4th lecture

A BIRD'S EYE VIEW OF THE STANDARD MODEL

Quantum mechanics	1925-26
Standard Model	1974-75

Trees

- have discussed relativistic quantum mechanics and its "improvement" by QFT (Pauli-Weisskopf)
- we need to go beyond free (= non-interacting) theories ⇝ try to treat interaction in QFT using perturbation theory...

· first problem: atomic electromagnetic transitions

$A^* \longrightarrow A + \gamma$

also treated in QM textbooks, because nonrelativistic QM can be applied here:

$H_{int} = e\int d^3r \; \vec{j}(\vec{r})\cdot\vec{A}(\vec{r},t)$, with $k=(\omega,\vec{k})$, $\omega^2-\vec{k}^2=0$

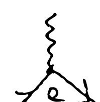
$$\vec{A}(\vec{r},t) \sim \sum_{\vec{k},\mu}\left(a(\vec{k},\mu)\,\vec{\varepsilon}(\vec{k},\mu)\,e^{ik\cdot x} + a^+(\vec{k},\mu)\,\vec{\varepsilon}^{\,*}(\vec{k},\mu)\,e^{-ik\cdot x}\right)_{k^2=0}$$

between $|in\rangle \sim |A^*\rangle\otimes|0\rangle$ and $|out\rangle \sim |A\rangle\otimes|\vec{p},\lambda\rangle$

⇝ matrix element $\langle A|e\vec{j}(\vec{r})|A^*\rangle\cdot\langle\vec{p},\lambda|\vec{A}(\vec{r},t)|0\rangle$

(first for hydrogen by Gordon 1929) $\sim e\frac{i\hbar}{2m}\langle A|\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi|A^*\rangle\cdot\vec{\varepsilon}^{\,*}(\vec{p},\lambda)\,e^{-ip\cdot x}$

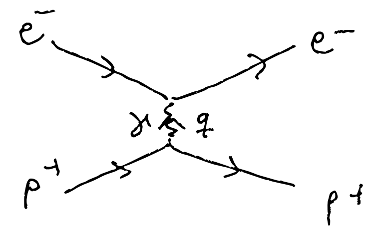
relativistic treatment: $H_{int}\longrightarrow V_{int} = e\,j^\mu A_\mu$, $j^\mu=(\rho,\vec{j})$, $A_\mu=(\varphi,\vec{A})$

for electrons ($e j^\mu$)  and for protons ($|e| j^\mu$) 

• Scattering processes (via two interactions)

– Mott scattering $e^-p^+ \to e^-p^+$

(Mott 1929 for $E_e \ll m_p \leadsto$
can replace j_p by $j_p^0 \sim V_p$ electrost. field)



Feynman diagram encodes an analytic expression:

$\longrightarrow$ external state with incoming or outgoing momentum, polaris., ...

vertex with coefficient e or $|e|$ $\leadsto$ order in perturb. thy.

$\leadsto$ virtual photons = electromagn. field with $q^2 \neq 0 \Leftrightarrow \omega^2 \neq \vec{q}^{\,2}$

nonrelativistic limit: p^+ at rest, v_e small $\leadsto$
scattering off external Coulomb potential

scattering amplitude $f(\vec{q})$ for momentum transfer $\vec{q} = \vec{p}_e^{\,\prime} - \vec{p}_e$

to lowest order in e: $f(\vec{q}) = \dfrac{2m\alpha}{\vec{q}^{\,2}}$ $\quad (\alpha = e^2/4\pi,\ m = m_e)$

differential cross section: $\dfrac{d\sigma}{d\Omega} = |f(\vec{q})|^2 = \dfrac{4m^2\alpha^2}{|\vec{q}^{\,2}|^2}$ Rutherford formula ($\alpha \to Z\alpha$)

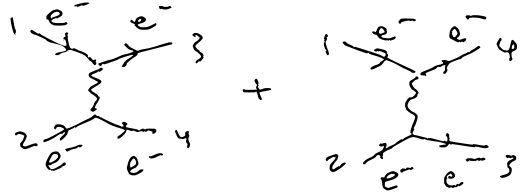
can be done even classically:

$f(\vec{q}) = -\dfrac{m}{2\pi}\int d^3r\, e^{-i\vec{q}\cdot\vec{r}}\, \dfrac{-\alpha}{r} = \dfrac{m}{2\pi}\, \dfrac{4\pi\alpha}{\vec{q}^{\,2}}$ ✓

relativistic generalization:

