

Formelsammlung

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta, \quad \sin(\alpha + \beta) = \cos \alpha \sin \beta + \sin \alpha \cos \beta,$$

$$e^{i\varphi} = \cos \varphi + i \sin \varphi, \quad (1+x)^\alpha = \sum_n \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!} x^n$$

$$\frac{d}{dx} x^n = n x^{n-1}, \quad \frac{d}{dx} \ln|x| = \frac{1}{x}, \quad \frac{d}{dx} e^x = e^x, \quad \frac{d}{dx} \sin x = \cos x, \quad \frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \cosh(x) = \sinh(x), \quad \frac{d}{dx} \sinh(x) = \cosh(x), \quad \frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

$$\text{Taylor-Formeln: } f(x) = \sum_{n=0}^{N-1} \frac{1}{n!} (x-x_0)^n \left(\frac{d}{dx}\right)^n f|_{x_0} + \frac{1}{N!} (x-x_0)^N \left(\frac{d}{dx}\right)^N f|_{\bar{x}}$$

$$f(x^1, x^2, \dots, x^n) = \sum_{l=0}^{N-1} \frac{1}{l!} (x^{i_1} - x_0^{i_1})(x^{i_2} - x_0^{i_2}) \dots (x^{i_l} - x_0^{i_l}) \partial_{i_1} \partial_{i_2} \dots \partial_{i_l} f|_{(x_0^1, x_0^2, \dots, x_0^n)} \\ + \frac{1}{N!} (x^{i_1} - x_0^{i_1})(x^{i_2} - x_0^{i_2}) \dots (x^{i_N} - x_0^{i_N}) \partial_{i_1} \partial_{i_2} \dots \partial_{i_N} f|_{(\bar{x}^1, \bar{x}^2, \dots, \bar{x}^n)}$$

$$\text{Kettenregeln: } \frac{d}{dx} f(g(x)) = \frac{dg}{dx} \frac{df}{dg}|_{g(x)}, \quad \frac{\partial z^i(y(x))}{\partial x^j} = \frac{\partial y^l(x)}{\partial x^j} \frac{\partial z^i(y)}{\partial y^l}|_{y(x)}$$

$$\int_{x\text{-Bereich}} d^n x f(x) = \int_{z\text{-Bereich}} d^n z \left| \det \frac{\partial x}{\partial z} \right| f(x(z)), \quad x\text{-Bereich} = x(z\text{-Bereich})$$

$$\text{Produktregeln: } \frac{d}{dx}(u \cdot v) = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}, \quad \int dx u \cdot \frac{dv}{dx} = u \cdot v \Big| - \int dx \frac{du}{dx} \cdot v$$

$$\text{Kugelkoordinaten: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r \sin \theta \cos \varphi \\ r \sin \theta \sin \varphi \\ r \cos \theta \end{pmatrix}, \quad \begin{aligned} \dot{\vec{r}} &= \dot{r} \vec{e}_r + \dot{\theta} r \vec{e}_\theta + \dot{\varphi} r \sin \theta \vec{e}_\varphi \\ \dot{r}^2 &= \dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2 \theta \dot{\varphi}^2, \\ dx dy dz &= r^2 dr d\cos \theta d\varphi \\ 0 \leq \theta &\leq \pi, \quad -1 \leq \cos \theta \leq 1 \end{aligned}$$

$$\text{Zylinderkoordinaten: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \rho \cos \varphi \\ \rho \sin \varphi \\ z \end{pmatrix}, \quad \begin{aligned} \dot{r}^2 &= \dot{\rho}^2 + \rho^2 \dot{\varphi}^2 + \dot{z}^2 \\ dx dy dz &= \rho d\rho d\varphi dz, \quad 0 \leq \varphi \leq 2\pi \end{aligned}$$

$$\text{Summationskonvention: } i \in \{1, 2, \dots, n\} \quad a_i b^i := a_1 b^1 + a_2 b^2 + \dots + a_n b^n$$

$$\text{Vektoren: } \vec{a} = \vec{e}_i a^i, \quad \text{Skalarprodukt: } \vec{e}_i \cdot \vec{e}_j = g(\vec{e}_i, \vec{e}_j) = g_{ij} \stackrel{\text{ONB}}{=} \delta_{ij}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \alpha = g(\vec{e}_i, \vec{e}_j) a^i b^j \stackrel{\text{ONB}}{=} \sum_i a^i b^i =: a^i b^i$$

$$\vec{a} = \vec{a}_\parallel + \vec{a}_\perp = \frac{(\vec{a} \cdot \vec{b})}{(\vec{b} \cdot \vec{b})} \vec{b} + \left(\vec{a} - \frac{(\vec{a} \cdot \vec{b})}{(\vec{b} \cdot \vec{b})} \vec{b} \right)$$

$$\dim = 3: \vec{a} \times \vec{b} = -(\vec{b} \times \vec{a}), \quad (\vec{a} \times \vec{b})^i = \varepsilon^{ijk} a^j b^k, \quad (\vec{a} \times \vec{b})^1 = a^2 b^3 - a^3 b^2, \quad \text{und zyklisch}$$

$$(\vec{a} \times \vec{b}) \perp \vec{a} \text{ und } \perp \vec{b}, |\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \alpha$$

$$\text{Vektorfelder: } (\text{grad } f)^i = \frac{\partial}{\partial x^i} f, (\text{rot } A)^i = \varepsilon^{ijk} \frac{\partial}{\partial x^j} A_k, \text{div } \vec{B} = \partial_x B^x + \partial_y B^y + \partial_z B^z$$

