ON THE LOSS OF REALITY IN QUANTUM PHYSICS

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What is “real” in modern physics?

“Reality” has turned out to be an unsuitable term

Classical properties of quantum systems are usually unfixed and only determined (become “real”) through measurement

Pragmatic use of language: photons or quarks are taken to be “real objects”, because that simplifies communication

Sociological aspect: new objects in physics evolve from hypothetical via schematic until real

Physicists have agreed on hard (often statistical) criteria for the acceptance of novel entities as being real

By the inventors of quantum theory:

“In a nutshell, I can indeed describe the whole thing as an act of desperation. Because by nature I am a peaceful person and disinclined to precarious adventures.”

Max Planck

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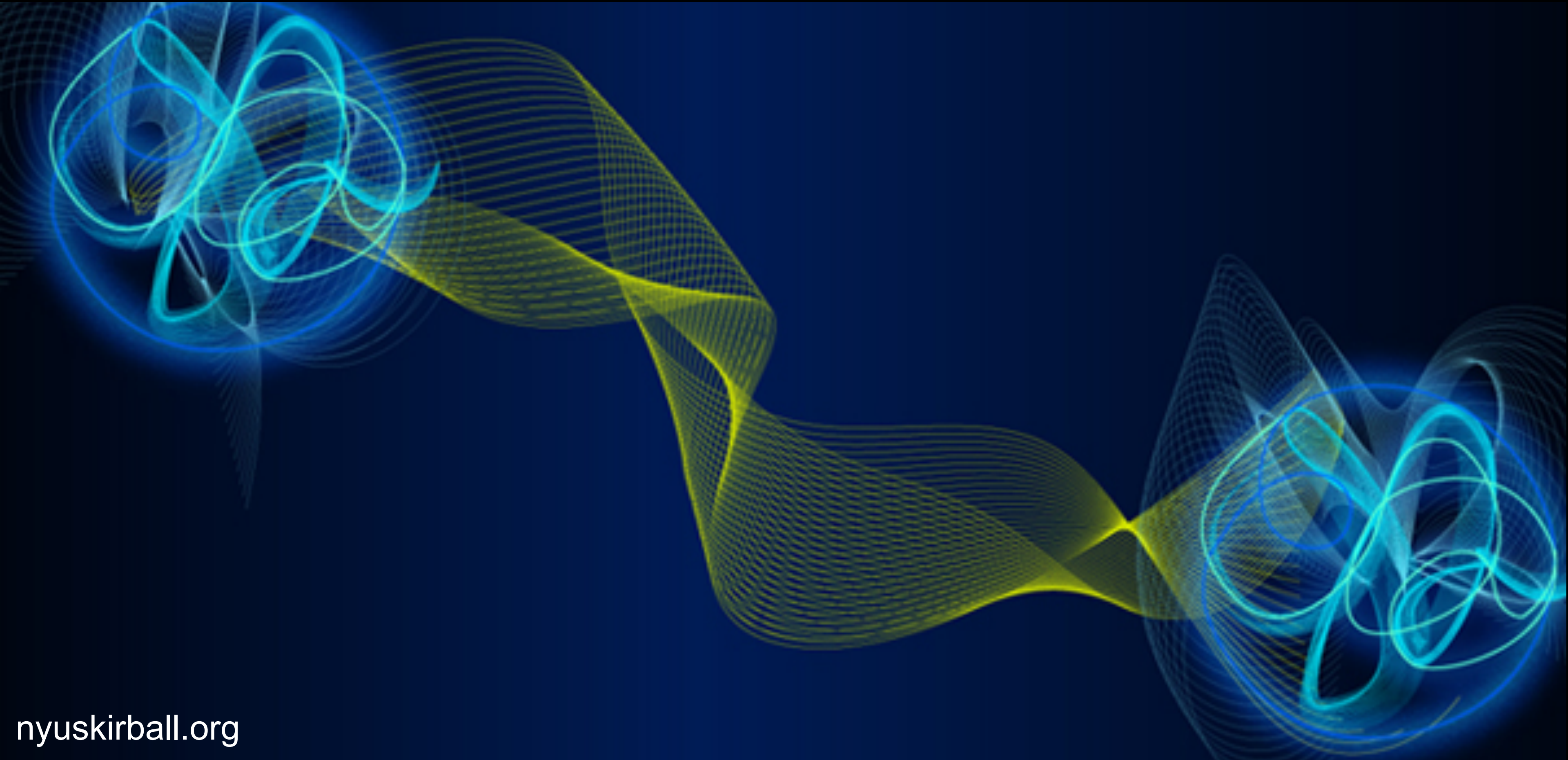
“I think now I understand what's going on: When I look with my left eye, then I notice a particle, but when I look with the right eye, I see a wave. As soon as I open both eyes, I go crazy!”

