Spontaneous Partial Supersymmetry Breaking in N=2 Supergravity and String Theory

Hagen Triendl

II. Institut für theoretische Physik
Universität Hamburg

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in collaboration with Jan Louis and Paul Smith
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Why are we interested in spontaneous $\mathcal{N} = 2 \to \mathcal{N} = 1$?

- $\mathcal{N} = 2$ supergravity in $4d$ is very restrictive. Which $\mathcal{N} = 1$ effective theories can be constructed from $\mathcal{N} = 2$ theories?
- For Minkowski vacua, spontaneous $\mathcal{N} = 2 \to \mathcal{N} = 1$ breaking is ruled out for supergravities constructed from superconformal tensor calculus. [Cecotti, Girardello, Porrati '84]

"Two into one won't go!"

- Ways to evade the no-go theorem have been found for simple examples, but the general picture remains unclear.

  [Ferrara, Girardello, Porrati '95; Fre, Girardello, Pesando, Trigiante '96]

- In *global* $\mathcal{N} = 2$ theories, spontaneous $\mathcal{N} = 2 \to \mathcal{N} = 1$ breaking requires electric and magnetic FI-terms [Antoniadis, Partouche, Taylor '95]

Which role do magnetic charges play in the local case?

Discussion in string theory

- Effective theories of type II flux compactifications are naturally $\mathcal{N} = 2$ gauged supergravities in $4d$
- Flux, torsion & non-geometric fluxes correspond to electric & magnetic gaugings in $4d$ [Polchinski, Strominger '95; Michelson '95]
- Another no-go theorem: [Gibbons '84; Maldacena, Nuñez '00]
  No stable Minkowski vacua in compactifications with flux/torsion in the absence of negative-energy sources
- In string theory, worldsheet instanton corrections might spoil the simple examples of spontaneous $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ breaking found in supergravity [Mayr '00]

What is the physical reason for no-go theorems?

How are these no-go theorems evaded?

When is spontaneous $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ (in Minkowski space) possible?

$\mathcal{N} = 2$ *gauged supergravity* *[de Wit, Lauwers and van Proeyen '85; Andrianopoli et al. '96]*

- **Gravity mult.:** 1 metric $g_{\mu\nu}$, 2 gravitini $\psi_{\mu\mathcal{A}}$, 1 vector A^0_μ
- **Vector mult.:** 1 vector A^i_μ, 2 fermions $\lambda^{i\mathcal{A}}$, 1 complex scalar t^i
- Scalars t^i parametrize *special Kähler manifold* $\mathcal{M}_\mathrm{v}$, characterized by a holomorphic prepotential $\mathcal{F}(t)$
- **Hyper mult.:** 2 fermions ζ_α, 4 real scalars q^u
- Scalars q^u parametrize *quaternionic-Kähler* manifold $\mathcal{M}_\mathrm{h}$, characterized by their $SU(2)$ curvature two-forms $K^x = \mathrm{d}\omega^x + \frac{1}{2}\epsilon^{xyz}\omega^y \wedge \omega^z$
- **Gauging** of isometries k_λ:

$$\partial_\mu q^u \to D_\mu q^u = \partial_\mu q^u - A_\mu{}^I \Theta_I{}^\lambda k^u_\lambda + B_{\mu I}\Theta^{I\,\lambda} k^u_\lambda$$

  Charges: $\Theta_I{}^\lambda$ (electric), $\Theta^{I\,\lambda}$ (magnetic) [de Wit, Samtleben, Trigiante '05]
- **Killing prepotentials:**

$$k^u_\lambda K^x_{uv} = \nabla P^x_\lambda$$

Partial super-Higgs mechanism *[Ferrara, Nieuwenhuizen '83]*

- One gravitino must become massive, forming an $\mathcal{N} = 1$ massive gravitino multiplet
- Thus, need at least one additional vector multiplet
- Super-Higgs must break $SU(2)$ R-symmetry, thus need at least one hypermultiplet

$$\left(2, \tfrac{3}{2}, \tfrac{3}{2}, 1\right) + \left(1, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0\right) + \left(\tfrac{1}{2}, \tfrac{1}{2}, 0, 0, 0, 0\right)$$

$$\Downarrow$$

$$\left(2, \tfrac{3}{2}\right) + \left(\tfrac{3}{2}\left(+\tfrac{1}{2}\right), 1\left(+0\right), 1\left(+0\right), \tfrac{1}{2}\right) + 2\left(\tfrac{1}{2}, 0, 0\right)$$