ε -Tensor: $\varepsilon_{i_1 i_2 \dots i_n} = \text{sign}(\pi) \varepsilon_{i_{\pi(1)} i_{\pi(2)} \dots i_{\pi(n)}}$, $i = 1, 2 \dots n$, antisymmetrisch, $\varepsilon_{12\dots n} = 1$
 $\dim = 3$:

$$\varepsilon_{ijk} \varepsilon_{lmn} = \delta_{il} \delta_{jm} \delta_{kn} + \delta_{im} \delta_{jn} \delta_{kl} + \delta_{in} \delta_{jl} \delta_{km} - \delta_{il} \delta_{jn} \delta_{km} - \delta_{in} \delta_{jm} \delta_{kl} - \delta_{im} \delta_{jl} \delta_{kn}$$

$$\varepsilon_{ijk} \varepsilon_{mnk} = \delta_{im} \delta_{jn} - \delta_{in} \delta_{jm}, \varepsilon_{ikl} \varepsilon_{jkl} = 2\delta_{ij}, \varepsilon_{ijk} \varepsilon_{ijk} = 6$$

Kronecker- δ : $a^i = \delta^i_j a^j$, $\delta^i_i = n = \text{Laufbereich}$

Matrizen: $(M \cdot N)^i_j = M^i_k N^k_j$, das heißt: Zeile \times Spalte, $(M^T)^i_j = M^j_i$

$$M^{i_1}_{j_1} M^{i_2}_{j_2} \dots M^{i_n}_{j_n} \varepsilon_{i_1 i_2 \dots i_n} = \varepsilon_{j_1 j_2 \dots j_n} \det M,$$

$$\det M = \det M^T, \det(M \cdot N) = \det M \cdot \det N, \det M^{-1} = \frac{1}{\det M},$$

$$\frac{\partial \det M}{\partial M^i_j} = \det M (M^{-1})^j_i, \text{Sp} M = M^i_i, \text{Sp}(A \cdot B) = \text{Sp}(B \cdot A), \text{Sp}(A \cdot B \cdot A^{-1}) = \text{Sp}(B)$$

$$\det(1 + N) = 1 + \text{Sp } N + O(N^2) \quad N^i_j = u^i w_j \Rightarrow \det(1 + N) = 1 + u^i w_i$$

Drehungen: $O^T = O^{-1}$, $O^i_k O^j_l \delta^{kl} = \delta^{ij}$, $\det O = 1$, $\dim = 3$: $\text{Sp } O = 2 \cos \varphi + 1$

$$D_x = \begin{pmatrix} 1 & & \\ & c & -s \\ & s & c \end{pmatrix}, D_y = \begin{pmatrix} c & s \\ & 1 & \\ -s & & c \end{pmatrix}, D_z = \begin{pmatrix} c & -s \\ s & c \\ & & 1 \end{pmatrix}$$

Eigenwerte: $M\vec{v} = \lambda\vec{v} \Leftrightarrow (M - \lambda\mathbf{1})\vec{v} = 0$, $\vec{v} \neq 0 \Rightarrow \det(M - \lambda\mathbf{1}) = 0$

$$M = M^* = M^T \Rightarrow M\vec{v}_i = \lambda_i \vec{v}_i, \vec{v}_i \cdot \vec{v}_j = \delta_{ij}, \lambda_i = \lambda_i^*$$

Eigenvektoren reelle Orthonormalbasis, Eigenwerte reell

$$\int dx e^{-x^2} = \sqrt{\pi}, \Gamma(s) = \int_0^\infty dt t^{s-1} e^{-t}, \Gamma(n+1) = n!, \Gamma(\frac{1}{2}) = \sqrt{\pi}, \Gamma(s+1) = s\Gamma(s)$$

$$\text{Volumen } S^{n-1}: 2\pi^{\frac{n}{2}} / \Gamma(\frac{n}{2})$$

Differentialformen: $dx^i \wedge dx^j = -dx^j \wedge dx^i$, $\omega = \frac{1}{p!} \omega_{i_1 \dots i_p}(x) dx^{i_1} \wedge \dots \wedge dx^{i_p}$, $i = 1 \dots D$

