5th Lecture

From nuclei to gluon jets (cont'd)

recall:

the constituent quark model with 3 flavors (u, d, s) & 3 colors (r, b, g) explains...

- the hadron zoo via qqq ($\bar{q}\bar{q}\bar{q}$) & $q\bar{q}$ bound states
- the long lifetimes of weakly decaying particles
- the parton substructure of baryons ("Bjorken scaling")

... but the confinement postulate remains unclear

- What about field theory? Useless? (Landau pole problem)
  - some theorists did not abandon it
  - the correct field theoretic description had already been invented in 1954 by Yang & Mills

- Yang-Mills theory (in detail later...)
  - a multi-photon generalization of electrodynamics
  - its "photons" are not neutral but "YM charged"
  - therefore elementary interactions between them
  - was unsuccessfully applied to ρ mesons
  - its massless gauge bosons prevented application to strong or weak interactions (short-ranged → mass)
  - Princeton talk by Yang & Pauli's reaction
  - quantization difficult, led to inconsistencies
  - Feynman tried (early 60s) YM as a model for quantum gravity (gauge bosons ⟺ gravitons)
  - Feynman's insights triggered "ghost method" of Faddeev & Popov (1967) ⟶ perturbation theory became consistent

- Yang-Mills theory applied to strong interactions
  turning point in 1973:

  × Pati/Salam, Fritzsch/Gell-Mann/Leutwyler, Weinberg
  → colored quarks exchange colored gluons "g"
  → gluons are the charged "photons" of an SU(3) version of YM
  → 8 gluons in SU(3) adjoint rep., quarks in SU(3) fund. rep. ← (r, g, b)

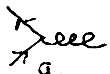
  (original YM was SU(2) → 3 "photons", 2 "colors")

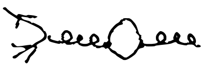
  breakthrough in 1974:

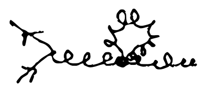
  × Gross/Wilczek, Politzer (see hep-th/9808154)
  → computed one-loop renormalization of YM coupling
  → color charge is <u>anti</u>-screened, <u>grows</u> with distance!
  → effective interaction weakens with $r \searrow \iff E \nearrow$
  → "asymptotic freedom": quarks are quasi-free inside hadrons
  → "infrared slavery": effective string $q \sim \bar{q}$ ↔ → $q \sim \bar{q}$ $q \sim \bar{q}$ confinement!

- asymptotic freedom settled the picture
  - explained the quasi-free partons of SLAC experiment
  - enabled perturbation theory at high energies
  - made confinement plausible (not proved until today)
  - made YM self-consistent:

which asymptotic states
millenium problem of Clay Mathem. Institute
→ win $1 million



+  + geometric series

+  +  + geom. series

recall QED: $e_0^2 \approx \dfrac{e_R^2}{1 - \frac{e_R^2}{6\pi^2} \ln \frac{\Lambda}{m}}$

Λ ← cutoff

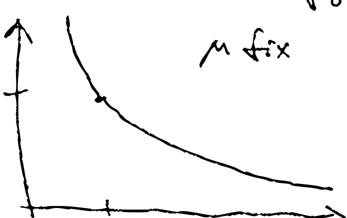
m ← electron mass

replace m → energy scale μ
no scale in massless QCD

G/W & P result: $g_0^2 \approx \dfrac{g^2(\mu)}{1 + b \frac{g^2(\mu)}{6\pi^2} \ln \frac{\Lambda}{\mu}}$ with $b = \frac{33}{2} - N_f > 0$

$\frac{33}{2}$ ↑ gluon contribution!

N_f ↑ # q flavors (6)



how to take $\Lambda \to \infty$?