- Coupling of vectors to scalars via gaugings:
  need two isometries k_1 and k_2
- *Which charges realize spontaneous $\mathcal{N} = 2 \to \mathcal{N} = 1$ breaking?*

Supersymmetry variation of fermions

$$\begin{array}{lrl} \text{Gravitino:} & \delta_\epsilon \Psi_{\mu\mathcal{A}} = & D_\mu \epsilon_\mathcal{A} - S_{\mathcal{AB}} \gamma_\mu \epsilon^\mathcal{B} + \dots \\ \text{Gaugino:} & \delta_\epsilon \lambda^{i\mathcal{A}} = & W^{i\mathcal{AB}} \epsilon_\mathcal{B} + \dots \\ \text{Hyperino:} & \delta_\epsilon \zeta_\alpha = & N_\alpha^\mathcal{A} \epsilon_\mathcal{A} + \dots \end{array}$$

Conditions for $\mathcal{N} = 1$ Minkowski vacua ($D_\mu \epsilon_\mathcal{A} = 0$)

Lorentz-symmetry of vacuum implies:
$S_{\mathcal{AB}} \epsilon_1^\mathcal{B} = W^{i\mathcal{AB}} \epsilon_{1\,\mathcal{B}} = N_\alpha^\mathcal{A} \epsilon_{1\,\mathcal{A}} = 0$ and $S_{\mathcal{AB}} \epsilon_2^\mathcal{B} \neq 0$

$$\begin{array}{lcrll} S_{\mathcal{AB}} & = & V^\Lambda & \Theta_\Lambda{}^\lambda & P_\lambda^x (\sigma^x)_{\mathcal{AB}} \\ W^{i\mathcal{AB}} & = & g^{i\bar{\jmath}} (\nabla_{\bar{\jmath}} \bar{V}^\Lambda) & \Theta_\Lambda{}^\lambda & P_\lambda^x (\sigma^x)^{\mathcal{AB}} \\ N_\alpha^\mathcal{A} & = & \bar{V}^\Lambda & \Theta_\Lambda{}^\lambda & U_{\alpha u}^\mathcal{A} k_\lambda^u \end{array}$$

Electric/magnetic-covariant: $V^\Lambda = (X^I, \mathcal{F}_I)$; $\Theta_\Lambda{}^\lambda = (\Theta_I{}^\lambda, -\Theta^{I\,\lambda})$

“Two into one won’t go” *[Cecotti, Girardello, Porrati ’84]*

- Locality of the theory ensures that we can find an $Sp(n_{\rm v}+1)$-frame s.t. all charges are electric, i.e. $\Theta^{I\,\lambda}=0$.
- Let us use special coordinates s.t. $V^\Lambda = (1, t^i, \mathcal{F}_0(t), \mathcal{F}_i(t))$
- Then, $S_{\mathcal{A}\mathcal{B}}\epsilon_1^{\mathcal{B}} = W^{i\mathcal{A}\mathcal{B}}\epsilon_{1\,\mathcal{B}} = 0$ are equivalent to

$$\Theta_I{}^\lambda P^x_\lambda \sigma^x_{\mathcal{A}\mathcal{B}}\,\epsilon_1^{\mathcal{B}} = 0\ .$$

- Since $\Theta_I{}^\lambda P^x_\lambda$ is real, this is just an $su(2)$ variation of ϵ_1.
- Hence, the solution must fulfill $\Theta_I{}^\lambda P^x_\lambda = 0$, thus there is no $\mathcal{N}=1$ solution.

End of the story?