parametrisierte Untermannigfaltigkeit: $x^i(\lambda^1, \dots, \lambda^p)$, $dx^i = d\lambda^a \frac{\partial x^i}{\partial \lambda^a}$, $a = 1 \dots p \leq D$

$$dx^{i_1} \wedge \dots \wedge dx^{i_p} = \frac{\partial x^{i_1}}{\partial \lambda^{a_1}} \dots \frac{\partial x^{i_p}}{\partial \lambda^{a_p}} d\lambda^{a_1} \wedge \dots \wedge d\lambda^{a_p} = \frac{\partial x^{i_1}}{\partial \lambda^{a_1}} \dots \frac{\partial x^{i_p}}{\partial \lambda^{a_p}} \epsilon^{a_1 \dots a_p} d^p \lambda$$

$$\int_M \omega = \int_{\lambda\text{-Bereich}} d^p \lambda \epsilon^{a_1 \dots a_p} \frac{\partial x^{i_1}}{\partial \lambda^{a_1}} \dots \frac{\partial x^{i_p}}{\partial \lambda^{a_p}} \frac{1}{p!} \omega_{i_1 \dots i_p}(x(\lambda)), M = \{x(\lambda), \lambda \in \lambda\text{-Bereich}\}$$

Tangentialvektoren: $t_a^i = \frac{\partial x^i}{\partial \lambda^a}$, $a = 1 \dots p$,

$$\text{Metrik: } g_{ab} = \vec{t}_a \cdot \vec{t}_b, \text{Größe: } = \int_{\lambda\text{-Bereich}} d^p \lambda \sqrt{\det g..}$$

$$p = 1: \text{Weg } \Gamma: \int_\Gamma \omega = \int_{\lambda_a}^{\lambda_e} d\lambda \frac{dx^i}{d\lambda} \omega_i(x(\lambda)), \text{Arbeit: } \int_\Gamma d\vec{x} \cdot \vec{F}(\vec{x}) = \int_{\lambda_1}^{\lambda_2} d\lambda \frac{d\vec{x}}{d\lambda} \cdot \vec{F}(\vec{x}(\lambda))$$

Tangentialvektor $t^i = \frac{dx^i}{d\lambda}$, Weglänge $l = \int_{\lambda_1}^{\lambda_2} d\lambda \sqrt{t^2}$

$p = 2$: Fläche F : $\int_F \omega = \int_{\lambda\text{-Bereich}} d\lambda^1 d\lambda^2 \frac{1}{2} \left(\frac{\partial x^i}{\partial \lambda^1} \frac{\partial x^j}{\partial \lambda^2} - \frac{\partial x^j}{\partial \lambda^1} \frac{\partial x^i}{\partial \lambda^2} \right) \omega_{ij}(x(\lambda))$

$D = 3$: $\omega_{12} = -\omega_{21} = \Omega^3 \dots, \omega_{ij} = \epsilon_{ijk} \Omega^k$

$\int_F \omega = \int_{\lambda\text{-Bereich}} d\lambda^1 d\lambda^2 t_1^i t_2^j \Omega^k \epsilon_{ijk} = \int_{\lambda\text{-Bereich}} d\lambda^1 d\lambda^2 (\vec{t}_1 \times \vec{t}_2) \cdot \vec{\Omega} = \int_{\lambda\text{-Bereich}} d^2 \vec{f} \cdot \vec{\Omega}$

äußere Ableitung: $d = dx^i \frac{\partial}{\partial x^i}$, p -Form $\rightarrow (p+1)$ -Form

$\omega = \frac{1}{p!} \omega_{i_1 \dots i_p}(x) dx^{i_1} \wedge \dots \wedge dx^{i_p} \rightarrow d\omega = \frac{1}{p!} (\partial_{i_0} \omega_{i_1 \dots i_p}(x)) dx^{i_0} \wedge dx^{i_1} \wedge \dots \wedge dx^{i_p}$

$p = 0$: $df = \partial_i f dx^i = \text{grad } f \cdot d\vec{x}$

$p = 1 D = 3$: $d(\vec{A} \cdot d\vec{x}) = \text{rot } \vec{A} \cdot \vec{df} = (\text{rot } \vec{A})_x dy dz + (\text{rot } \vec{A})_y dz dx + (\text{rot } \vec{A})_z dx dy$

$p = 2 D = 3$: $d(\vec{B} \cdot \vec{df}) = d(B_x dy dz + B_y dz dx + B_z dx dy) = \text{div } \vec{B} d^3 x$

$d^2 = 0$, $\text{rot grad} = 0$, $\text{div rot} = 0$

sternförmig: $d\eta = 0 \Rightarrow \eta = d\omega$, $\text{rot } \vec{A} = 0 \Rightarrow \vec{A} = \text{grad } f$, $\text{div } \vec{B} = 0 \Rightarrow \vec{B} = \text{rot } \vec{C}$

Integralsätze: $\int_M d\omega = \int_{\partial M} \omega$

$p = 1$: $\int_{\text{Weg}} \text{grad } f \cdot d\vec{x} = f|_{\text{Randpunkte}}$

