Perturbative spectra in gauge theories with gravity duals

Christoph Sieg

Niels Bohr International Academy
Niels Bohr Institute

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C.S., A. Torrielli:	0505071
F. Fiamberti, A. Santambrogio, C.S., D. Zanon:	0712.3522
	0806.2095
	0806.2103
	0811.4594
F. Fiamberti, A. Santambrogio, C.S.:	0908.0234
J. Minahan, O. Ohlsson Sax, C.S.:	0908.2463
	0912.3460

Outline

AdS/CFT correspondence

$\lambda \gg 1$ ← → $\lambda \ll 1$

energy E
(semicl.) strings

anom. dim. γ
comp. operators
$\mathcal{O}_L \propto \mathrm{tr}\, \phi_{i_1} \ldots \phi_{i_L}$

[Bena, Polchinski, Roiban]
[Kazakov, Marshakov, Minahan, Zarembo]

integrable systems
(asymptot.) Bethe ansätze

[Minahan, Zarembo]

[Beisert, Dippel, Staudacher]
[Arutyunov, Frolov, Staudacher]

string

gauge

dressing phase
[Beisert, Hernandez, Lopez]
[Beisert, Eden, Staudacher]

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$\mathcal{O}_S \propto \mathrm{tr}\,\phi\,D^S\,\phi$

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string — gauge

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$[\mathcal{D}, Q_i] = 0$

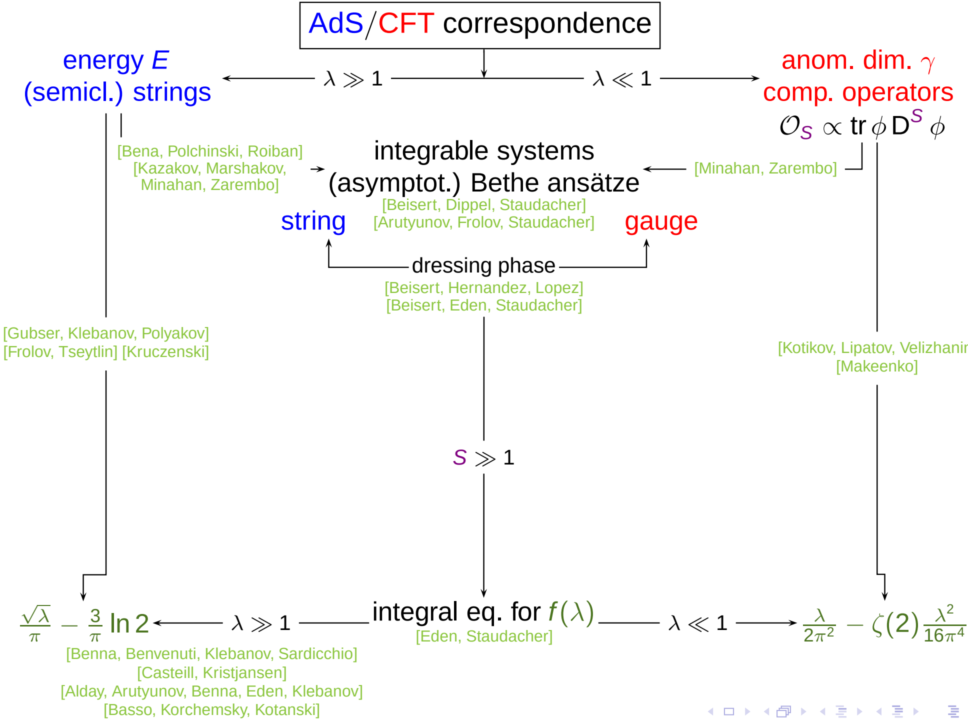
dilatation operator

[Beisert, Kristjansen, Staudacher]

$$\mathcal{D}\,\vec{\mathcal{O}}_L = \gamma\,\vec{\mathcal{O}}_L$$

Feynman graph computations
in the flavour $SU(2)$ subsector:

1-loop: [Berenstein, Maldacena, Nastase]
2-loops: [Gross, Mikhailov, Roiban]
checks at higher loops:
[Gross, Mikhailov, Roiban]
[Beisert, McLoughlin, Roiban]
[Fiamberti, Santambrogio, CS, Zanon]
[Fiamberti, Santambrogio, CS]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[Gubser, Klebanov, Polyakov]
[Frolov, Tseytlin] [Kruczenski]
[Kotikov, Lipatov, Velizhanin
[Makeenko]
$S \gg 1$
$\frac{\sqrt{\lambda}}{\pi} - \frac{3}{\pi} \ln 2$
$\lambda \gg 1$
integral eq. for $f(\lambda)$
[Eden, Staudacher]
$\lambda \ll 1$
$\frac{\lambda}{2\pi^2} - \zeta(2) \frac{\lambda^2}{16\pi^4}$
[Benna, Benvenuti, Klebanov, Sardicchio]
[Casteill, Kristjansen]
[Alday, Arutyunov, Benna, Eden, Klebanov]
[Basso, Korchemsky, Kotanski]

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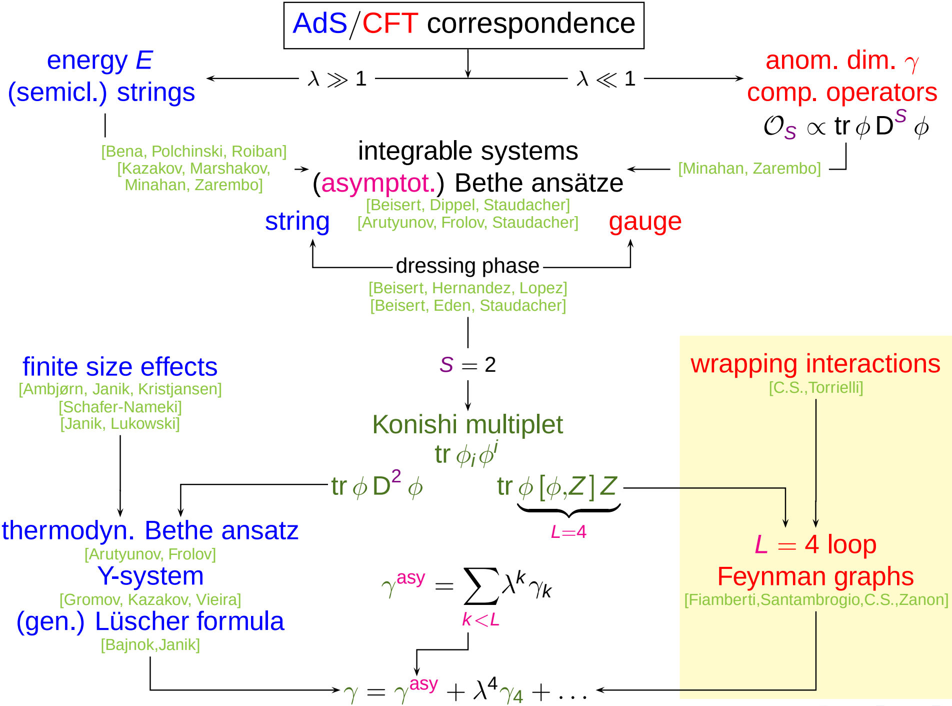
$S = 2$

Konishi multiplet
$\mathrm{tr}\, \phi_i \phi^i$
$\mathrm{tr}\, \phi\, \mathrm{D}^2\, \phi$ $\underbrace{\mathrm{tr}\, \phi\, [\phi, Z]\, Z}_{L=4}$

$$\gamma^{\mathrm{asy}} = \sum_{k<L} \lambda^k \gamma_k$$



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$S = 2$
finite size effects
[Ambjørn, Janik, Kristjansen]
[Schafer-Nameki]
[Janik, Lukowski]
wrapping interactions
[C.S.,Torrielli]
Konishi multiplet
$\mathrm{tr}\,\phi_i\phi^i$
$\mathrm{tr}\,\phi\,D^2\,\phi$
$\mathrm{tr}\,\phi\,[\phi,Z]\,Z$
$L=4$
thermodyn. Bethe ansatz
[Arutyunov, Frolov]
Y-system
[Gromov, Kazakov, Vieira]
(gen.) Lüscher formula
[Bajnok,Janik]
$L = 4$ loop
Feynman graphs
[Fiamberti,Santambrogio,C.S.,Zanon]
$\gamma^{\mathrm{asy}} = \sum_{k<L} \lambda^k \gamma_k$
$\gamma = \gamma^{\mathrm{asy}} + \lambda^4\gamma_4 + \dots$

AdS_4/CFT_3 (ABJM) correspondence

[Aharony, Bergman, Jafferis, Maldacena]