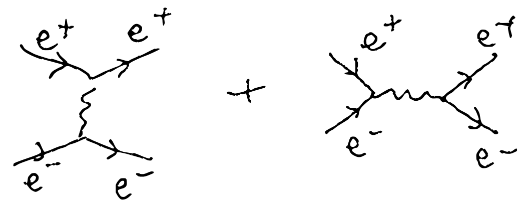
$\vec{q} \to -q^2 = \vec{q}^{\,2} - q_0^2$, additional electron polarization factors via computation of the Feynman diagram

– Møller scattering
$e^-e^- \to e^-e^-$



· mid-30s when $|\vec{p}_i| \sim m_i$

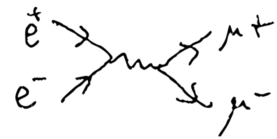
– Bhabha scattering
$e^+e^- \to e^+e^-$



· more technical but analogous

· two diagrams → interference

– $e^+e^- \to \mu^+\mu^-$



cross section for (initial & final) energies >> masses is simple:

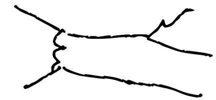
$$d\sigma = \frac{\alpha^2}{16E^2}\left(1+\cos^2\theta\right)d\Omega$$

$2E$ = center-of-mass energy
$\theta = \measuredangle\,(e^+e^-\text{ axis}, \mu^+\mu^-\text{ axis})$

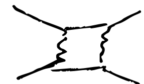
is simpler than Møller or Bhabha scattering
but was computed only in 1955 by Berestetsky & Pomeranchuk
will serve as a model for $e^+e^- \to$ new particles later

Loops, Feynman technique, divergences & renormalization

- so far, only tree diagrams



- higher orders may contain loops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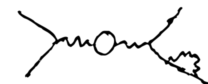
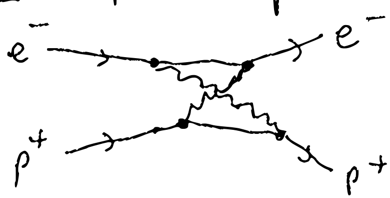


  ↳ "loop corrections"

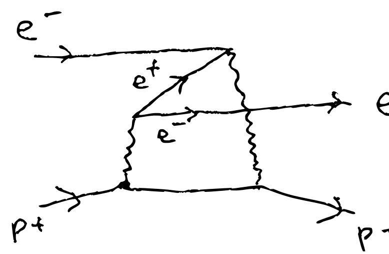
- before Feynman: "old-fashioned" perturbation theory (nonrelativistic quantum mechanics adopted to relativ. particles)

<u>ex</u> $ep \to ep$

3-mom. conserv. at vertex



$+$



$+ \ldots$

all lines are "on-shell"

- Feynman technique: only <u>one</u> diagram (at this order)

4-mom. conserv. at vertex

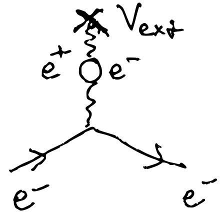
only topology matters

price to pay: internal lines off-shell → "virtual particles", e.g. γ in $e^+e^- \to \mu^+ e^-$ in CMS has $\vec{q} = 0$ but $q_0 = 2E$

Feynman diagrams provide enormous technical simplification!

- more serious problem: some loops diverge!

Consider one-loop correction to electron scattered off V_{ext}



$\sim \int_0^\infty \frac{dp^2}{p^2+m^2}$ is log divergent $\sim \ln \frac{\Lambda}{m}$ with cutoff $p^2 < \Lambda^2$

can be absorbed into a modification of $V_{ext} = \frac{e_0^2}{4\pi r}$ to

$$\frac{e_1^2}{4\pi r} = \frac{e_0^2}{4\pi r}\left(1 - \frac{e_0^2}{6\pi^2} \ln \frac{\Lambda}{m}\right) + \text{finite terms}, \quad e_0 = \text{"bare charge"}$$

$\longrightarrow$ screening

sign makes QED inconsistent at very high energies:

$e_1^2 \approx 0$ for $\frac{e_0^2}{6\pi^2} \ln \frac{\Lambda_*}{m} \approx 1 \iff \Lambda_* = m\, e^{3\pi/2\alpha_0} \approx 10^{250}\, g$ (universe $\sim 10^{57} g$)

observed is "renormalized charge" (physical charge)