Wolfgang Pauli to Werner Heisenberg

ILLUSTRATION:
Spooky quantum action at a distance



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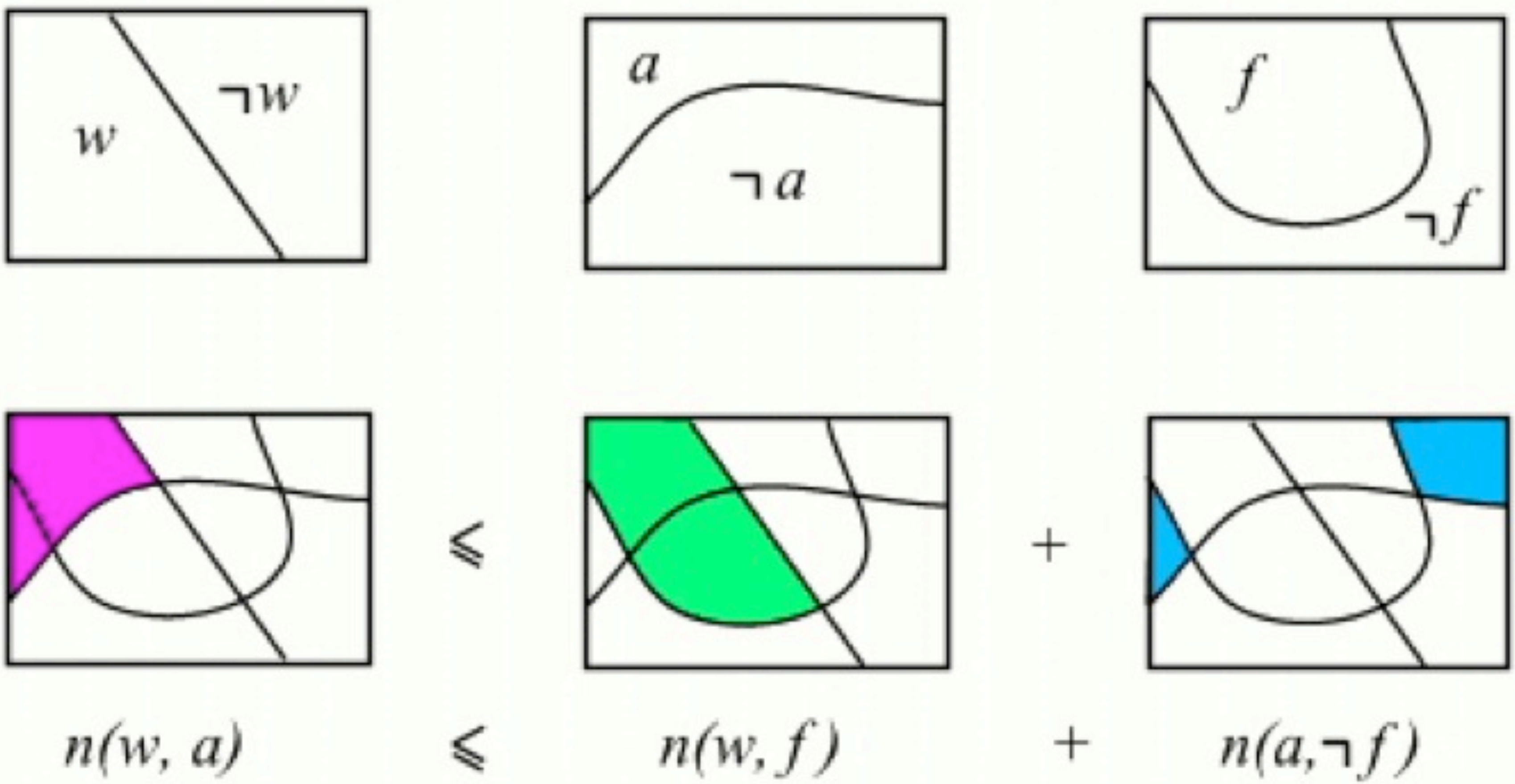
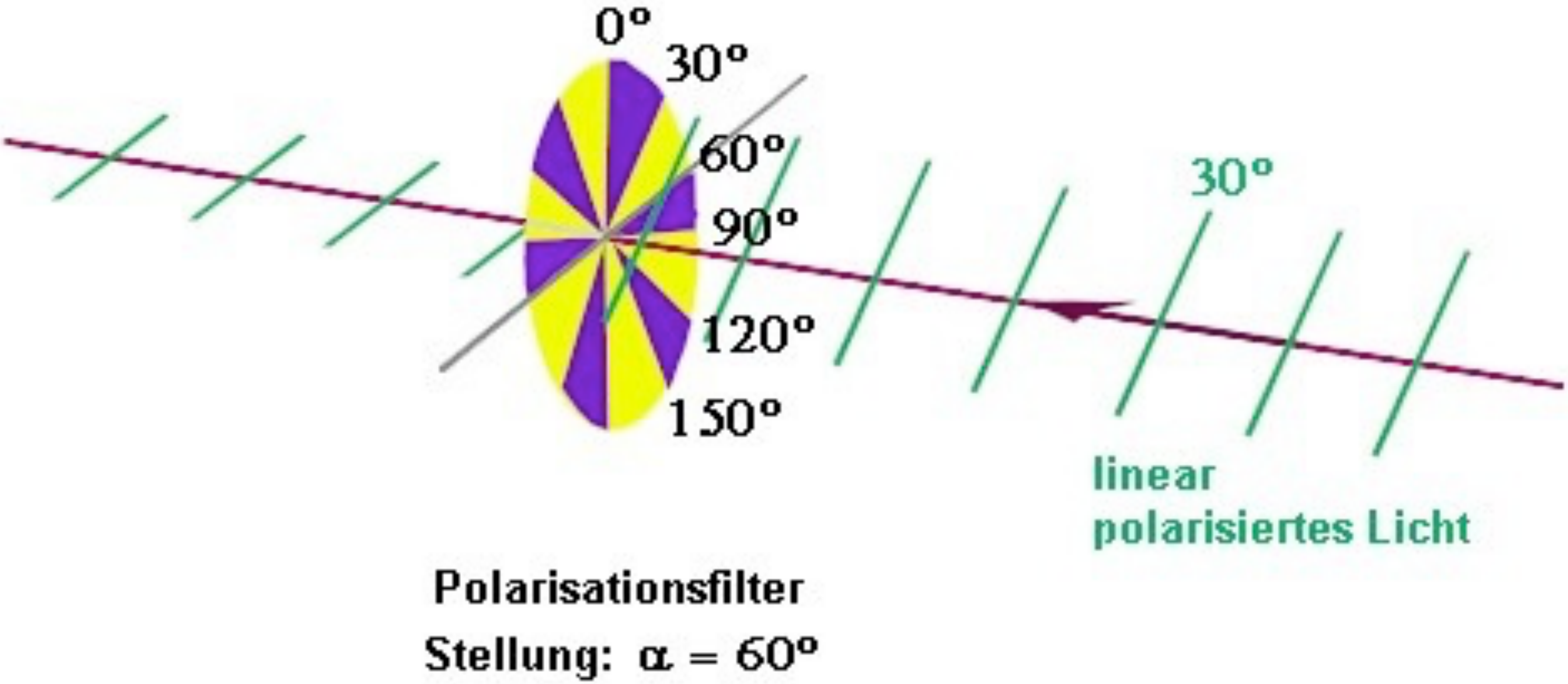
John Steward Bell (1928-1990)



