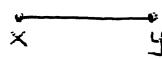


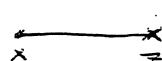
Quantum Field Theory: Exercise Session 3

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The Feynman rules for ϕ^4 theory read:



$$= i\Delta_F(x - y)$$



$$= - \int d^4z \Delta_F(x - z) J(z)$$



$$= -i \int d^4x d^4y J(x) \Delta_F(x - y) J(y)$$



$$= \left(-i \frac{\lambda}{4!} \right) \int d^4z$$

where $\Delta_F(x - z)$ is the Feynman propagator and $J(x)$ is a source for $\phi(x)$.

The generating functional is given by:

$$Z[J] = \frac{\exp \left[(-i \frac{\lambda}{4!}) \int d^4z \left(\frac{1}{i} \frac{\delta}{\delta J(z)} \right)^4 \right] \exp \left[-\frac{i}{2} \int d^4x d^4y J(x) \Delta_F(x - y) J(y) \right]}{\left\{ \exp \left[(-i \frac{\lambda}{4!}) \int d^4z \left(\frac{1}{i} \frac{\delta}{\delta J(z)} \right)^4 \right] \exp \left[-\frac{i}{2} \int d^4x d^4y J(x) \Delta_F(x - y) J(y) \right] \right\}_{J=0}}$$

and the n -point function is given by

$$G_n(x_1, x_2, \dots, x_n) = i^{-n} \left[\frac{\delta^n Z[J]}{\delta J(x_1) \delta J(x_2) \dots \delta J(x_n)} \right]_{J=0}$$

1. In terms of Feynman diagrams $Z[J]$ can be written as

$$\begin{aligned} Z[J] &= \frac{\left[1 - i \frac{\lambda}{4!} (3 \text{ } \infty \text{ } + 6 \text{ } \times \text{ } \circlearrowleft \text{ } + \text{ } \times \text{ } \times \text{ } + \text{ } \times \text{ } \times \text{ }) + \mathcal{O}(\lambda^2) \right] \exp \left[\frac{1}{2} \text{ } \times \text{ } \times \text{ } \right]}{\left[1 - i \frac{\lambda}{4!} 3 \text{ } \infty \text{ } + \mathcal{O}(\lambda^2) \right]} \\ &= \left[1 - i \frac{\lambda}{4!} (6 \text{ } \times \text{ } \circlearrowleft \text{ } + \text{ } \times \text{ } \times \text{ }) + \mathcal{O}(\lambda^2) \right] \exp \left[\frac{1}{2} \text{ } \times \text{ } \times \text{ } \right] \end{aligned}$$

Show, at least qualitatively, that the vacuum diagrams cancel also at order λ^2 .

2. The four-point function reads

$$G_4(x_1, x_2, x_3, x_4) = \left(\begin{array}{c} \text{Diagram 1: } 1 \xrightarrow{\text{---}} 2 \\ \text{Diagram 2: } 3 \xrightarrow{\text{---}} 4 \\ + \end{array} + \begin{array}{c} \text{Diagram 3: } 1 \downarrow \\ 3 \downarrow \\ 2 \downarrow \\ 4 \end{array} + \begin{array}{c} \text{Diagram 4: } 1 \nearrow \\ 3 \nearrow \\ 2 \nearrow \\ 4 \end{array} \right)$$

$$- \frac{i\lambda}{2} \left(\begin{array}{c} \text{Diagram 5: } 1 \xrightarrow{\text{---}} 2 \\ 3 \xrightarrow{\text{---}} 4 \\ + \end{array} + \begin{array}{c} \text{Diagram 6: } 1 \xrightarrow{\text{---}} 2 \\ 3 \xrightarrow{\text{---}} 4 \\ + \end{array} + \begin{array}{c} \text{Diagram 7: } 1 \downarrow \\ 3 \downarrow \\ 2 \downarrow \\ 4 \end{array} + \begin{array}{c} \text{Diagram 8: } 1 \downarrow \\ 3 \downarrow \\ 2 \downarrow \\ 4 \end{array} \right. \\ \left. + \begin{array}{c} \text{Diagram 9: } 1 \nearrow \\ 3 \nearrow \\ 2 \nearrow \\ 4 \end{array} + \begin{array}{c} \text{Diagram 10: } 1 \nearrow \\ 3 \nearrow \\ 2 \nearrow \\ 4 \end{array} \right) \\ - i\lambda \begin{array}{c} \text{Diagram 11: } 1 \times 2 \\ 3 \times 4 \end{array} + \mathcal{O}(\lambda^2)$$

What diagrams appear in G_4 at order λ^2 ? Take the symmetry factors into account!

3. Show, at least to order λ , that the functional $W[J] = -i \ln Z[J]$ generates only the connected Feynman diagrams of $G_4(x_1, x_2, x_3, x_4)$,