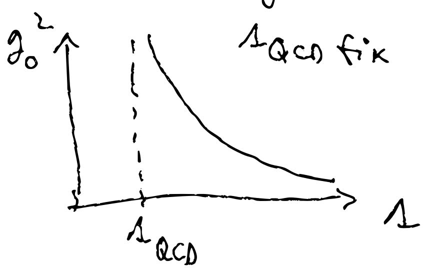
$$g_0^2 \approx \frac{g^2(\mu)}{1 + b\,\frac{g^2(\mu)}{6\pi^2}\ln\frac{\Lambda}{\mu}}$$

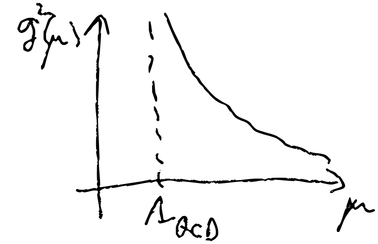
$$\approx \frac{6\pi^2}{\frac{6\pi^2}{g^2(\mu)} + b\ln\frac{\Lambda}{\mu}} = \frac{6\pi^2}{b\ln\frac{\Lambda}{\mu e^{-\frac{6\pi^2}{b g^2(\mu)}}}} =: \frac{6\pi^2}{b\ln\frac{\Lambda}{\Lambda_{QCD}}}$$

with $\Lambda_{QCD} = \mu\, e^{-\frac{6\pi^2}{b\,g^2(\mu)}}$ $\Longleftrightarrow$ $g^2(\mu) = \frac{6\pi^2}{b\ln\frac{\mu}{\Lambda_{QCD}}}$

independent of μ!

$\rightsquigarrow$ $g^2(\Lambda) = g_0^2$





can take $\Lambda \to \infty$

- "dimensional transmutation"
  - no dimensionful parameter in original theory (put $m_q \to 0$)
  - effective dim'less coupling $g^2(\mu)$ develops energy dependence via renormalization
  - $g^2(\mu)$ becomes large at some value $\mu \approx \Lambda_{QCD}$
  - new energy scale "out of nothing", fundamental constant of QCD
  - replaces m_p (used previously), but of same size!
  - experimentally $\Lambda_{QCD} \approx 200$ MeV
  - higher-loop calculations slightly correct this value
  - experimental input: measure $g^2(\mu_0)$ at <u>one</u> scale μ_0
  - now can send $\Lambda \to \infty$ ($\sim g_0^2 \to 0$), comb. $\mu e^{-\frac{6\pi^2}{b\, g^2(\mu)}}$ fix
  - compare with experiment $\rightsquigarrow$ not the slightest doubt today that QCD is correct

- very impressive confirmation: hadron jets

$e^+e^- \longrightarrow$ lots of hadrons, bundled in "jets"

in two stages: first, fundamental process

$e^+e^- \to q\bar{q}$ or $q\bar{q}g$ or $q\bar{q}q\bar{q}$ or ...

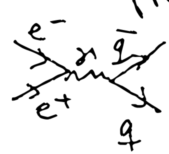
Second, "hadronization" (dressing to become colorless)

$q\bar{q} \to 2$ jets, $q\bar{q}g \to 3$ jets, $q\bar{q}q\bar{q} \to 4$ jets etc.

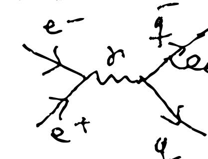
<u>ex</u>. 3 jets:

- cross sections calculable from QCD!

1^{st}, fundamental: e.g.



$\sim e^2$,



$\sim e g^2$ ← suppression

accurate in pert. thy.

2^{nd}, hadronization: less accurate, non-perturbative ...