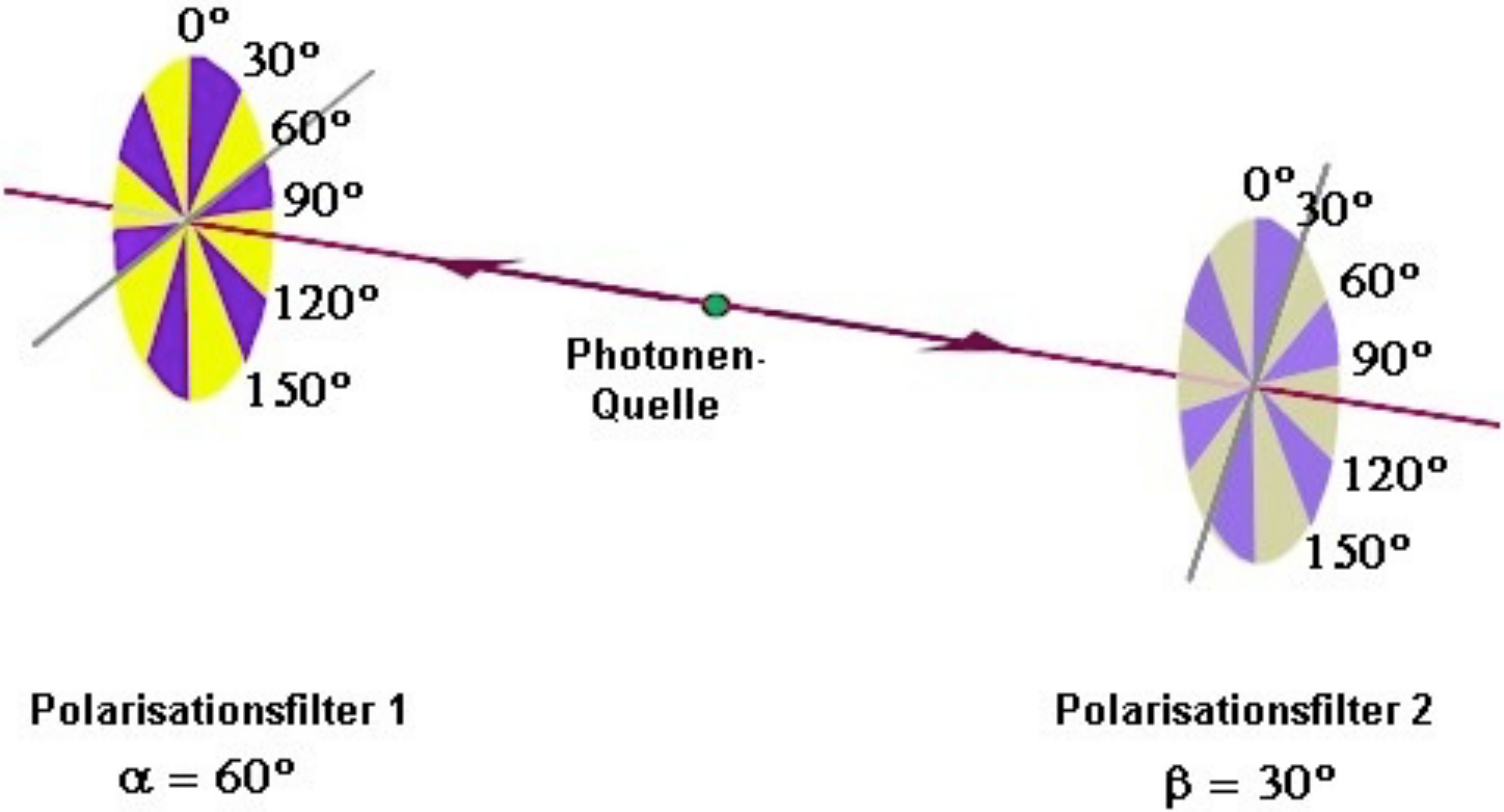
Bell’s Inequality

Embacher (2000)



Der Winkel zwischen der Polarisationsrichtung des einfallenden Lichts und der Ausrichtung des Polarisationsfilters beträgt in diesem Beispiel 30°.

Embacher (2000)



Embacher (2000)

|α−β|	P(α,β)	P(α,¬β)	P(¬α,β)	P(¬α,¬β)
0°	½	0	0	½
30°	?	?	?	?
60°	?	?	?	?
90°	0	½	½	0

The probabilities (P) for the passing through of:

- both photons (α,β)
- only the left photon (α,¬β)
- only the right photon (¬α,β)
- none of the photons (¬α,¬β)

apply Bell to the case of photon
`passing through' probabilities:

