

13. Präsenzübung zur Quantentheorie II, SS 2007

(zu bearbeiten am Donnerstag, 12.07.2007)

Aufgabe P21 *Wicksches Theorem für skalares Feld*

Das quantisierte skalare Feld lautet

$$\phi(x) = \int d\tilde{k} \{ a(\vec{k})e^{-ik \cdot x} + a^\dagger(\vec{k})e^{ik \cdot x} \} =: \phi^+(x) + \phi^-(x).$$

Zeigen Sie explizit (mit der Notation $\phi(i) := \phi(x_i)$):

a)

$$\begin{aligned} \phi(1)\phi(2) &= : \phi(1)\phi(2) : + [\phi^+(1), \phi^-(2)] \\ T\phi(1)\phi(2) &= : \phi(1)\phi(2) : + \langle 0 | T\phi(1)\phi(2) | 0 \rangle \end{aligned}$$

b)

$$\begin{aligned} \phi(1)\phi(2)\phi(3) &= : \phi(1)\phi(2)\phi(3) : + [\phi^+(1), \phi^-(2)] \phi(3) \\ &\quad + [\phi^+(2), \phi^-(3)] \phi(1) + [\phi^+(1), \phi^-(3)] \phi(2) \\ T\phi(1)\phi(2)\phi(3) &= \dots \end{aligned}$$

c)

$$T : \phi(1)\phi(2) : \phi(3) = \dots$$

d)

$$\begin{aligned} \phi(1)\phi(2)\phi(3)\phi(4) &= : \phi(1)\phi(2)\phi(3)\phi(4) : \\ &\quad + : \phi(1)\phi(2) : [\phi^+(3), \phi^-(4)] \\ &\quad + : \phi(1)\phi(3) : [\phi^+(2), \phi^-(4)] \\ &\quad + : \phi(2)\phi(3) : [\phi^+(1), \phi^-(4)] \\ &\quad + [\phi^+(1), \phi^-(2)] \{ : \phi(3)\phi(4) : + [\phi^+(3), \phi^-(4)] \} \\ &\quad + [\phi^+(2), \phi^-(3)] \{ : \phi(1)\phi(4) : + [\phi^+(1), \phi^-(4)] \} \\ &\quad + [\phi^+(1), \phi^-(3)] \{ : \phi(2)\phi(4) : + [\phi^+(2), \phi^-(4)] \} \\ T\phi(1)\phi(2)\phi(3)\phi(4) &= \dots \end{aligned}$$

e) Was ist die graphische Darstellung von $\langle 0 | T\phi(1)\phi(2)\phi(3) : \phi^3(4) : | 0 \rangle$ mit Hilfe von $\langle 0 | T\phi(1)\phi(2) | 0 \rangle =: 1 \bullet \text{---} \bullet 2$?

Anmerkung: Nützliche Abkürzungen sind $(12) := \theta(t_1 - t_2)$ und $[\phi^+(1), \phi^-(2)] \phi(3) =: [1, 2] 3$.