A way out

- $\mathcal{N} = 1$ solutions are possible if no special coordinates exist in the purely electric frame. [Ferrara, Girardello, Porrati '95]
- Drawback: No special coordinates means no prepotential, thus the tools of special geometry are not usable.
- Alternative description: By electric/magnetic-duality, one can use a frame *with* special coordinates but with both *electric* and *magnetic* charges.
- Then, $S_{\mathcal{AB}}\epsilon_1^{\mathcal{B}} = W^{i\mathcal{AB}}\epsilon_{1\,\mathcal{B}} = 0$ correspond to

$$(\Theta_I{}^{\lambda} - \mathcal{F}_{IJ}\Theta^{J\,\lambda})P^x_{\lambda}\sigma^x_{\mathcal{AB}}\,\epsilon_1^{\mathcal{B}} = 0\ .$$

- These linear equations can be easily solved for $\Theta_I{}^{\lambda}$ and $\Theta^{J\,\lambda}$.

Two into one can go!

- General solution:

$$\Theta_I{}^1 = -\,\mathrm{Im}\left(P_2(q_0)\,\mathcal{F}_{IJ}(t_0)\,C^J\right), \qquad \Theta^{I\,1} = -\,\mathrm{Im}\left(P_2(q_0)\,C^I\right),$$
$$\Theta_I{}^2 = \mathrm{Im}\left(P_1(q_0)\,\mathcal{F}_{IJ}(t_0)\,C^J\right), \qquad \Theta^{I\,2} = \mathrm{Im}\left(P_1(q_0)\,C^I\right),$$

  with $P_\lambda = P^x_\lambda\,(\epsilon_1^{\mathcal{A}}\sigma^x_{\mathcal{AB}}\epsilon_1^{\mathcal{B}})$ and C^I a complex vector.
- Locality:

$$\bar{C}^I\,(\mathrm{Im}\,\mathcal{F})_{IJ}(t_0)\,C^J = 0\,.$$

We find a solution

- where $\mathcal{F}$ does not exist in purely the electric frame,
- that can be constructed for *any* $\mathcal{M}_{\mathrm{v}} \times \mathcal{M}_{\mathrm{h}}$,
- that can be constructed at *any* point of $\mathcal{M}_{\mathrm{v}} \times \mathcal{M}_{\mathrm{h}}$.

Hyperino variation?

Example: special quaternionic-Kähler manifolds [Ferrara, Sabharwal '90]

- Special quaternionic-Kähler manifolds are fibrations over special Kähler manifolds ("c-map")
- They arise naturally in type II compactifications to $\mathcal{N} = 2$ in $4d$.
- They admit $2n_{\rm h} + 1$ shift isometries $k_{\tilde{\Lambda}}, k_{\tilde{\phi}}$ in the fibre obeying

$$[k_{\tilde{\Lambda}}, k_{\tilde{\Sigma}}] = \Omega_{\tilde{\Lambda}\tilde{\Sigma}} k_{\tilde{\phi}} \qquad \text{(Heisenberg algebra)}$$

  with $\Omega_{\tilde{\Lambda}\tilde{\Sigma}}$ being the $Sp(n_{\rm h})$-metric.
- Fibration structure singles out a certain $SU(2)$-frame in which also $N_\alpha^{\mathcal{A}} \epsilon_{1\,\mathcal{A}} = 0$ can be realized. [Cassani, Bilal '07]

Special quaternionic-Kähler manifolds and $\mathcal{N} = 1$

Solution:

$$\Theta_\Lambda{}^{\tilde{\Sigma}} = \mathrm{Re} \begin{pmatrix} \bar{\mathcal{F}}_{IJ} \bar{C}^J \, \mathcal{G}_{AB} D^B & \bar{\mathcal{F}}_{IJ} \bar{C}^J \, D^A \\ \bar{C}^I \, \mathcal{G}_{AB} D^B & \bar{C}^I \, D^A \end{pmatrix}$$

Locality:

$$\bar{C}^I \, (\mathrm{Im}\, \mathcal{F})_{IJ} \, C^J = 0$$

Commutativity:

$$\bar{D}^A \, (\mathrm{Im}\, \mathcal{G})_{AB} \, D^B = 0$$

Spontaneous $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ breaking

- can be realized for any $\mathcal{M}_\mathrm{v} \times \mathcal{M}_\mathrm{h}$
- can be realized at any point thereof.