$$P(\alpha,\beta) \leq P(\alpha,\gamma) + P(\beta,\neg\gamma)$$

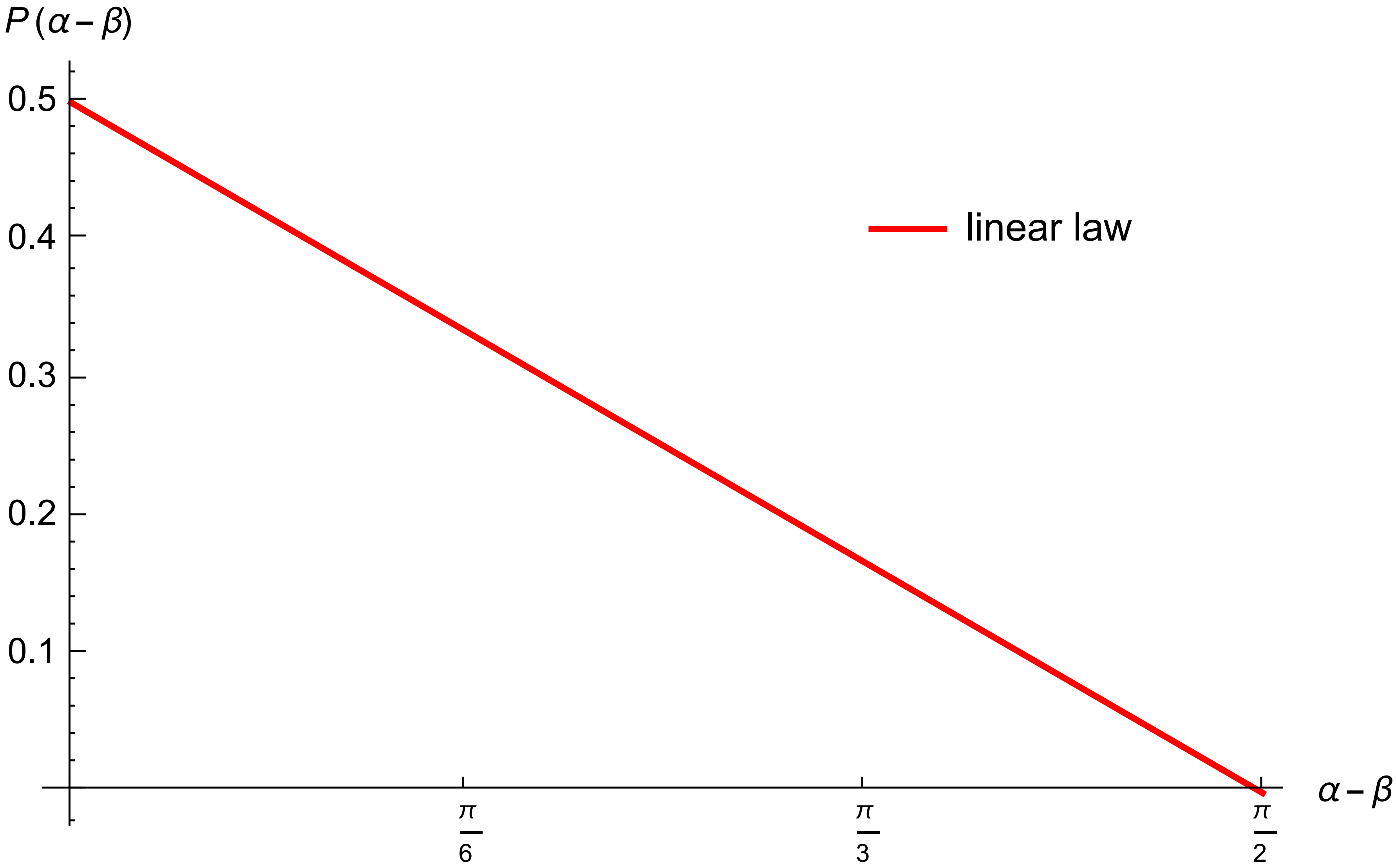
for a tripel (α,β,γ) of angles

maximal $P(\alpha,\beta)$ requires a linear law

$$P(\alpha,\beta) = \tfrac{1}{2} - {}^{|\alpha-\beta|}\!/_{\pi}$$

$$P(\alpha,\neg\beta) = {}^{|\alpha-\beta|}\!/_{\pi}$$

$$-|\alpha-\beta| \leq -|\alpha-\gamma| + |\beta-\gamma| \quad ✓$$



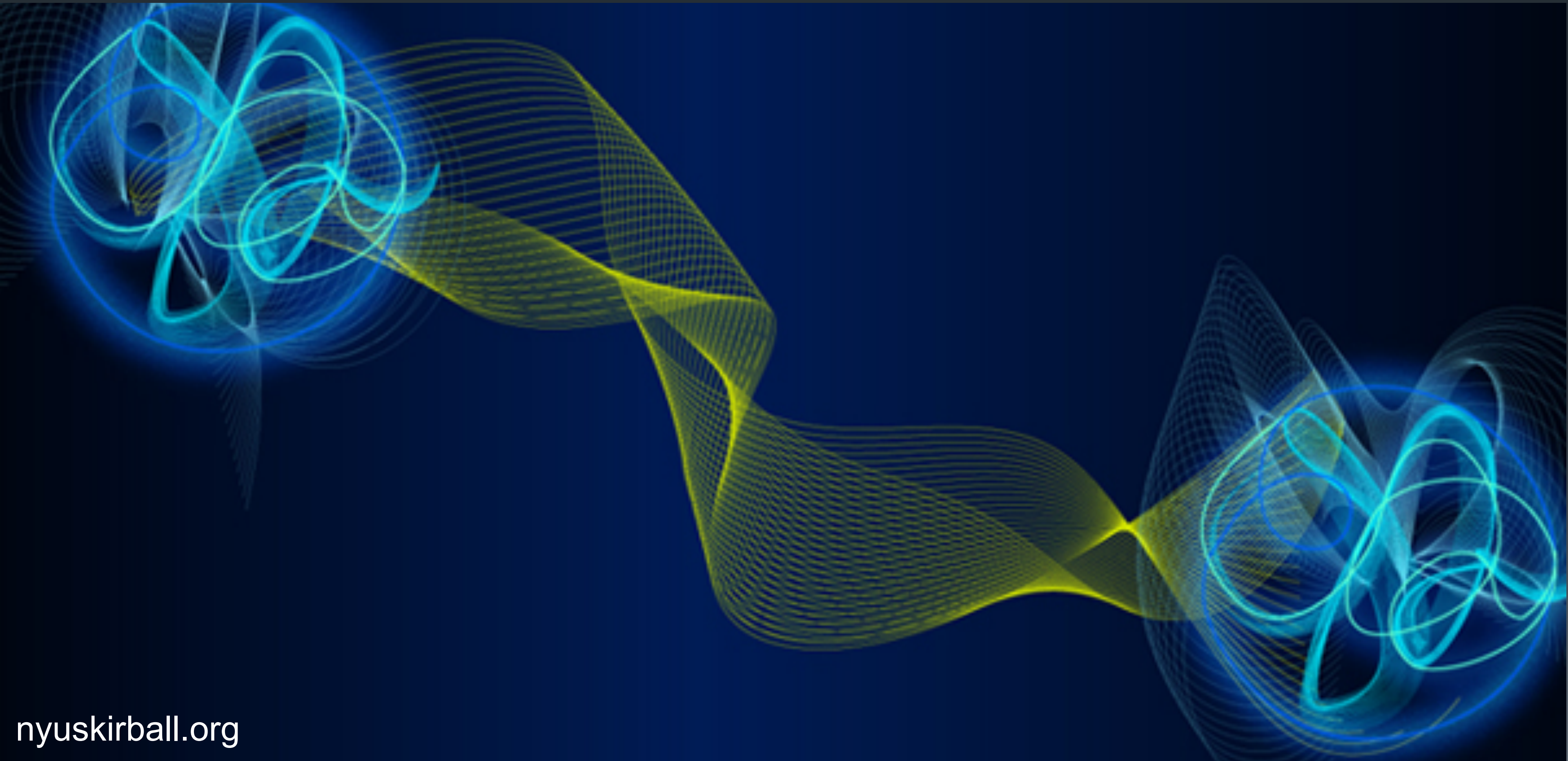
$P(\alpha-\beta)$
0.5
0.4
0.3
0.2
0.1
linear law
$\alpha-\beta$
$\frac{\pi}{6}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$

We can fill the table with the linear law

|α−β|	P(α,β)	P(α,¬β)	P(¬α,β)	P(¬α,¬β)
0º	½	0	0	½
30º	⅓	⅙	⅙	⅓
60º	⅙	⅓	⅓	⅙
90º	0	½	½	0

the maximum allowed by local realism

What does quantum theory tell ?



