2. Präsenzübung, **Statistische Physik**

zu bearbeiten am Donnerstag, 20.10.2011

Aufgabe P4 *Ideal gas*

The number of states accessible to an ideal gas of N particles and total energy within the interval $[U, U+\epsilon]$ is

$$g(N,U) \simeq \epsilon f(N) U^{\frac{3N}{2}}.$$

What is the most likely total energy U of the gas if it is in thermal equilibrium at temperature T?

Aufgabe P5 *Van der Waals gas*

A somewhat more realistic model of a gas, which takes into account the finite size of the particles as well as an attractive potential, has, at thermal equilibrium at temperature τ, the total energy

$$U = \frac{3}{2} N\tau - aN^2/V$$

where N is the number of particles, V is the volume in which the gas is confined, and a is a constant. Derive the most general form of the entropy $\sigma(N, V, U)$ that you can deduce from this equation.

Aufgabe P6 *Quantum harmonic oscillator*

We consider N identical quantum harmonic oscillators with angular frequency ω. The energy eigenvalues of one oscillator are $n_1\hbar\omega$, $n_1 = 0, 1, 2, \ldots, \infty$, and all have multiplicity one. If the ith oscillator is at level n_i, then the total energy is

$$U = n\hbar\omega$$

where $n = \sum_i n_i$.

a. We want to compute the number $g(N, n)$ of states of N oscillators with total energy $U = n\hbar\omega$. As directly counting it is a bit difficult, we will use a trick. Clearly $g(N, n)$ corresponds to the number of ways that the N positive integers $n_1, \ldots, n_N$ can sum up to n. Consider the function

$$f(t) := (1-t)^{-N} = \left(\sum_{n=0}^{\infty} t^n\right)^N = \sum_{n_1,\ldots,n_N=0}^{\infty} t^{\sum_i n_i}.$$

Show, by re-arranging the terms of the sum, that

$$f(t) = \sum_{n=0}^{\infty} g(N, n) t^n.$$

Use successive differentiations of $f(t)$ to extract the coefficients

$$g(N, n) = \frac{(N+n-1)!}{n!(N-1)!}.$$

b. Assuming both N and n are much larger than 1, find an approximate expression for the entropy $\sigma(N, n)$ using the Stirling approximation $\log N! \simeq N \log N - N$. You can also replace $N-1$ by N.

c. Show that the total energy $U = n\hbar\omega$ as function of the temperature τ is

$$U = \frac{N\hbar\omega}{e^{\hbar\omega/\tau} - 1}.$$