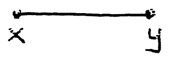
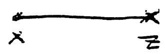
Quantum Field Theory: Exercise Session 3

Lecturer: Olaf Lechtenfeld

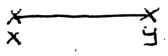
The Feynman rules for ϕ^4 theory read:



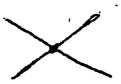
$$= i\Delta_F(x-y)$$



$$= -\int d^4z\, \Delta_F(x-z)\, J(z)$$



$$= -i\int d^4x\, d^4y\, J(x)\, \Delta_F(x-y)\, J(y)$$



$$= \left(-i\frac{\lambda}{4!}\right)\int d^4z$$

where $\Delta_F(x-z)$ is the Feynman propagator and $J(x)$ is a source for $\phi(x)$.

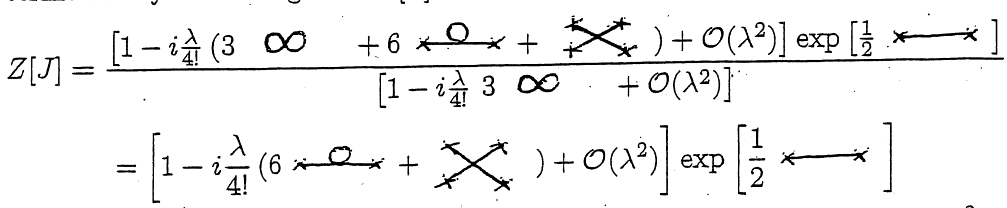
The generating functional is given by:

$$Z[J] = \frac{\exp\left[\left(-i\frac{\lambda}{4!}\right)\int d^4z\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^4\right]\exp\left[-\frac{i}{2}\int d^4x\, d^4y\, J(x)\, \Delta_F(x-y)\, J(y)\right]}{\left\{\exp\left[\left(-i\frac{\lambda}{4!}\right)\int d^4z\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^4\right]\exp\left[-\frac{i}{2}\int d^4x\, d^4y\, J(x)\, \Delta_F(x-y)\, J(y)\right]\right\}_{J=0}}$$

and the n-point function is given by

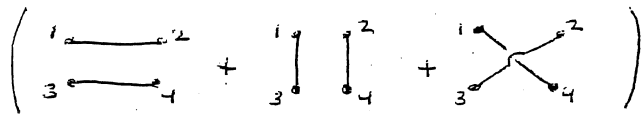
$$G_n(x_1, x_2, ..., x_n) = i^{-n}\left[\frac{\delta^n Z[J]}{\delta J(x_1)\,\delta J(x_2)\ldots\delta J(x_n)}\right]_{J=0}$$

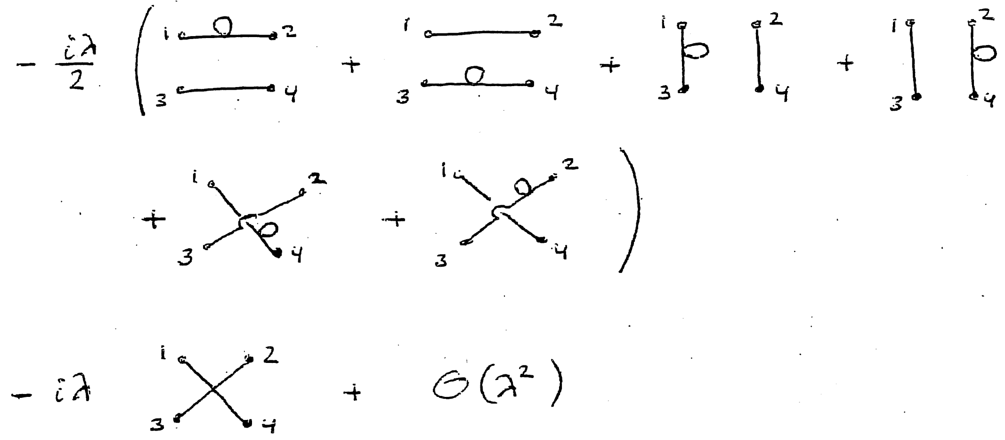
1. In terms of Feynman diagrams $Z[J]$ can be written as



Show, at least qualitatively, that the vacuum diagrams cancel also at order λ^2.

2. The four-point function reads

$G_4(x_1, x_2, x_3, x_4) =$ 



What diagrams appear in G_4 at order λ^2? Take the symmetry factors into account!

3. Show, at least to order λ, that the functional $W[J] = -i \ln Z[J]$ generates only the connected Feynman diagrams of $G_4(x_1, x_2, x_3, x_4)$,