$p = 2 D = 3$: $\int_{\text{Fläche}} \text{rot } \vec{A} \cdot \vec{df} = \oint_{\text{Randkurve}} d\vec{x} \cdot \vec{A}$, Rechtsschraube in $d\vec{f}$ -Richtung

$p = 3 D = 3$: $\int_{\text{Volumen}} \text{div } \vec{B} d^3 x = \oint_{\text{Randfläche}} \vec{df} \cdot \vec{B}$, $d\vec{f}$ nach außen gerichtet

komplexe Funktionen: $z = x + iy$, $f(z) = u(x, y) + iv(x, y)$,

komplex differenzierbar: $df(z) = dz \frac{df}{dz} \Leftrightarrow (\partial_x u = \partial_y v, \partial_y u = -\partial_x v)$ Cauchy Riemann

$\int_{\Gamma} dz f(z) = \int_{\Gamma} (dx u - dy v) + i(dy u + dx v) = \int_{\lambda\text{-Bereich}} d\lambda ((\dot{x} u - \dot{y} v) + i(\dot{y} u + \dot{x} v))$

$\oint_{\Gamma} f(z) dz = 0$ falls f im umlaufenen Gebiet komplex differenzierbar,

$\oint_{\Gamma} f(z) dz = 2\pi i \sum_{z_i} \text{Res } f(z)$, Γ im Gegenzeigersinn, $f(z) = \sum_{l=-\infty}^{\infty} c_l (z - z_i)^l$, $\text{Res } f = c_{-1}$

δ -Funktion: $\int dx' \delta(x - x') t(x') = t(x)$, $\delta(ax) = \frac{1}{|a|} \delta(x)$, $\delta(f(x)) = \sum_{x_i: f(x_i)=0} \frac{\delta(x-x_i)}{|\frac{df}{dx}|}$

$\int dx \frac{d}{dx} \delta(x) t(x) = -\frac{d}{dx} t(0)$, $f(x) \delta(x) = f(0) \delta(x)$, $f(x) \frac{d}{dx} \delta(x) = f(0) \frac{d}{dx} \delta(x) - \frac{df}{dx}(0) \delta(x)$

$\theta(x) = \{0 \text{ falls } x < 0, 1 \text{ falls } x > 0\}$, $\frac{d}{dx} \theta(x) = \delta(x)$, $\int dk e^{ikx} = 2\pi \delta(x)$

$\lim_{\epsilon \rightarrow 0^+} \frac{1}{x+i\epsilon} = \text{P.V.} \frac{1}{x} - i\pi \delta(x)$, $\int dx \text{P.V.} \frac{1}{x} t(x) = \frac{1}{2} \int dx \frac{t(x)-t(-x)}{x}$

Entwicklung im Intervall: $I = \{x : -\frac{L}{2} \leq x \leq \frac{L}{2}\}$, Skalarprodukt $(f, g) = \int_I dx f^*(x) g(x)$

f_l vollst. ONSystem: $(f_k, f_l) = \delta_{kl}$, $g(x) = \sum_l f_l(x) (f_l, g)$, $(g, g) = \sum_l |(f_l, g)|^2 \forall g$

Fourierreihe: $f_l = \frac{1}{\sqrt{L}} e^{2\pi i l \frac{x}{L}}$ $l \in \mathbf{Z}$, $g(x) = \sum_{-\infty}^{\infty} \frac{1}{\sqrt{L}} e^{2\pi i l \frac{x}{L}} g_l$, $g_l = \int_{-\frac{L}{2}}^{+\frac{L}{2}} dx \frac{1}{\sqrt{L}} e^{-2\pi i l \frac{x}{L}} g(x)$

reelle Fourierreihe: $g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(2\pi n \frac{x}{L}) + \sum_{n=1}^{\infty} b_n \sin(2\pi n \frac{x}{L})$

$$a_n = \frac{2}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \cos(2\pi n \frac{x}{L}) \cdot g(x), \quad b_n = \frac{2}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \sin(2\pi n \frac{x}{L}) \cdot g(x)$$

Fourierdarstellung: $f(x) = \int dk \frac{1}{\sqrt{2\pi}} e^{ikx} \tilde{f}(k), \quad \tilde{f}(k) = \int dx \frac{1}{\sqrt{2\pi}} e^{-ikx} f(x)$

$$\widetilde{\frac{df}{dx}}(k) = ik \tilde{f}(k), \quad (\widetilde{f \cdot g})(k) = \int \frac{dp}{\sqrt{2\pi}} \tilde{g}(p) \tilde{f}(k-p)$$

partielle Differentialoperatoren: (x^i kartesische Koord.) $\Delta = \sum_{i=1}^3 \partial_{x^i} \partial_{x^i}, \quad \square = \partial_{x^0} \partial_{x^0} - \Delta$

