13. Präsenzübung, **Statistische Physik**

zu bearbeiten am Donnerstag, 19.1.2012

<u>Aufgabe P26</u> *Decimation of 2D Ising model*

We consider the 2D Ising model on a square lattice with constant nearest neighbour and next-nearest neighbour interactions and external magnetic fields. If H is the Hamiltonian, we have

$$-H/\tau = K \sum_{nn} \sigma_i \sigma_j + L \sum_{nnn} \sigma_i \sigma_j + h \sum_i \sigma_i.$$

The first sum is over all the nearest neighbour interactions (edges of the square lattice) and the second one is over all next-nearest neighbour interactions (diagonals).

Assume that N is the number of spins, but do not worry about boundary conditions. Consider the sublattices A and B containing each half the spins and organized as the two colors of a checkerboard. Let S_A, respectively S_B, denote all possible configurations of the spins in sublattice A, respectively B.

Compute the effective Hamiltonian H' up to order 2 in K, order 1 in L and order 1 in h, so that the partition function is

$$Z = \sum_{S_A S_B} e^{-H/\tau} \simeq \sum_{S_B} e^{-H'/\tau}.$$

Observe that H and H' have the same form up to a constant term, and write down the renormalization equations for the parameters K, L and h.