end{pmatrix}$

choose "unitary gauge": $\langle\phi\rangle = v\begin{pmatrix}0\\1\end{pmatrix}$ & substitute $\phi = \begin{pmatrix}0\\v + H/2\end{pmatrix}$

$U = \mathbb{1}$

$$|D_\mu\phi|^2 = \phi^\dagger \overleftarrow{D}_\mu^\dagger D^\mu \phi \xrightarrow[H=0]{\text{vacuum:}} (0\ v)\left(g\hat{W}_\mu^\dagger - \tfrac{i}{2}g'B_\mu\right)\left(g\hat{W}^\mu + \tfrac{i}{2}g'B^\mu\right)\begin{pmatrix}0\\v\end{pmatrix}$$

$$= (0\ v)\left(g\left(+\tfrac{i}{2}\sigma_a\right)W_\mu^a - \tfrac{i}{2}g'\mathbb{1}B_\mu\right)\left(g\left(-\tfrac{i}{2}\sigma_b\right)W^{\mu b} + \tfrac{i}{2}g'\mathbb{1}B^\mu\right)\begin{pmatrix}0\\v\end{pmatrix}$$

$\sigma_a\sigma_b = \delta_{ab}\mathbb{1} + i\varepsilon_{abc}\sigma_c$

$$= v^2\frac{g^2}{4}\left((W_\mu^1)^2 + (W_\mu^2)^2\right) + v^2\frac{1}{4}\left(gW_\mu^3 + g'B_\mu\right)^2$$

mass term for $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2)$

$$\boxed{m_W^2 = \frac{1}{2}v^2g^2}$$

$\frac{1}{2}m_Z^2 Z_\mu Z^\mu$

mass term for combination

$$Z_\mu = \frac{gW_\mu^3 + g'B_\mu}{\sqrt{g^2+g'^2}}$$

$$\boxed{m_Z^2 = \frac{1}{2}v^2(g^2+g'^2)}$$

- orthogonal combination

$$A_\mu = \frac{-g'W_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}}$$ stays massless $\rightsquigarrow$ photon!

- remaining part of ϕ is real Higgs field H with $\boxed{m_H^2 = \lambda v^2}$

- Symmetry breaking (without gauge fields):

$U(2) = SU(2)\times U_Y(1) \longrightarrow U_{em}(1)$, VEV $\langle\phi\rangle$ invariant

"weak hypercharge"

under $\phi \mapsto e^{i\alpha(1+\sigma_3)}\phi : U_{em}(1)$

but 3 of 4 gauge fields eat 3 would-be Goldstone bosons

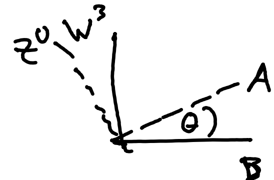
- counting degrees of freedom:

unbroken: $(3\times 2)_{W^a} + 2_B + 4_\phi = 12$

broken: $(3\times 3)_{W^\pm, Z} + 2_A + 1_H = 12$

- $\dfrac{g}{\sqrt{g^2+g'^2}} = \cos\theta_W$, $\dfrac{g'}{\sqrt{g^2+g'^2}} = \sin\theta_W \rightarrow g'/g = \tan\theta_W$

θ_W is called Weinberg angle, rotates basis $\begin{pmatrix} W^3 \\ B \end{pmatrix}$ to basis $\begin{pmatrix} Z^0 \\ A \end{pmatrix}$



- gauge boson masses can be expressed as

$$m_W = \frac{gv}{\sqrt{2}}, \quad m_Z = \frac{gv}{\sqrt{2}\cos\theta_W} = \frac{m_W}{\cos\theta_W}, \quad m_H = \sqrt{\lambda}\, v$$

- tomorrow's exercise:

fermion gauge interactions

$\mathcal{L} \sim i\bar{\psi}\gamma^\mu D_\mu \psi$

what is D_μ for any particular $\psi \in \{e, \nu, u, d, \dots\}$

know: interaction with $W^\pm_\mu$, Z_μ should be L/R-asymmetric

Weinberg & Salam in 1967/68 found right solution:

- left-handed fermion couple to $\hat{W}_\mu$ & B_μ

$\hat{W}_\mu \to$ SU(2) doublet

$B_\mu \to$ carry $U_Y(1)$ charge Y_L

- right handed fermions couple to $\hat{W}_\mu$ & B_μ

$\hat{W}_\mu \to$ they do <u>not</u> couple! $\Leftrightarrow$ SU(2) singlet

$B_\mu \to$ carry another charge Y_R

- look at 1st generation ($\nu = \nu_e$): $\begin{pmatrix}\nu\\e\end{pmatrix}_L$, ν_R, e_R; $\begin{pmatrix}u\\d\end{pmatrix}_L$, u_R, d_R

$$\left.\begin{aligned} \rightsquigarrow D_\mu \begin{pmatrix}\nu\\e\end{pmatrix}_L &= \left(\partial_\mu + g\hat{W}_\mu + \frac{i}{2}g' Y_L B_\mu\right)\begin{pmatrix}\nu\\e\end{pmatrix}_L \\ D_\mu e_R &= \left(\partial_\mu + \frac{i}{2}g' Y_R B_\mu\right) e_R \end{aligned}\right\} \Rightarrow \text{table of values } \{Y_L, Y_R\}$$