$$\Delta = \partial_r^2 + \frac{2}{r} \partial_r + \frac{L^2}{r^2}, \quad L^2 = \frac{1}{\sin^2 \theta} \partial_\varphi^2 + \partial_\theta^2 + \cot \theta \partial_\theta, \quad \text{grad} = \vec{e}_r \partial_r + \vec{e}_\varphi \frac{1}{r \sin \theta} \partial_\varphi + \vec{e}_\theta \frac{1}{r} \partial_\theta$$

$$\Delta \phi(\vec{x}) = -4\pi \rho(\vec{x}), \quad \phi(\vec{x}) = \int d^3 y \frac{\rho(\vec{y})}{|\vec{x}-\vec{y}|}, \quad \Delta \frac{1}{|\vec{x}|} = -4\pi \delta^3(\vec{x})$$

$$(\partial_t - D \Delta) \phi(t, \vec{x}) = 0, \quad \phi(t, \vec{x}) = \frac{1}{\sqrt{4\pi D t^3}} \int d^3 y e^{-\frac{(\vec{x}-\vec{y})^2}{4Dt}} \phi(0, \vec{y}), \quad \vec{j} = -D \text{ grad} \phi$$

$$\text{Mittelwert: } M_{t,\vec{x}}[v] = \frac{1}{4\pi} \int_{\cos \theta = -1}^{\cos \theta = +1} d \cos \theta \int_0^{2\pi} d\varphi v(\vec{x} + t \vec{n}(\cos \theta, \varphi))$$

$$\square \phi(t, \vec{x}) = 4\pi \rho(t, \vec{x}) \quad \phi(0, \vec{x}) = u(\vec{x}) \quad \dot{\phi}(0, \vec{x}) = v(\vec{x}) \quad c = 1$$

$$\phi(t, \vec{x}) = t M_{t,\vec{x}}[v] + \frac{\partial}{\partial t} (t M_{t,\vec{x}}[u]) + \int_{K_{|t|,\vec{x}}} d^3 y \frac{\rho(t - \text{sign}(t)|\vec{x}-\vec{y}|, \vec{y})}{|\vec{x}-\vec{y}|}, \quad K_{r,\vec{x}} = \{\vec{y} : |\vec{x}-\vec{y}| \leq |r|\}$$

$$\phi_{\text{ret}}(x^0, \vec{x}) = \int d^3 y \frac{\rho(x^0 - |\vec{x}-\vec{y}|, \vec{y})}{|\vec{x}-\vec{y}|} = 2 \int d^4 y \theta(x^0 - y^0) \delta((x-y)^2) \rho(y)$$

$$\square \frac{1}{2\pi} \theta(x^2) \delta(x^2) = \delta^4(x^0, \vec{x}), \quad x^2 = (x^0)^2 - \vec{x}^2$$

Relativitätstheorie: $a = (a^0, a^1, a^2, a^3), \quad a \cdot b = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 = a^m b^n \eta_{mn}$

$m \neq n : \eta_{mn} = 0, \quad \eta_{00} = 1, \quad \eta_{11} = \eta_{22} = \eta_{33} = -1, \quad \eta_{mn} = \eta^{mn}, \quad a_m = \eta_{mn} a^n, \quad a^n = \eta^{nr} a_r$

Lorentztransformation: $x \rightarrow x', \quad x'^m = \Lambda^m_n x^n, \quad x' \cdot y' = x \cdot y, \quad \Lambda^m_k \Lambda^n_l \eta_{mn} = \eta_{kl}$

kontragrediente Transformation: $\partial'_n = \frac{\partial}{\partial x'^n} = \left(\Lambda^{-1T} \right)_n^m \frac{\partial}{\partial x^m} = \Lambda_n^m \partial_m, \quad \Lambda^{-1T} = \eta \Lambda \eta^{-1}$

$$\Lambda_{2 \times 2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \begin{pmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{pmatrix}, \quad p = \begin{pmatrix} \frac{E}{c} \\ \vec{p} \end{pmatrix} = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}} \begin{pmatrix} c \\ \vec{v} \end{pmatrix}, \quad p^2 = m^2 c^2, \quad \frac{dx^m}{d\tau} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \begin{pmatrix} c \\ \vec{v} \end{pmatrix}$$

Vektorfeld: $j'^m(x'(x)) = \Lambda^m_n j^n(x), \quad A'^m(x'(x)) = \Lambda^m_n A^n(x)$

Tensorfeld: $F'^{mn}(x'(x)) = \Lambda^m_k \Lambda^n_l F^{kl}(x)$

Wirkung: $W[q] = \int dt \mathcal{L}(q(t), \dot{q}(t), t), \quad \text{Energie: } E = \sum E_n(q) \dot{q}^n \Rightarrow \mathcal{L} = \sum \frac{1}{n-1} E_n(q) \dot{q}^n$