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The quantum formalism

Linearly polarized one-photon states

$$|\alpha\rangle \ \in S^1 \subset \mathbb{C}P^1 \qquad \text{with polarization label} \quad \alpha \in S^1$$

in homogeneous coordinates on $\mathbb{R}^2 \subset \mathbb{C}^2$ with orthonormal basis $\{|h\rangle, |v\rangle\}$ of horizontally and vertically polarized states:

$$|\alpha\rangle \ = \ \cos\alpha\,|h\rangle + \sin\alpha\,|v\rangle$$

Two-photon states (not all) $\qquad a_{=}^2 + a_{\dashv}^2 + a_{\vdash}^2 + a_{||}^2 = 1$

$$|\vec{\alpha}\rangle\rangle \ = \ a_{=}|hh\rangle\rangle + a_{\dashv}\,|hv\rangle\rangle + a_{\vdash}\,|vh\rangle\rangle + a_{||}\,|vv\rangle\rangle \ \in S^3 \subset \mathbb{R}^4$$

are *separable* iff $a_{=}a_{||} - a_{\dashv}a_{\vdash} = 0 \quad \Leftrightarrow \quad a_{=} = c_1 c_2\,,\ a_{\dashv} = c_1 s_2$ etc., i.e.

$$|\vec{\alpha}\rangle\rangle \ = \ |\beta\rangle \otimes |\gamma\rangle \ = \ \big(c_1|h\rangle + s_1|v\rangle\big) \otimes \big(c_2|h\rangle + s_2|v\rangle\big) \ \in S^1 \times S^1$$

Other states are called *entangled*

Probabilities for a photon in state $|\psi\rangle$ to pass / not pass a filter with orientation α

$$P(\alpha) \;=\; \langle\psi|\Pi_{+}^{\alpha}|\psi\rangle \qquad \text{and} \qquad P(\neg\alpha) \;=\; \langle\psi|\Pi_{-}^{\alpha}|\psi\rangle$$

with the observables (in the basis $\{|h\rangle, |v\rangle\}$)

$$\Pi_{+}^{\alpha} \;=\; |\alpha\rangle\langle\alpha| \;=\; \begin{pmatrix} \cos\alpha\,\cos\alpha & \cos\alpha\,\sin\alpha \\ \cos\alpha\,\sin\alpha & \sin\alpha\,\sin\alpha \end{pmatrix} \;=\; \frac{1}{2}\begin{pmatrix} 1+\cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & 1-\cos 2\alpha \end{pmatrix}$$

$$\Pi_{-}^{\alpha} \;=\; 1 - |\alpha\rangle\langle\alpha| \;=\; \begin{pmatrix} \sin\alpha\,\sin\alpha & -\cos\alpha\,\sin\alpha \\ -\cos\alpha\,\sin\alpha & \cos\alpha\,\cos\alpha \end{pmatrix} \;=\; \frac{1}{2}\begin{pmatrix} 1-\cos 2\alpha & -\sin 2\alpha \\ -\sin 2\alpha & 1+\cos 2\alpha \end{pmatrix}$$

being mutually orthogonal hermitian projectors

$$(\Pi_{\pm}^{\alpha})^2 = \Pi_{\pm}^{\alpha}\,, \quad \Pi_{+}^{\alpha}\Pi_{-}^{\alpha} = 0\,, \quad \Pi_{+}^{\alpha} + \Pi_{-}^{\alpha} = \mathbb{1}$$

Probability for a photon in state $|\gamma\rangle$ to pass through a filter with orientation α is thus

$$P_{|\gamma\rangle}(\alpha) \;=\; \langle\gamma|\Pi_{+}^{\alpha}|\gamma\rangle \;=\; \begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix}^{\top} \begin{pmatrix} \cos\alpha\,\cos\alpha & \cos\alpha\,\sin\alpha \\ \cos\alpha\,\sin\alpha & \sin\alpha\,\sin\alpha \end{pmatrix} \begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix} \;=\; \cos^2(\alpha-\gamma)$$

Let us consider two-photon states $|\psi\rangle\rangle = |\vec{\alpha}\rangle\rangle \in S^3$ of the kind introduced earlier.

To compute the probability for the left photon to pass a filter with orientation α, the relevant observable is $\Pi_+^\alpha \otimes \mathbb{1}$ (the right photon is not observed!), so evaluate

$$P(\alpha, \cdot) \;=\; \langle\langle\psi|\Pi_+^\alpha \otimes \mathbb{1}\,|\psi\rangle\rangle$$

If we measure the left photon with a filter of orientation α
and we also measure the right photon with a filter of orientation β,
the observable to quantify the probability for both to pass through is $\Pi_+^\alpha \otimes \Pi_+^\beta$

Hence, the different probabilities for the left and right photon to pass / not pass are

$$\begin{aligned}
P(\alpha, \beta) &= \langle\langle\psi|\Pi_+^\alpha \otimes \Pi_+^\beta|\psi\rangle\rangle \\
P(\alpha, \neg\beta) &= \langle\langle\psi|\Pi_+^\alpha \otimes \Pi_-^\beta|\psi\rangle\rangle \\
P(\neg\alpha, \beta) &= \langle\langle\psi|\Pi_-^\alpha \otimes \Pi_+^\beta|\psi\rangle\rangle \\
P(\neg\alpha, \neg\beta) &= \langle\langle\psi|\Pi_-^\alpha \otimes \Pi_-^\beta|\psi\rangle\rangle
\end{aligned}$$