Type II A ST $AdS_4 \times \mathbb{CP}_3$ $\Leftrightarrow$ 3-dim. $\mathcal{N} = 6$ CS theory

energy E ← $\lambda \gg 1$ — $\lambda \ll 1$ → anom. dim. γ

(semicl.) strings — comp. operators

$\mathcal{O}_J \propto \operatorname{tr} \phi_{n_1} \dots \phi_{n_J}$

[Arutyunov, Frolov]
[Stefanski]
[Gromov, Vieira]
→ integrable systems ← [Minahan, Zarembo] [Bak, Rey]

(asymptot.) Bethe ansätze

[Gromov, Vieira]
[Ahn, Nepomechie]

magnon dispersion relation

[Beisert, Dippel, Staudacher]
[Beisert]
[Arutyunov, Frolov, Zamaklar]

$$E = \sqrt{Q^2 + 4h(\lambda)^2 \sin^2 \tfrac{p}{2}} - Q$$

↓

$$h(\lambda)^2 = ?$$

↓

$$h(\lambda)^2 = \frac{\lambda}{4\pi^2}$$

unexpectedly simple in AdS_5/CFT_4

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BMN limit
giant magnons
[Nishioka, Takayanagi]
[Gaiotto, Giombi, Yin]
[Grignani, Harmark, Orselli]

quantum corr.
[McLoughlin, Roiban, Tseytlin]

two loops
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$$\frac{\lambda}{2} - \sqrt{\frac{\lambda}{2}}\frac{\ln 2}{\pi} \longleftarrow h(\lambda)^2 = ? \longrightarrow \lambda^2$$

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four loops
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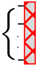
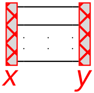
Renormalization of composite operators

composite operator $\mathcal{O}_L = L$  of length L (with L scalar fields)

two-point functions of composite operators: tree level

$$\left(\mathcal{O}_L^A(x),\ 1\ ,\mathcal{O}_L^B(y)\right) =$$



$$= \frac{\delta^{AB}}{(x-y)^{2\Delta}}\,, \qquad \Delta = L$$

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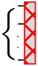
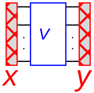
composite operator $\mathcal{O}_L = L$  of length L (with L scalar fields)

two-point functions of composite operators: with loop corrections

$$\left(\mathcal{O}_L^A(x), V_{2L}, \mathcal{O}_L^B(y)\right) =$$



$$= \frac{\delta^{AB}}{(x-y)^{2\Delta}}\,, \qquad \Delta = L + \gamma + \dots$$

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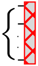
composite operator $\mathcal{O}_L = L$  of length L (with L scalar fields)

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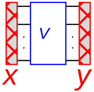

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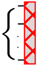
renormalization of composite operators in a CFT in $D = 4 - 2\varepsilon$ dimensions

$$\mathcal{O}^a_{L,\text{ren}} = \mathcal{Z}^a{}_b \mathcal{O}^b_{L,\text{bare}}\,, \qquad \mathcal{D} = \mu \frac{\mathrm{d}}{\mathrm{d}\mu} \ln \mathcal{Z}(\lambda \mu^{2\varepsilon})$$

Renormalization of composite operators

composite operator $\mathcal{O}_L = L$  of length L (with L scalar fields)

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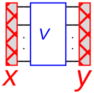

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anomalous dimensions:
eigenvalues of the dilatation operator $\mathcal{D} = \sum_{k \geq 1} \lambda^k \mathcal{D}_k$

$$\mathcal{D}\,\vec{\mathcal{O}}_L = \gamma\,\vec{\mathcal{O}}_L$$

Bethe ansatz in the flavour $SU(2)$ subsector

complex fields: $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$, $\psi = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4)$, $Z = \frac{1}{\sqrt{2}}(\phi_5 + i\phi_6)$

ψ only as internal flavour in Feynman diagrams

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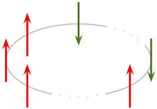
map to integrable spin chain of length L

$$\mathcal{O}_L = \mathrm{tr}(\underbrace{\phi\ldots\phi}_{M}\underbrace{ZZZ\ldots Z}_{L-M}) \quad \leftrightarrow$$



BPS operator $\mathrm{tr}(Z\ldots Z)$	$\leftrightarrow$	ferromagnetic vaccum
impurities ϕ	$\leftrightarrow$	spin excitations (magnons)
dilatation operator $\mathcal{D}$	$\leftrightarrow$	Hamiltonian H
anomalous dimensions γ	$\leftrightarrow$	energies E