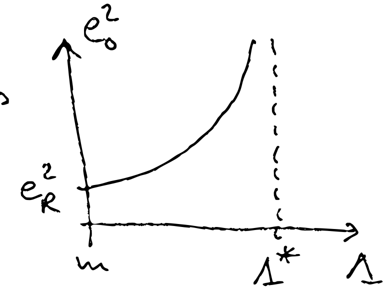
$$e_R^2 \approx e_0^2 (1 + \sim\!O\!\sim + \sim\!O\!\sim\!O\!\sim + \ldots) = \frac{e_0^2}{\left(1 + \frac{e_0^2}{6\pi^2} \ln \frac{\Lambda}{m}\right)}$$

infinite subclass of diagrams — geom. series

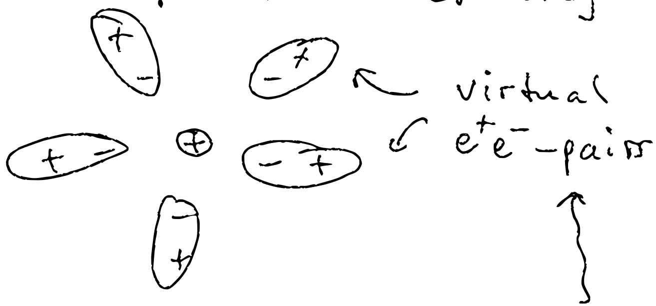
- leads to so-called "Landau pole": reverse to

$$e_0^2 \approx \frac{e_R^2}{1 - \frac{e_R^2}{6\pi^2} \ln \frac{\Lambda}{m}}$$

keep e_R^2 fix $\rightsquigarrow$



interpretation: screening



moving into the "cloud": $m^{-1} \sim$ Compton wavelength

$$\left.\begin{array}{ll} r \gtrsim m^{-1} & \text{see } \ln \frac{\Lambda}{m} \\ r \lesssim m^{-1} & \text{see } \ln \Lambda r \end{array}\right\} \Rightarrow e^2_{phys}(r)$$

despite the inconsistency (Landau pole), in QED can absorb all singular loop diagrams by renormalizing the charge ($e_0 \to e_R$) and electron mass ($m_0 \to m_R$).

From nuclei to gluon jets

history of attempts to apply QFT to strong interactions

- Yukawa theory 1935:

nuclear interactions mediated by mesons (like QED by γ)

- meson-nucleon coupling $g \gg e$ (strong interaction $\rightarrow$ large coupling)
- in nucleus no antinucleons $\rightsquigarrow$ purely attractive interaction
  $\rightsquigarrow$ mediating field quanta have even spin $\rightsquigarrow$ spin 0 simplest
- short range ~ 1 fm $\rightsquigarrow$ mesons have mass $\mu \approx (1\text{ fm})^{-1} \approx 200$ MeV

heuristic argument in favor of the last point (detail later...)

QED: $f_{e^+e^-}(\vec{q}) \sim \tilde{V}_{\text{coulomb}}(\vec{q}) \sim \frac{1}{\vec{q}^{\,2}} \overset{\text{relativ.}}{\rightsquigarrow} \frac{-1}{q^2} = \frac{1}{\vec{q}^{\,2} - q_0^2}$ scattering amplitude

$\rightsquigarrow (-q_0^2 + \vec{q}^{\,2}) \cdot f(q) = \text{const.} \overset{\text{F.T.}}{\rightsquigarrow} (\partial_t^2 - \Delta)\tilde{f}(x) \equiv \Box \tilde{f}(x) \sim \delta(x)$

QCD: add mass term $\rightsquigarrow$ KFG eq.: $(\partial_t^2 - \Delta + \mu^2)\tilde{f}(x) \sim \delta(x)$

$\rightsquigarrow (-q_0^2 + \vec{q}^{\,2} + \mu^2) f(q) = \text{const.} \overset{\text{nonrel.}}{\rightsquigarrow} f(\vec{q}) \sim \frac{1}{\vec{q}^{\,2} + \mu^2} \overset{\text{F.T.}}{\rightsquigarrow} -\frac{g}{4\pi r} e^{-\mu r}$

- confirmation in 1947 with discovery of π meson ✓

but: – many more mesons & baryons found ⇝ their role?

– coupling strength $g^2/4\pi \simeq 14 \simeq 2000\times\left(e^2/4\pi = \alpha\right)$

perturb. thy. is unreliable | Landau pole for $\Lambda_* \simeq m_p\, e^{\frac{3\pi}{2\cdot 14}} \simeq 1\,\text{GeV}$ (!!!)

- many theorists abandoned QFT for fund. interactions

mid-50s till mid-60s: other attempts (bootstrap)

a growing zoo of "elementary" particles, but...