In certain decays of exited atomic states,
a pair of photons is emitted (in opposing directions) in the entangled state

$$|\psi\rangle\!\rangle \;=\; \frac{1}{\sqrt{2}}\Big(|hh\rangle\!\rangle + |vv\rangle\!\rangle\Big) \;\neq\; |\beta\rangle \otimes |\gamma\rangle$$

In this state, the probability for the left photon to pass through a filter α is

$$P(\alpha,\cdot) \;=\; \tfrac{1}{2}\Big(\langle\!\langle hh| + \langle\!\langle vv|\Big)\Pi_+^{\alpha}\otimes\mathbb{1}\,\Big(|hh\rangle\!\rangle + |vv\rangle\!\rangle\Big) \;=\; \tfrac{1}{2}\Big(\langle h|\Pi_+^{\alpha}|h\rangle + \langle v|\Pi_+^{\alpha}|v\rangle\Big) \;=\; \tfrac{1}{2}$$

The interesting calculation is that of $P_{++} := P(\alpha,\beta)$ and $P_{+-} := P(\alpha,\neg\beta)$:

$$\begin{aligned}
P_{+\pm} \;&=\; \tfrac{1}{2}\big(\langle\!\langle hh| + \langle\!\langle vv|\big)\Pi_+^{\alpha}\otimes\Pi_\pm^{\beta}\big(|hh\rangle\!\rangle + |vv\rangle\!\rangle\big)\\
&=\; \tfrac{1}{2}\big(\langle h|\Pi_+^{\alpha}|h\rangle\langle h|\Pi_\pm^{\beta}|h\rangle + \langle v|\Pi_+^{\alpha}|v\rangle\langle v|\Pi_\pm^{\beta}|v\rangle + \langle h|\Pi_+^{\alpha}|v\rangle\langle h|\Pi_\pm^{\beta}|v\rangle + \langle v|\Pi_+^{\alpha}|h\rangle\langle v|\Pi_\pm^{\beta}|h\rangle\big)\\
&=\; \tfrac{1}{8}\big((1+\cos 2\alpha)(1\pm\cos 2\beta) + (1-\cos 2\alpha)(1\mp\cos 2\beta) \pm \sin 2\alpha\,\sin 2\beta \pm \sin 2\alpha\,\sin 2\beta\big)\\
&=\; \tfrac{1}{4}\big(1 \pm \cos 2\alpha\,\cos 2\beta \pm \sin 2\alpha\,\sin 2\beta\big) \;=\; \tfrac{1}{4}\big(1 \pm \cos 2(\alpha-\beta)\big) \;=\; \begin{cases}\tfrac{1}{2}\cos^2(\alpha-\beta)\\ \tfrac{1}{2}\sin^2(\alpha-\beta)\end{cases}
\end{aligned}$$

Likewise, due to symmetry, $P_{-\mp} \;=\; P_{+\pm} \;=\; \tfrac{1}{4}\Big(1 \pm \cos 2(\alpha-\beta)\Big)$

Let us fill the table with the quantum law

$\lvert\alpha-\beta\rvert$	$P(\alpha,\beta)$	$P(\alpha,\neg\beta)$	$P(\neg\alpha,\beta)$	$P(\neg\alpha,\neg\beta)$
0°	½	0	0	½
30°	⅜	⅛	⅛	⅜
60°	⅛	⅜	⅜	⅛
90°	0	½	½	0