Euler-Lagrange-Ableitung: $\frac{\delta \mathcal{L}}{\delta q^m} = \partial_{q^m} \mathcal{L} - \frac{d}{dt} \partial_{\dot{q}^m} \mathcal{L}, \quad \text{Euler-Lagrange-Gleichung: } \frac{\delta \mathcal{L}}{\delta q^m} = 0$

Symmetrie und Erhaltungsgröße: $\delta q^m \frac{\delta \mathcal{L}}{\delta q^m} + \frac{d}{dt} Q = 0$

$\delta q^m \partial_{q^m} \mathcal{L} + \left(\frac{d}{dt} \delta q^m \right) \partial_{\dot{q}^m} \mathcal{L} + \frac{d}{dt} K(q, \dot{q}, t) = 0$ Erhaltungsgröße: $Q = \delta q^m \partial_{\dot{q}^m} \mathcal{L} + K$

$$q^1 \text{ zyklisch: } \frac{\partial \mathcal{L}}{\partial q^1} = 0 \Rightarrow \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}^1} = 0, \quad E = \dot{q}^i \frac{\partial \mathcal{L}}{\partial \dot{q}^i} - \mathcal{L}, \quad \partial_t \mathcal{L} = 0 \Rightarrow \frac{dE}{dt} = 0$$

$$\text{eindim. Bew.: } E = \frac{1}{2} m \dot{x}^2 + V(x), \quad t(x) - t(x_0) = \int_{x_0}^x \frac{dx'}{\sqrt{\frac{2}{m}(E - V(x'))}}, \quad \omega_{\text{klein}}^2 = \frac{V''(\bar{x})}{m}$$

$$\text{harmonischer Oszillator: } V(x) = \frac{1}{2} \kappa x^2, \quad \ddot{x} + \omega^2 x = 0, \quad \omega^2 = \frac{\kappa}{m}, \quad x(t) = A \cos(\omega t + \varphi)$$

$$\text{gekoppelte Osz.: } \ddot{x}^i + (\Omega^2)^{ij} x^j = 0, \quad (\Omega^2 - \omega_j^2) n_{(j)} = 0, \quad x^i(t) = \sum_j a_j n_{(j)}^i \cos(\omega_j t + \varphi_j)$$

$$\text{Virialsatz: } V(\lambda \vec{r}) = \lambda^n V(\vec{r}), \quad \text{skalierte Bahn: } \vec{r}_\lambda(t) = \lambda \vec{r}(\lambda^{\frac{n}{2}-1} t), \quad 2\bar{E}_{\text{kin}} = n\bar{E}_{\text{pot}}$$

$$\text{Kepler: } V(\vec{r}) = -\frac{\alpha}{|\vec{r}|}, \quad r(\varphi) = \frac{a(1-e^2)}{1+e \cos \varphi}, \quad e = \sqrt{1 + \frac{2EL^2}{m\alpha^2}}, \quad \frac{T^2}{(2\pi)^2} = \frac{a^3}{G_N(m_{\text{Sonne}} + m_{\text{Planet}})}$$

$$\text{Nichtinertialsystem: } \vec{x}_{\text{inertial}} = \vec{R}_{\text{Ursprung}} + O \vec{r}_{\text{Labor}}, \quad \dot{O} = O \Omega, \quad \Omega = -\Omega^T, \quad \Omega \vec{r} = \vec{\Omega} \times \vec{r}$$

$$\mathcal{L} = \frac{1}{2} m \dot{\vec{r}}^2 + m \vec{\Omega} \cdot (\vec{r} \times \dot{\vec{r}}) + \frac{1}{2} m (\vec{\Omega} \times \vec{r})^2 - O^{-1} \vec{R} \cdot \vec{r} - \hat{V}(r)$$

$$m \ddot{\vec{r}} = -\text{grad } \hat{V} - m O^{-1} \ddot{\vec{R}} - 2m \Omega \times \dot{\vec{r}} - m \dot{\Omega} \times \vec{r} - m \vec{\Omega} \times (\vec{\Omega} \times \vec{r})$$

$$\text{starrer Körper: } \vec{x}_\alpha = \vec{R} + O \vec{r}_\alpha, \quad \dot{\vec{x}}_\alpha = \dot{\vec{R}} + O(\vec{\Omega} \times \vec{r}_\alpha), \quad \vec{\omega} = O \vec{\Omega}, \quad \vec{y}_\alpha = O \vec{r}_\alpha$$

$$\text{Masse: } M = \int d^3x \rho(x), \quad \text{Schwerpunkt: } \vec{R} = \frac{1}{M} \int d^3x \rho(x) \vec{x}, \quad \text{Impuls: } \vec{P} = M \dot{\vec{R}}$$

$$\text{Trägheitstensor: } \theta^{ij} = \int d^3x \rho(x) (\delta^{ij} \vec{x}^2 - x^i x^j), \quad \theta^{ij} = O^i_k O^j_l \vartheta^{kl}, \quad \theta^{11} + \theta^{22} \geq \theta^{33}$$

