Tutorial 7 - Fundamental Interactions

Olaf Lechtenfeld, Gabriel Picanço

Jul 2021

1 Fixing Weak Hypercharges

In the Electroweak theory, we couple gauge bosons to fermions in a chiral fashion. Axial currents that are conserved in a classical level can acquire non-zero divergences (that is, not being conserved anymore) through one-loop corrections to the three-gauge-boson vertex function. Every fermion which couples to the gauge bosons involved will circulate in a triangle. Those are anomalous contributions and should somehow be equal to zero, otherwise we would lose gauge invariance. In principle, there are ten diagrams that can give us anomalous contributions:

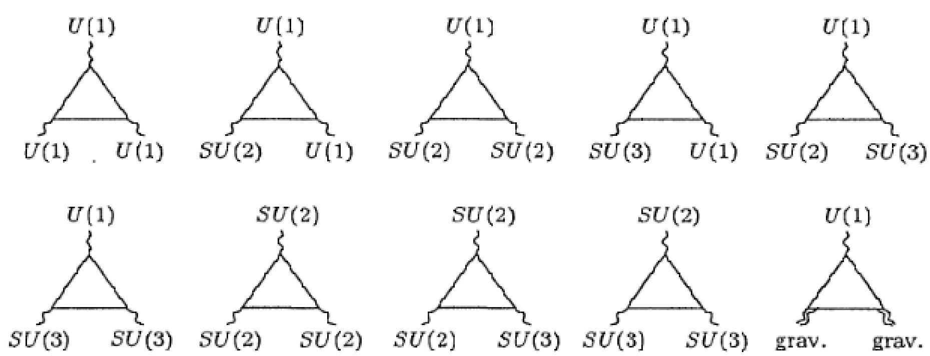

Fig. 1: EW diagrams contributing to gauge anomalies[1]

Each loop is proportional to a "generalized trace" that will represent a usual trace over the 3 matrix contributions of different representations but will also sum over all of the different particles that can generate each diagram. Remember that we are working with the gauge group SU(2)$\times$ U(1)$_Y\times$ SU(3)$_C$. In this analysis one can think about only one generation of quarks and leptons. Let us use the gauge anomalies cancellation to find the values of the weak hypercharges for the Standard Model.

[1]from M. E. Peskin, D. V. Schroeder; An introduction to Quantum Field Theory, Addison-Wesley Publishing Company, 1995

a) Argue why the diagrams with only one SU(2) or only one SU(3) boson vanish. Now argue why the diagram with three SU(2) bosons also vanishes.

The diagrams that are left are the ones with one U(1) gauge boson and two SU(2) or two SU(3) bosons, the one with three U(1) bosons and the gravitational anomaly with one U(1) boson.

b) Considering the gauge redundancies, in principle how many different hypercharges can we have in one generation of quarks and leptons? Imposing that the hypercharge of the right-handed neutrino is 0 to simplify from the beginning and knowing that the left-handed and right-handed fermions contribute to the anomalies equations with opposite sign, write equations for the 4 gauge anomalies that still need to be canceled in terms only of the hypercharges of each type.

Hint: don't worry about the numerical contribution of the matrix elements regarding SU(2), SU(3) or the graviton, just think about who can participate in any given interaction and their hypercharge values.

c) Is the system fully determined? If not, explain. Now reduce the system of equations. Assume that the hypercharge of the left-handed electron is non-zero (only by the equations it could be zero, but we would have a solution that is not realized in nature). Use the remaining freedom to set it to -1 and find the other hypercharge values. Check if they are the same as in Smilga's table 12.1.

2 Mass spectrum

Consider a complex scalar field with potential

$$V = -m^2(\phi^2 + (\phi^*)^2) + \lambda(\phi^*\phi)^2, \tag{1}$$

with m^2 and λ positive. Find the mass spectrum of the theory.

3 Optional: SU(5) Great Unification Theory

We've seen that it is possible to unify the electromagnetic and weak interactions in one description using the gauge group SU(2)$\times$ U$(1)_Y$. If we also want to describe QCD, we use the gauge group SU(2)$\times$ U$(1)_Y\times$ SU$(3)_C$, which corresponds to the Standard Model of Particle Physics. It could be possible to unify the Electroweak and Strong interactions in a single interaction (with a single coupling constant) for very high energy scales. Theories that try to do that are called Great Unification Theories (GUT's).

As SU(2)$\times$ U$(1)_Y\times$ SU$(3)_C$ has rank 4, any GUT has to be based on a semi-simple Lie group of rank at least 4. There is only one such group with rank 4 that is compatible with the strong and electroweak interactions, the group

SU(5) (we could also take groups with higher ranks, for example SO(10) which has rank 5).

a) How many gauge bosons are there in the SU(5) theory? How many of those need to correspond to the known gauge bosons for the Standard Model?

b) The rest of the (in principle) massless gauge bosons are new and combine to produce interactions that are not present in the Standard Model, as quark-quark and quark-lepton annihilation (X and Y bosons). Those processes violate baryonic number, so they are a problem to the theory, as we don't see baryonic number violation in experiments. What can we try to make the SU(5) gauge theory be consistent with the experiments known nowadays? Here you don't need to compute anything, just describe what change one could make and why it could fix the inconsistencies.

c) In the great unification scenario there is also an issue regarding the coupling constants of the interactions of the Standard Model. Explain it and relate it to the answer you gave in the previous item.