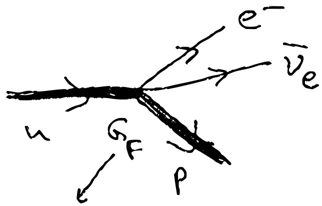
<u>ex</u>: 2-jet prediction: $e^+e^- \to q\bar{q}$ analogous to $e^+e^- \to \mu^+\mu^-$

$$d\sigma_{e^+e^- \to q\bar{q}} = \sum_q \frac{\alpha^2}{16 E^2} \cdot 3 Z_q^2 \left(1+\cos^2\theta\right) d\Omega$$

← angular distribution is seen ✓

From β decay to the Higgs boson

- Fermi's theory of weak interactions (1934)



Fermi's constant

$G_F \approx 1.2 \times 10^{-5} (\mathrm{GeV})^{-2}$

$[G_F] = E^{-2}$ or L^2

"current-current" interaction (4-fermion-interaction)

$$V_{int} = e\, j^\mu A_\mu \longrightarrow G_F\, j^\mu J_\mu$$

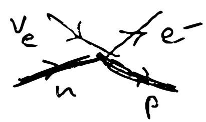
j^μ = leptonic current $(e\, \nu_e)$ } → later

J_μ = baryonic current $(n\, p)$

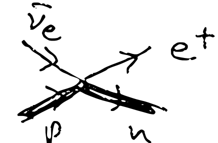
⇝ gave accurate predictions at low energies

⇝ predicts also "crossed channels"

$\nu_e n \longrightarrow e^- p$



, $\bar{\nu}_e p \longrightarrow e^+ n$



total cross section (neutrino-nucleon) $\sigma^{tot}_{\nu N} \sim G_F^2 E_\nu^2$ for $E_\nu < m_N$ [dimensional analysis]

typically $E_\nu \sim 1$ MeV (reactors, sun) ⇝ mean free path for ν in matter ≈ 100 lys

neutrino scattering first observed by Cowan/Reines 1956

ν source = nuclear reactor, $5 \cdot 10^{13}\, \nu/s$ ⇝ 3 ν interactions/hr

- parity violation & V-A theory

– by 1956 exp. had shown P violation in weak decays

ex. of parity-odd scalar: helicity $h = \frac{\vec{S}\cdot\vec{p}}{|\vec{p}|}$

– experiments revealed: $h_\nu = -\frac{1}{2}$, $h_{\bar\nu} = +\frac{1}{2}$ [Lee & Yang, Wu]

⤳ "neutrinos are left-handed"

⤳ left-right asymmetry at a fundamental level!

[side remark:
combination CP maps left-handed ν ⟶ right-handed $\bar\nu$
is it a fundamental symmetry? not quite!
its established violation requires 3 quark generation]

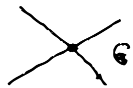
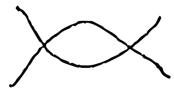
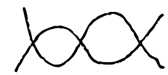
– Fermi's currents are vector-like (parity-odd)
to account for P violation the required modification is

$j^\mu \to j^\mu_V - j^\mu_A$, $J^\mu \to c_V J^\mu_V - c_A J^\mu_A$ | V = (polar) vector, A = axial vector

⤳ "V-A theory" 1957 by Marshak/Sudarshan, Feynman/Gell-Mann
explained well the polarisations & angular asymmetries

- resolution of non-renormalizability

  - the Fermi and V-A theory are non-renormalizable

    loops:  G_F ,  $\sim G_F^2 \Lambda^2$,  $\sim G_F^3 \Lambda^4$, ...

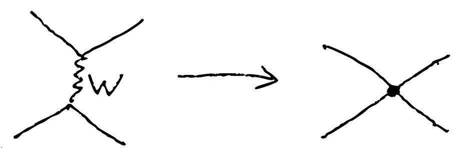
    ⇝ but only $\ln \Lambda$ divergences are curable

  - related: cross sections grow quadratically with energy E

    ⇝ not much beyond $E \sim G_F^{-1/2}$ they exceed unity

    ⇝ scattering probability $\gtrsim 1$ violates "unitarity"

  - way out: 4-fermion interaction is only "effective",

    a low-energy approximation to 

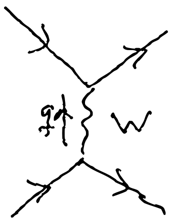
    via exchange of an "intermediate vector boson" (IVB) W_μ