- a "Mendeleev table" for hadrons in 60s by Gell-Mann:

→ all grouped into multiplets (=representations) of SU(3)

main message: understandable if hadrons = quark bound states (later...)

new constituent

at the time, $\underline{3}$ quarks (u,d,s) sufficed, hence SU($\underline{3}$) "flavor"

- terminology: mass hierarchy

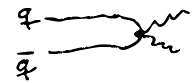
leptons ("light") $\ll$ mesons ("intermediate") $\lesssim$ baryons ("heavy")

$e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau \xleftarrow{\text{decay}} \pi, K, \rho, K^*, D, \ldots \longleftrightarrow p, n, \Lambda, \Sigma, \ldots$

- constituent quark model:

baryons $= qqq$ (antibaryons $= \bar{q}\bar{q}\bar{q}$), mesons $= q\bar{q}$

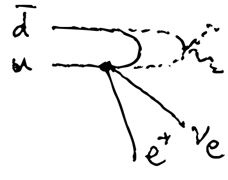
e.g. $|\pi^0\rangle = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right)$



$\pi^0 \to 2\gamma$

via $q\bar{q} \to 2\gamma$

e.g. $|\pi^+\rangle = |u\bar{d}\rangle$



$\pi^+ \to e^+\nu_e$ (or $e^+\nu_e\pi^0$)
$\hookrightarrow 2\gamma$

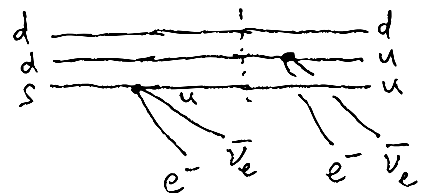
via $u \to d e^+ \nu_e$

semileptonic weak

e.g. $|\Sigma^-\rangle = |dds\rangle$

$|n\rangle = |ddu\rangle$

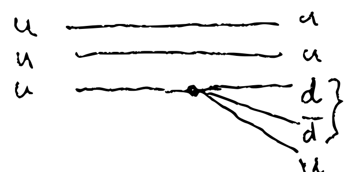
$|p\rangle = |duu\rangle$



$\Sigma^- \to e^-\bar{\nu}_e n$
$\hookrightarrow p e^-\bar{\nu}_e$

via $\left.\begin{matrix} s \\ d \end{matrix}\right\} \to u e^- \bar{\nu}_e$

e.g $|\Delta^{++}\rangle = |uuu\rangle$



$\}\ p$

$\Delta^{++} \to p\pi^+$

hadronic strong

$\}\ \pi^+$ via $u \to u d \bar{d}$

long lifetimes of weakly decaying hadrons appeared "strange"

$\to$ quarks carry fractional electric charges:

$q_u = \frac{2}{3}|e|$, $q_d = -\frac{1}{3}|e| = q_s$

first confirmation: detection of Ω^- particle

$$\begin{array}{cccc} \Delta^- & \Delta^0 & \Delta^+ & \Delta^{++} \\ & \Sigma^- & \Sigma^0 & \Sigma^+ \\ & \Xi^- & \Xi^0 & \\ & & \Omega^- \leftarrow & \end{array} \qquad \begin{array}{cccc} uuu & uud & udd & ddd \\ uus & uds & dds & \\ uss & dss & & \\ sss & & & \end{array}$$

... but quarks were not found in nature ...

additional problem with Pauli principle (q are fermions)

$\Delta^{++} = uuu$ has spin $= 3/2$, but $|uuu\rangle \otimes |\uparrow\uparrow\uparrow\rangle$ — $S_3 = +3/2$ state, ground state $l = 0$

cannot exist due to permutation symmetry

- solution: a new degree of freedom called "color", each quark flavor (u, d, s) comes in 3 colors (r, g, b)

$$\leadsto |\Delta^{++}\rangle_{+3/2} = |uuu\rangle \otimes |\uparrow\uparrow\uparrow\rangle \otimes |[rgb]\rangle$$

totally antisym.: $\begin{vmatrix} 1r & 2r & 3r \\ 1g & 2g & 3g \\ 1b & 2b & 3b \end{vmatrix}$

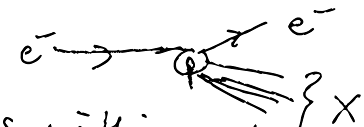
- postulate: only colorless combinations of quarks exist in nature, quarks are "confined" inside hadrons

many found this crazy $\rightarrow$ disbelief in quarks

- changed in 1969

deep inelastic scattering $ep \to ep$ at SLAC

inclusive cross section



ignore

showed pointlike constituents within protons

dubbed "parton" ($\Leftarrow$ Rutherford)

partons have same quantum numbers as quarks

$\Longrightarrow$ they <u>are</u> quarks!

"interago ergo sum"

but what about confinement & field theory?