String realisations

Solution:

$$\Theta_{\Lambda}{}^{\tilde{\Sigma}} = \mathrm{Re} \begin{pmatrix} \bar{\mathcal{F}}_{IJ} \bar{C}^J \, \mathcal{G}_{AB} D^B & \bar{\mathcal{F}}_{IJ} \bar{C}^J \, D^A \\ \bar{C}^I \, \mathcal{G}_{AB} D^B & \bar{C}^I \, D^A \end{pmatrix}$$

- Moduli space of hypermultiplet scalars in $SU(3) \times SU(3)$ structure compactifications of type II string is special quaternionic-Kähler.
- The stringy realisation of the solution for the charges always includes *"non-geometric fluxes"* and by this evades the Gibbons-Maldacena-Nuñez no-go theorem.
- The solution is completely mirror-symmetric.

String realisations

Solution:

$$\Theta_\Lambda{}^{\tilde{\Sigma}} = \mathrm{Re}\begin{pmatrix} \bar{\mathcal{F}}_{IJ}\bar{C}^J\,\mathcal{G}_{AB}D^B & \bar{\mathcal{F}}_{IJ}\bar{C}^J\,D^A \\ \bar{C}^I\,\mathcal{G}_{AB}D^B & \bar{C}^I\,D^A \end{pmatrix}$$

- Worldsheet instantons do not change these results because they just correct the holomorphic prepotentials $\mathcal{F}$ and $\mathcal{G}$ which are kept arbitrary in our analysis.
- Spacetime instantons break all *but* the gauged isometries.

  [Kashani-Poor, Tomasiello '05]
- Flux quantization might put serious constraints on the existence of $\mathcal{N} = 2 \to \mathcal{N} = 1$ breaking in string theory.

$\mathcal{N} = 1$ *AdS vacua*

- The same analysis can be done for AdS space, giving a general solution for $\mathcal{N} = 1$ vacua.
- In principle, one adds only some inhomogeneity to the equation coming from the gravitino variation.
- However, for the c-map case, the solution differs drastically since usually a different supersymmetry generator remains unbroken.
- In this way, the Minkowski solution forces the cosmological constant to vanish, without any fine-tuning.
- In contrast to the Minkowski case, one can realize $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ breaking without the need of non-geometric fluxes (as already known in the literature)

The N=1 low-energy effective theory

- By integrating out the massive gravitino multiplet one should obtain the $\mathcal{N} = 1$ effective action.
- Integrating out the massive gravitino multiplet corresponds to performing the quotient $\mathcal{M}_q = \mathcal{M}_{\mathrm{h}} / < k_1, k_2 >$
- It turns out that $\mathcal{M}_q$ is Kähler with Kähler two-form $K = \mathrm{d}\omega^x \epsilon_1^{\mathcal{A}} \sigma^x_{\mathcal{A}\mathcal{B}} \epsilon_2^{\mathcal{B}}$
- The Killing prepotentials give the holomorphic superpotential

$$W = \mathrm{e}^{-K/2} X^I (\Theta_I{}^\lambda - \mathcal{F}_{IJ} \Theta^{J\,\lambda}) P^x_\lambda \epsilon_1^{\mathcal{A}} \sigma^x_{\mathcal{A}\mathcal{B}} \epsilon_1^{\mathcal{B}}$$

  and the D-terms

$$D_i = (\nabla_i X^I)(\Theta_I{}^\lambda - \mathcal{F}_{IJ} \Theta^{J\,\lambda}) P^x_\lambda \epsilon_1^{\mathcal{A}} \sigma^x_{\mathcal{A}\mathcal{B}} \epsilon_2^{\mathcal{B}}$$

- Integrating out the massive graviphoton leads to a holomorphic gauge kinetic function.

Conclusions

- Spontaneous partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking in Minkowski space is possible for any $\mathcal{N} = 2$ moduli space $\mathcal{M}_{\rm v} \times \mathcal{M}_{\rm h}$ as long as two appropriate Killing vectors exist on $\mathcal{M}_{\rm h}$.
- Such Killing vectors exist for any special quaternionic manifold.
- In string realizations, flux quantization might put constraints on $\mathcal{M}_{\rm v} \times \mathcal{M}_{\rm h}$ to allow for $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ breaking.
- Similar story for $\mathcal{N} = 1$ AdS vacua, but the solution for charges is very different.
- The $\mathcal{N} = 2$ quantities descend to the usual $\mathcal{N} = 1$ quantities in the effective action.

Two into one can go!