note that $P(30°) = \frac{3}{8} > \frac{1}{3}$

Check Bell's inequality

$P(\alpha,\beta) \;\leq\; P(\alpha,\gamma) + P(\beta,\neg\gamma)$

for the quantum law

$P(\alpha,\beta) \;=\; \frac{1}{2}\cos^2(\alpha-\beta)$

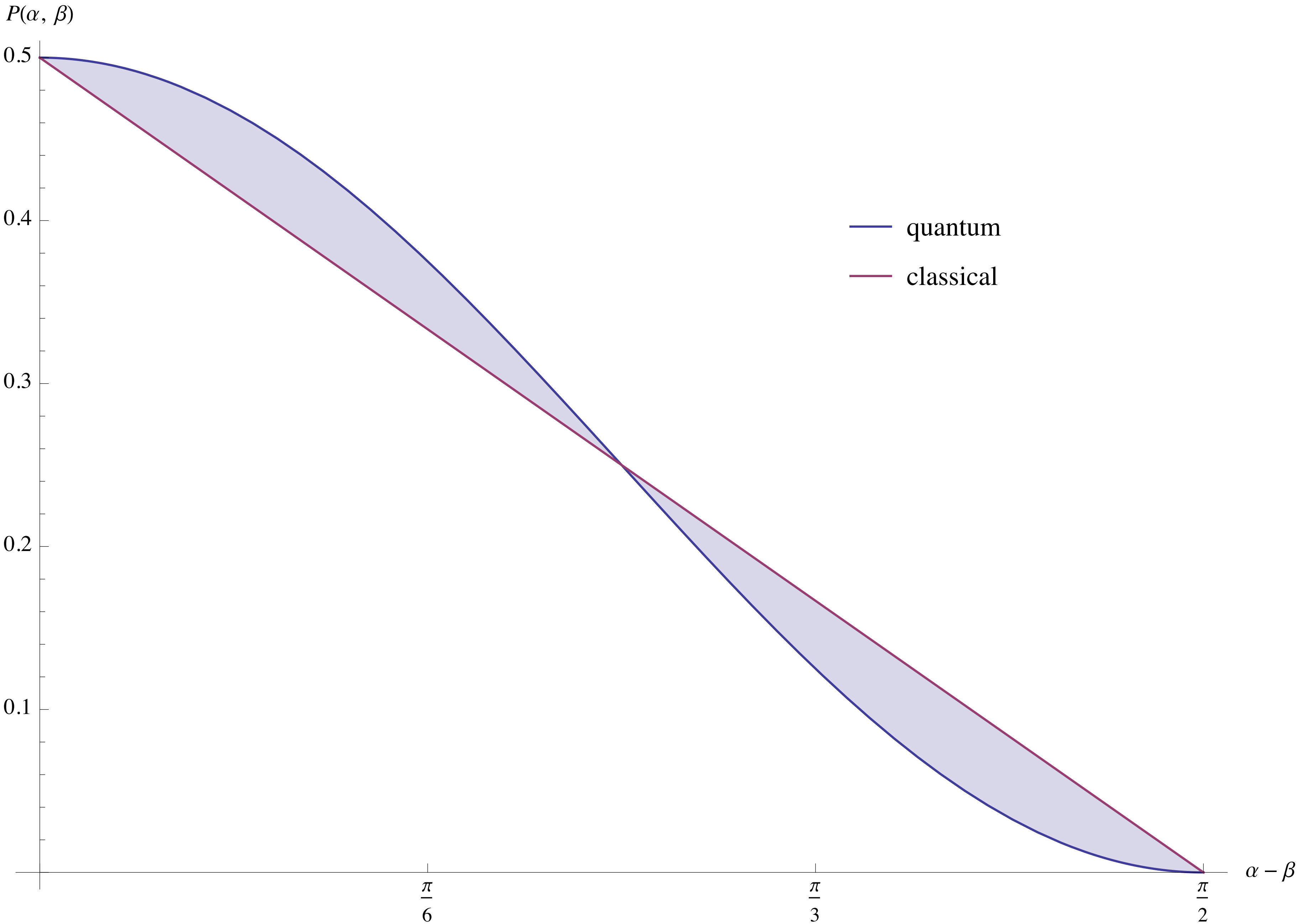
with a tripel (α,β,γ) of angles

choose $\alpha=0°$, $\beta=30°$, $\gamma=60°$

$$P(0°,30°) \overset{?}{\leq} P(0°,60°) + P(30°,\neg 60°)$$

$\lvert\alpha - \beta\rvert$	$P(\alpha,\beta)$	$P(\alpha,\neg\beta)$	$P(\neg\alpha,\beta)$	$P(\neg\alpha,\neg\beta)$
0º	½	0	0	½
30º	⅜	⅛	⅛	⅜
60º	⅛	⅜	⅜	⅛
90º	0	½	½	0

⅜ ≤ ⅛ + ⅛ ☛ inequality is violated !


P(α, β)
0.5
0.4
0.3
0.2
0.1
quantum
classical
π/6
π/3
π/2
α − β

Experimental reality: states cannot be entangled perfectly:

$$|\psi\rangle\rangle \;=\; \sqrt{p_1}\,|hh\rangle\rangle + \sqrt{p_2}\,|vv\rangle\rangle \qquad \text{with} \qquad p_1 + p_2 = 1$$

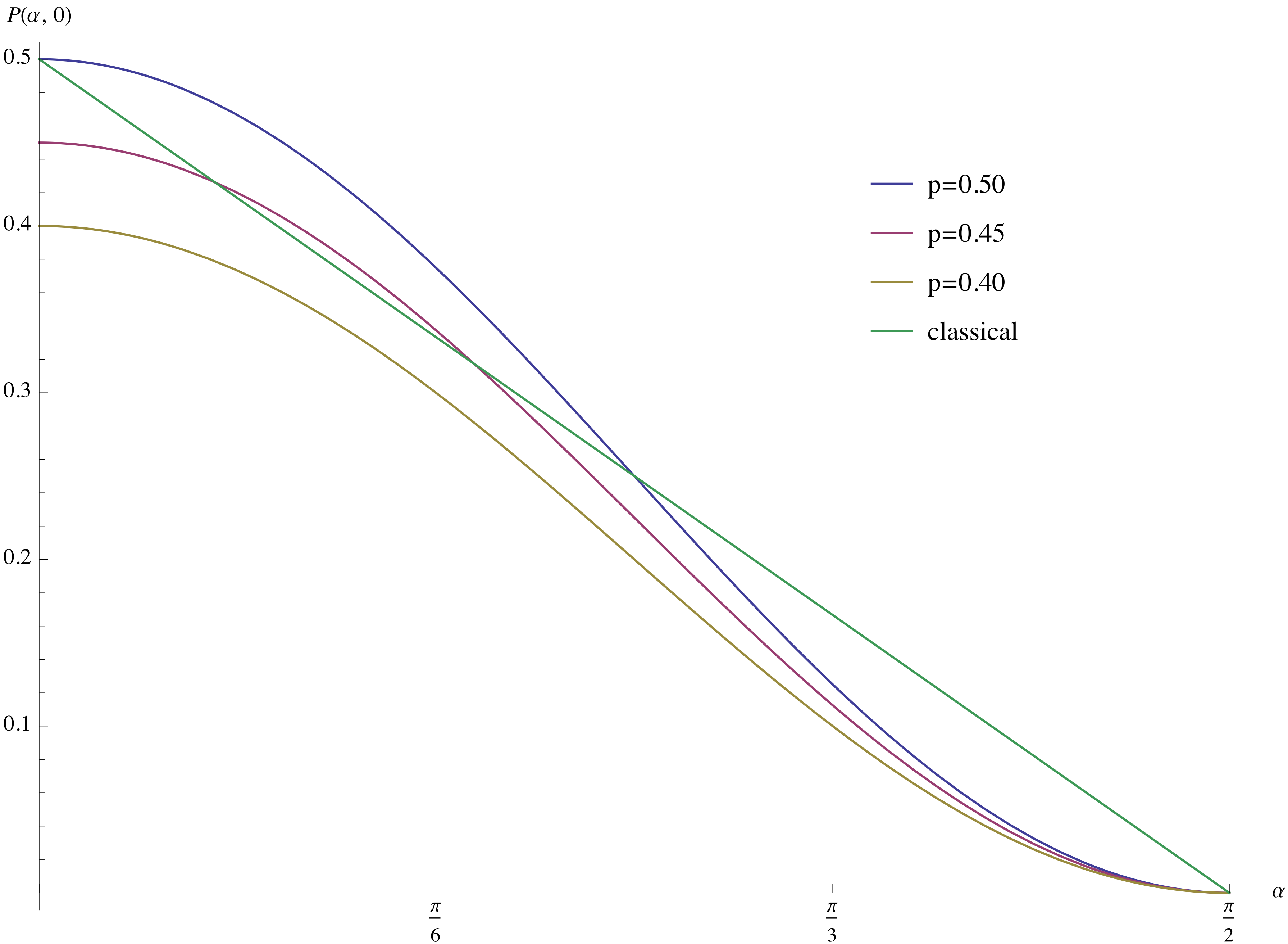
previous (maximal) case was $p_1 = p_2 = \frac{1}{2}$