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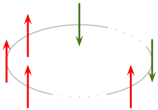

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impurities ϕ $\leftrightarrow$ spin excitations (magnons)

dilatation operator $\mathcal{D}$ $\leftrightarrow$ Hamiltonian $H = H_{XXX_{1/2}} + \dots$

anomalous dimensions γ $\leftrightarrow$ energies E

operator mixing problem solved by the asymptotic Bethe ansatz

$$\sum_{j=1}^{M} p_j = 0\,, \quad e^{i p_j L} = \prod_{k \neq j}^{M} \hat{S}(u_j, u_k) e^{2i\theta(u_j, u_k)}\,, \quad E = \sum_{j=1}^{M} \left(\sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p_j}{2}} - 1 \right)$$

momentum conservation

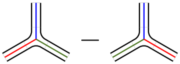
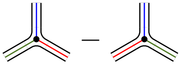
matrix part

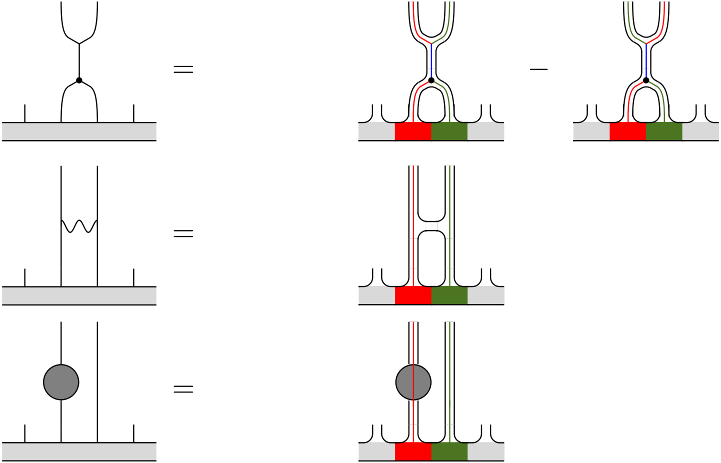
dressing phase

two-particle S-matrix

single magnon dispersion relation

One loop

$i\,\mathrm{tr}(\psi\,[Z,\phi]) = i\Big($  $\Big)\,, \quad -i\,\mathrm{tr}(\bar\psi\,[\bar\phi,\bar Z]) = -i\Big($  $\Big)$



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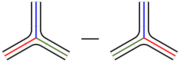
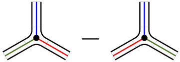
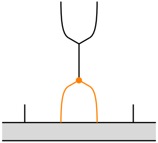
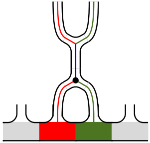
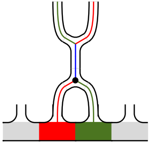
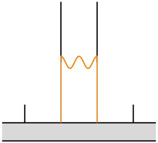
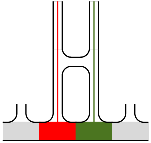
$i\,\mathrm{tr}(\psi\,[Z,\phi]) = i\Big($  $\Big)$, $\quad -i\,\mathrm{tr}(\bar{\psi}\,[\bar{\phi},\bar{Z}]) = -i\Big($  $\Big)$

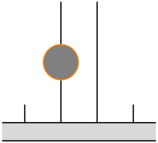


$= -\frac{\lambda}{(4\pi)^2\varepsilon}\Bigg($  $-$  $\Bigg)$



$=$ finite





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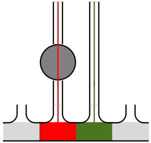
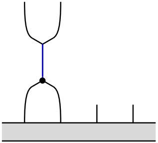
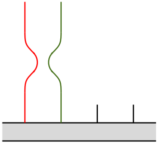
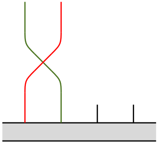
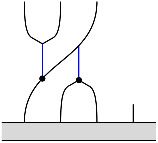
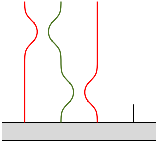
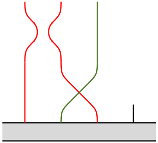
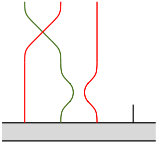
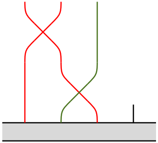


$$\mathcal{D}_1 = 2\frac{\lambda}{(4\pi)^2}\left(1 - \sum_{i=1}^{L} \mathrm{P}_{i\,i+1}\right)$$