    → split 

  - modelled after QED & QCD: Yang-Mills theory!

recall scattering amplitude for Yukawa theory $\sim \frac{1}{\mu^2 - q^2}$
"meson exchange"

analogous ~~for~~ W exchange & dim'less coupling g



$\sim \frac{g^2}{m_W^2 - q^2}$

$\xrightarrow{q^2 \ll m_W^2} g^2/m_W^2 \to G_F$

$\xrightarrow{q^2 \gg m_W^2} -g^2/q^2 \to$ suppression

$q^2 \ll m_W^2$: $\sigma \sim G_F^2 \cdot E^2 \sim g^4 E^2 / m_W^4$ $\qquad [g] = 1$ dimensionless

$q^2 \gg m_W^2$: $\sigma \sim \frac{g^4}{(q^2)^2} \cdot E^2 \sim g^4/E^2$ $\qquad$ does not grow

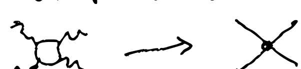
(using $q^2 \sim E^2$)

- general story:

fundamental $\underset{\text{integrate in}}{\overset{\text{integrate out}}{\rightleftarrows}}$ effective

<u>ex</u>: Born-Oppenheimer approximation (slow ↔ fast) (1927)

<u>ex</u>: Euler-Heisenberg vertex for light-light scattering (1936)



effective 4γ vertex, not yet observed

- the question of the W mass
  - YM is a gauge theory → gauge inv. forbids direct masses
  - still, a W mass can be "dynamically generated" by a special, gauge-invariant interaction with a particular scalar field (the Higgs!) to be invented!
  - not so exotic, already known in electrodynamics: vacuum dispersion $\omega^2 - c^2\vec{k}^2 = 0$ of EM waves modified in <u>media</u>:
    - × glass (refraction index n): $n^2\omega^2 - c^2\vec{k}^2 = 0$
    - × plasma: $\omega^2 - c^2\vec{k}^2 = \Pi(\omega, \vec{k})$, but $\Pi(\omega, \vec{k}=0) \neq 0$
      ⇝ $\omega^2 = \Pi(\omega, 0)$ solved by plasma frequency $\omega_{pe}^2 = \frac{ne^2}{m^2}$ of stationary oscillation ($\vec{k}=0$!) → effective photon mass!
    - × Meißner effect in superconductors
      ⇝ EM penetrates slightly into superconductor, with e^{-mr} fall-off
      ⇝ photon becomes massive in superconductor

- the Glashow – Salam – Weinberg (GSW) model
  1961 1968 1967

  - based of the non-Abelian Higgs model (1964)

  - a theory of weak <u>and</u> electromagnetic interactions because $W^{\pm}$ carries not only weak but also electric charge

  - prediction: massless γ, massive $W^{\pm}$, massive Z^0 (!) with mass values:

$$m_W = \frac{1}{\sin\theta_W}\sqrt{\frac{\pi\alpha}{G_F\sqrt{2}}} \approx \frac{37.3}{\sin\theta_W}\,\text{GeV} \approx 80.38\ \text{GeV}$$

exp. input $\sin^2\theta_W \approx 0.231$

$$m_Z = \frac{m_W}{\cos\theta_W} \approx \frac{74.6}{\sin 2\theta_W}\,\text{GeV} \approx 91.19\ \text{GeV}$$

} confirmed directly CERN 1983

and Weinberg angle θ_W to be determined experimentally

  - rough estimate already from $g^2/m_W^2 \approx G_F$ and $g \approx e$ → $m_{W,Z} \approx 100$ GeV

  - Z^0 brings about "neutral current" processes

e.g. $\nu p \to \nu p$ or $\to X$

not in Fermi theory
discovered at CERN 1973

final prediction: Higgs boson discovered at CERN 2012
$m_H \approx 125$ GeV