$$E_{\text{kin}} = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2} \vec{\omega} \cdot \theta \vec{\omega} = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2} \vec{\Omega} \cdot \vartheta \vec{\Omega}, \quad \vec{L} = \vec{R} \times \vec{P} + \theta \vec{\omega} = \vec{R} \times \vec{P} + O(\vartheta \vec{\Omega})$$

$$\frac{d\vec{P}}{dt} = \sum_\alpha \vec{F}_\alpha, \quad \frac{d\vec{L}}{dt} = \sum_\alpha \vec{x}_\alpha \times \vec{F}_\alpha, \quad \text{freier starrer Körper: } \theta_1 \dot{\Omega}_1 = (\theta_2 - \theta_3) \Omega_2 \Omega_3$$

$$\text{konjugierter Impuls: } p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}^i}, \quad \text{Hamiltonfunktion: } \mathcal{H}(q, p, t) = p_j \dot{q}^j(q, p) - \mathcal{L}(q, \dot{q}(q, p), t)$$

$$\text{Hamiltonsche G.: } \dot{q}^i = \frac{\partial \mathcal{H}}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial \mathcal{H}}{\partial q^i}, \quad \text{Poissonklammer: } \{A, B\} = \frac{\partial A}{\partial q^i} \frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i} \frac{\partial B}{\partial q^i}$$

$$\{aA + bB, C\} = a\{A, C\} + b\{B, C\}, \quad \{A, B\} = -\{B, A\}, \quad \{A, BC\} = \{A, B\}C + B\{A, C\}$$

$$\partial \{A, B\} = \{\partial A, B\} + \{A, \partial B\}, \quad 0 = \{A, \{B, C\}\} + \{B, \{C, A\}\} + \{C, \{A, B\}\}$$

$$\text{Erhaltungsgrößen: } \frac{d}{dt} \phi = \{\phi, \mathcal{H}\} + \partial_t \phi = 0, \quad \frac{d\phi_1}{dt} = 0 = \frac{d\phi_2}{dt} \Rightarrow \frac{d\{\phi_1, \phi_2\}}{dt} = 0$$

$$\text{kanonische, inf. Transformation: } \delta q^i = \{q^i, \phi\}, \quad \delta p_i = \{p_i, \phi\}, \quad \text{Symmetrie} \Leftrightarrow \frac{d}{dt} \phi = 0$$

$$\text{flächentreue Entwicklung: } \omega(t) = dq^i \wedge dp_i, \quad q^i(\lambda_1, \lambda_2, t), p_i(\lambda_1, \lambda_2, t) \Rightarrow \partial_t \omega = 0,$$

$$\text{Liouville: } \partial_t \omega^n = 0$$

$$\text{Maxwell-Gleichungen: } \text{div } \vec{B} = 0, \quad \text{rot } \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0, \quad \text{div } \vec{E} = 4\pi \rho, \quad \text{rot } \vec{B} - \frac{\partial}{\partial t} \vec{E} = \frac{4\pi}{c} \vec{j}$$

$$\vec{B} = \text{rot } \vec{A}, \quad \vec{E} = -\text{grad } \phi - \frac{\partial}{\partial t} \vec{A}, \quad \text{Eichtransformation: } \vec{A}' = \vec{A} - \text{grad } \chi, \quad \phi' = \phi + \frac{\partial}{\partial t} \chi$$

$$\text{Lorenz-Eichung: } \frac{\partial}{\partial t} \phi + \text{div } \vec{A} = 0 \Leftrightarrow \partial_m A^m = 0, \quad x^m = (ct, \vec{x}), \quad A^m = (\phi, \vec{A}), \quad j^m = (c\rho, \vec{j})$$

$$\text{Kontinuitätsgleichung (lokale Ladungserhaltung): } \partial_t \rho + \text{div } \vec{j} = 0 \Leftrightarrow \partial_m j^m = 0$$

kovariante Beschreibung: $F_{mn} = -F_{nm}$, $F_{0i} = E^i$, $F_{ij} = -\epsilon_{ijk} B^k$, $F^{mn} = \eta^{mk} \eta^{nl} F_{kl}$

$$\partial_m F^{mn} = \frac{4\pi}{c} j^n, \quad \partial_k F_{lm} + \partial_l F_{mk} + \partial_m F_{kl} = 0$$

$F_{mn} = \partial_m A_n - \partial_n A_m$ ist eichinvariant: $A'_m = A_m + \partial_m \chi \rightarrow F'_{mn} = F_{mn}$

$$\square A^m = \frac{4\pi}{c} j^m, \quad A^m(x^0, \vec{x}) = \frac{1}{c} \int d^3 y \frac{j^m(x^0 - |\vec{x} - \vec{y}|, \vec{y})}{|\vec{x} - \vec{y}|} + \int \frac{d^3 k}{2k^0} \{ a^m(\vec{k}) e^{-i k x} + a^{m*}(\vec{k}) e^{i k x} \}$$

$$k^0 = |\vec{k}|, \quad k^0 a^0(\vec{k}) - \vec{k} \vec{a}(\vec{k}) = 0, \quad k x = k^0 x^0 - \vec{k} \cdot \vec{x}$$