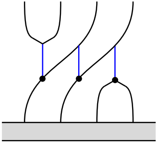
Chiral functions

 $\rightarrow \; -$  $+$ 

$$\chi(1) \; = \; - \; \{\} \; + \; \{1\}$$

 $\rightarrow$  $-$  $-$  $+$ 

$$\chi(1,2) \; = \; \{\} \; - \; \{1\} \; - \; \{1\} \; + \; \{1,2\}$$

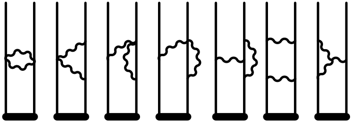
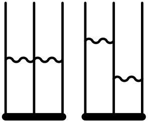
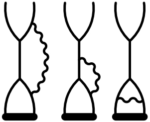
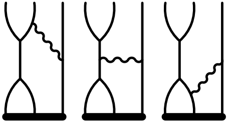
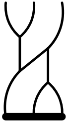


$$\{a_1,\ldots,a_n\} = \sum_{i=1}^{L} \mathrm{P}_{i+a_1\; i+a_1+1} \ldots \mathrm{P}_{i+a_n\; i+a_n+1}$$

$$\chi(1,2,3) \; = \; -\{\} + 3\{1\} - 2\{1,2\} - \{1,3\} + \{1,2,3\}$$

Two loops

- all diagrams (apart from reflections, one-loop wave function ren.)
- finiteness — Fiamberti, Santambrogio, CS, Zanon
- generalized finiteness $\Rightarrow$ absence of $\chi()$ to all orders — CS, to appear

	$R = 1$	$R = 2$	$R = 3$
$\chi()$			
$\chi(1)$	–		
$\chi(1,2)$	–	–	

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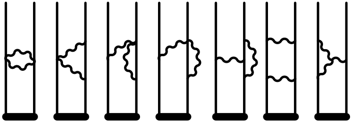
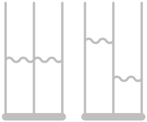
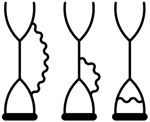
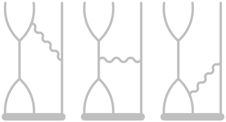
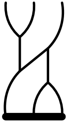
$\chi()$			
$\chi(1)$	–		
$\chi(1,2)$	–	–	

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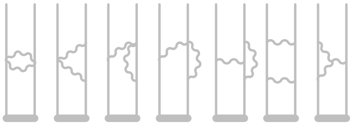
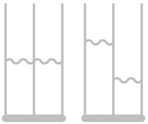
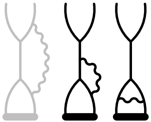
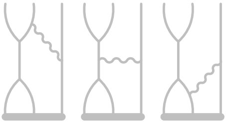
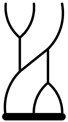
$\chi()$			
$\chi(1)$	–		
$\chi(1,2)$	–	–	

$$\mathcal{D}_2 = 4\chi(1) - 2[\chi(1,2) + \chi(2,1)]$$

Checks at higher loops

Feynman diagrams with $\chi(a_1, \dots, a_n)$ at k loops are simplest if:

purely chiral $\rightarrow n = k$

of maximum range $\rightarrow \max_{a_1,\dots,a_n} - \min_{a_1,\dots,a_n} = k - 1$

$$
\begin{aligned}
\mathcal{D}_4 = \; & +200\chi(1) \\
& -150[\chi(1,2)+\chi(2,1)] + 8(10+\epsilon_{3a})\chi(1,3) - 4\chi(1,4) \\
& +60[\chi(1,2,3)+\chi(3,2,1)] \\
& +(8+2\beta+4\epsilon_{3a}-4i\epsilon_{3b}+2i\epsilon_{3c}-2i\epsilon_{3d})\chi(1,3,2) \\
& +(8+2\beta+4\epsilon_{3a}+4i\epsilon_{3b}-2i\epsilon_{3c}+2i\epsilon_{3d})\chi(2,1,3)] \\
& -(4+4i\epsilon_{3b}+2i\epsilon_{3c})[\chi(1,2,4)+\chi(1,4,3)] \\
& -(4-4i\epsilon_{3b}-2i\epsilon_{3c})[\chi(1,3,4)+\chi(2,1,4)] \\
& -(12+2\beta+4\epsilon_{3a})\chi(2,1,3,2) \\
& +(18+4\epsilon_{3a})[\chi(1,3,2,4)+\chi(2,1,4,3)] \\
& -(8+2\epsilon_{3a}+2i\epsilon_{3b})[\chi(1,2,4,3)+\chi(1,4,3,2)] \\
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\end{aligned}$$