Repeat the computation:

$$\begin{aligned}
P_{+\pm} &= \tfrac{1}{2}\big(\langle\langle hh|\sqrt{p_1} + \langle\langle vv|\sqrt{p_2}\big)\Pi_+^\alpha \otimes \Pi_\pm^\beta\big(\sqrt{p_1}|hh\rangle\rangle + \sqrt{p_2}|vv\rangle\rangle\big) \\
&= \tfrac{1}{4}\big(p_1(1+\cos 2\alpha)(1\pm\cos 2\beta) + p_2(1-\cos 2\alpha)(1\mp\cos 2\beta) \pm 2\sqrt{p_1 p_2}\sin 2\alpha\,\sin 2\beta\big) \\
&= \tfrac{1}{4}\big(1 \pm \cos 2\alpha\,\cos 2\beta \pm 2\sqrt{p_1 p_2}\sin 2\alpha\,\sin 2\beta + (p_1 - p_2)(\cos 2\alpha \pm \cos 2\beta)\big)
\end{aligned}$$

this depends on α and β separately

plot $P_{\alpha,0} = P_{++}|_{\beta=0}$ for $p \equiv p_1 \leq \frac{1}{2}$



$P(\alpha, 0)$
0.5
0.4
0.3
0.2
0.1
p=0.50
p=0.45
p=0.40
classical
$\frac{\pi}{6}$
$\frac{\pi}{3}$
$\frac{\pi}{2}$
α

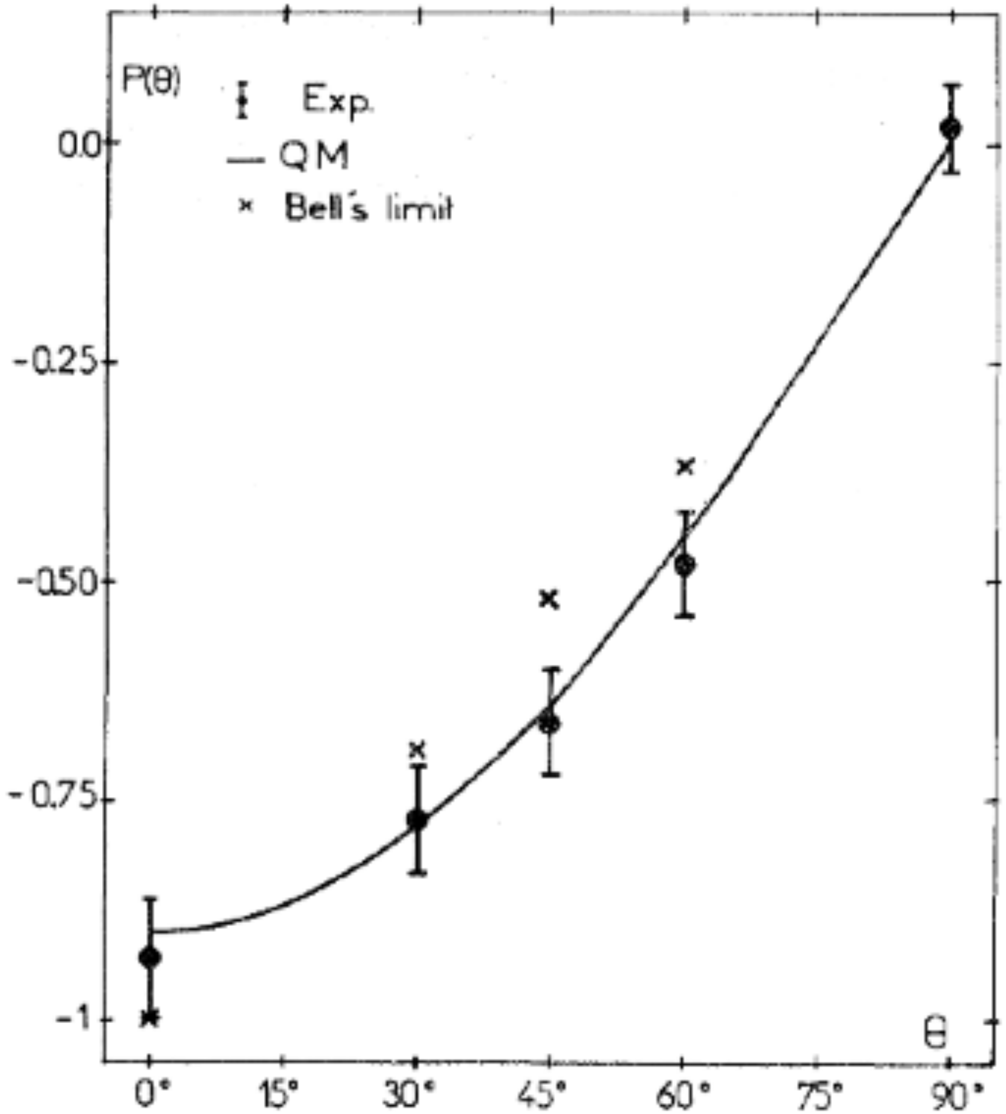


FIG. 10. Experimental results (see Table V) for $P_{\exp}(\theta)$ as compared to the limit of Bell and predictions of QM.

M. Lamehi-Rachti and W. Mittig, Phys. Rev. D 14 (1976) 2543

What is so exciting about this?

- $P(\alpha,\bullet) = P(\bullet,\beta) = \frac{1}{2}$, but $P(\alpha,\beta) \neq f(\alpha)\, g(\beta)$ ➔ correlations
- The correlations are nonlocal (filters may be widely separated)
- The correlations are acausal (delayed-choice experiments)
- If “polarization” is a real property of individual photons in an entangled state, passing / no passing is predetermined
- This explains correlations but also limits them! (Bell’s theorem)
- Quantum correlations exceed this limit ➔ local realism is dead

Resolution of the contradiction:

- Before passing the polarization filter the photons do <u>not</u> carry a well defined polarization with them
- Local hidden parameters (“elements of reality”) do not exist
- “Polarization” is not a classical property of the photons, hence Bell’s inequality need not hold
- Counterfactuality: unperformed experiments have no outcome
- Quantum correlations perfectly epitomize the quantum world


THANK YOU !