Multipolmomente: $Q = \int d^3 x \rho(\vec{x})$, $\vec{d} = \int d^3 x \vec{x} \rho(\vec{x})$, $Q^{ij} = \int d^3 x (3x^i x^j - \delta^{ij} \vec{x}^2) \rho(\vec{x})$

Energiedichte: $\frac{1}{8\pi} (\vec{E}^2 + \vec{B}^2)$ Poynting-Vektor: $\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{B}$

$$\text{Energiesatz: } \frac{\partial}{\partial t} \left[\frac{1}{8\pi} (\vec{E}^2 + \vec{B}^2) \right] + \text{div} \left[\frac{c}{4\pi} \vec{E} \times \vec{B} \right] + \vec{j} \cdot \vec{E} = 0$$

$$\text{Impuls: } \frac{\partial}{\partial t} \left[\frac{1}{4\pi c} \vec{E} \times \vec{B} \right]^j + \left(\frac{-1}{4\pi} \right) \partial_i \left[E^i E^j + B^i B^j - \frac{1}{2} \delta^{ij} (\vec{E}^2 + \vec{B}^2) \right] = - \left[\rho \vec{E} + \frac{1}{c} \vec{j} \times \vec{B} \right]^j$$

Energie-Impuls-T.: $T_{\text{Feld}}^{mn}(x) = -\frac{1}{4\pi} (F^{ml} F^n_l - \frac{1}{4} \eta^{mn} F^{kl} F_{kl})$, Lorentzk.: $\vec{F} = q(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$

ebene elektromagnetische Welle: $\vec{E}(t, \vec{x}) = \Re \vec{E}_0 e^{i(\vec{k} \vec{x} - \omega t)}$, $\vec{B} = \frac{\vec{k}}{|\vec{k}|} \times \vec{E}(t, \vec{x})$

$\frac{\omega}{c} = |\vec{k}|$, $\vec{k} \cdot \vec{E}_0 = 0$, $\vec{S} = \frac{c}{4\pi} \frac{\vec{k}}{|\vec{k}|} |\vec{E}(t, \vec{x})|^2$, \vec{S} , \vec{E} , \vec{B} orthogonales Rechtssystem

$$\vec{k} = \frac{\omega}{c} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} : \vec{E}_0 = \frac{|\vec{E}_0|}{\sqrt{1+a^2}} \begin{pmatrix} 1 \\ i a \\ 0 \end{pmatrix}, \quad -1 \leq a \leq 1, \text{ linear: } a = 0, \text{ zirkular: } a = \pm 1$$

skalare Kugelwelle: $\phi(t, \vec{x}) = \Re \frac{\phi_0}{|\vec{x}|} e^{i(k|\vec{x}| - \omega t)}$, $k = \frac{\omega}{c}$

Punktladung: Weltlinie $r^m(s) = (ct(s), \vec{r}(s))$, $j^m(x) = cq \int ds \frac{dr^m}{ds} \delta^4(x - r(s))$

Liènard-Wiechert: $A_{\text{ret}}^m(x) = q \frac{\frac{dr^m}{ds}}{\frac{dr^n}{ds} \cdot (x_n - r_n)} \Big|_{s = \bar{s}(x) : (x - r(\bar{s}))^2 = 0, x^0 > r^0(\bar{s})}$

Energieimpulstensor: $T^{mn}(x) = \int ds mc^2 \frac{\dot{r}^m \dot{r}^n}{\sqrt{\dot{r}^2}} \delta^4(x - r(s))$, $P^m = \frac{1}{c} \int d^3 x T^{0m}$

Bewegungsgleichung ohne Strahlungsrückwirkung: $mc \frac{d}{ds} \frac{\dot{r}^m}{\sqrt{\dot{r}^2}} = \frac{q}{c} F^{mn} \dot{r}_n$

Hertzscher Dipol: Fernzone $\vec{B}(t, \vec{x}) = \frac{1}{|\vec{x}|} \frac{1}{c^2} \frac{d^2}{dt^2} \vec{d}(t - \frac{|\vec{x}|}{c}) \times \frac{\vec{x}}{|\vec{x}|}$, $\vec{E} = \vec{B} \times \frac{\vec{x}}{|\vec{x}|}$

elektrische Dipolstrahlung: $\vec{S}(t, \vec{x}) = \frac{c}{4\pi} \frac{\vec{x}}{|\vec{x}|^3} \sin^2 \theta \left| \frac{1}{c^2} \frac{d^2}{dt^2} \vec{d}(t - \frac{|\vec{x}|}{c}) \right|^2$, $\theta = \theta(\vec{x}, \frac{d^2}{dt^2} \vec{d})$