$\beta = 4\zeta(3)$ is leading coeff. of the dressing phase

Beisert, McLoughlin, Roiban

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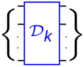
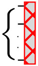
\end{aligned}$$

important for leading wrapping correction

Fiamberti, Santambrogio, CS, Zanon

Finite size effects / wrapping interactions

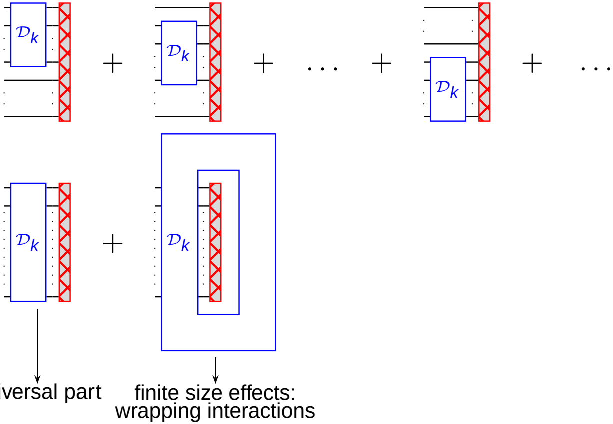
dilatation operator $\mathcal{D}_k = k+1$  $k+1$ at order k, i.e. $\sim \lambda^k$

composite operator $\mathcal{O}_L = L$  of length L

action of the dilatation operator:

$k < L$: $\mathcal{D}_k \mathcal{O}_L =$

$k \geq L$: $\mathcal{D}_k \mathcal{O}_L =$



$\mathcal{D}_k$ from integrability: only in the asymptotic limit $k < L$

Calculation of wrapping effects

Fiamberti, Santambrogio, C.S., Zanon: 0712.3522, 0806.2095, 0806.2103, 0811.4594

Fiamberti, Santambrogio, C.S.: 0908.0234

1. analyze the properties of the wrapping interactions   C.S., Torrielli: 0505071
2. use efficient formalism $\to \mathcal{N} = 1$ superfields
3. compute all k-loop diagrams? No!
   use known asymptotic dilatation operator $\mathcal{D}_k$
   - correct it for the application to $\mathcal{O}_L$ with $k = L$
   - need appropriate basis for flavour permutations
     $\to$ chiral functions
   - only have to compute subtraction and wrapping
4. compute the divergences of the loop integrals analytically
   - we improved the Gegenbauer polynomial x-space technique
     (correct treatment of traceless products in numerators)
   - we introduced recursion chains for the radial integrations,
     $\to$ at $k = 11$ loops: 225 975 instead of 39 916 800 terms

Our results as tests of AdS/CFT

Fiamberti, Santambrogio, C.S., Zanon: 0712.3522, 0806.2095

- four-loop result of $\mathcal{O}_4 = \text{tr}(\phi\,[\phi, Z]\,Z)$

$$\gamma_4 = -2496 + 576\zeta(3) - 1440\zeta(5)$$

- matches result from the string integrable model [Bajnok, Janik]
  now available also at five-loops [Bajnok, Hegedus, Janik, Lukowski]

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- five-loop result of $\mathcal{O}_5 = \mathrm{tr}(\phi\,[\phi, Z]\,ZZ)$

$$\gamma_5 = 6664 + 1152\zeta(3) + 3840\zeta(5) - 2240\zeta(7)$$

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Fiamberti, Santambrogio, C.S., Zanon: 0806.2103, 0811.4594

- $L \leq 11$ loop results of $\mathcal{O}_L = \mathrm{tr}(\phi Z \ldots Z)$ in β-deformed $\mathcal{N} = 4$ SYM
- match results from the string integrable model [Beccaria, De Angelis]

Conclusions and outlook

- perturbative computations important:
  first computation of the 4-loop anomalous dimension of the Konishi operator
- $\mathcal{N} = 1$ supergraphs is an efficient tool (finiteness theorems)
- refined tests at four and five loops
- wrapping increases transcendentality (degree of harmonic sums)
- all results confirm the duality and are in accord with the Y-system

- still non-trivial cancellations and simplifications
  $\longrightarrow$ more efficient formalism:
  required for calculations at higher orders and beyond the critical wrapping orders

Thank you!