DISCRETE SYMMETRIES

Besides the Poincaré group, i.e. translations and transformations from the group $SO(1, 3)$, there are three more, rather important transformations: Parity $\mathcal{P}$, Charge Conjugation $\mathcal{C}$ and Time Reversal $\mathcal{T}$.

[H1] Parity Violation [3 pts]
Show explicitly, that the Fermi-Lagrangian for the weak interaction,

$$\mathcal{L}_{\text{Fermi}} = G \bar{\psi}_1 \gamma^\mu \psi_2 \bar{\psi}_3 \gamma^\mu \psi_4$$

violates parity. Here, $\psi_i$, $i = 1, 2, 3, 4$, denote four different Dirac fields.

[H2] Charge Conjugation [3 pts]
The definition of charge conjugation $\mathcal{C}$ fixes the matrix $C$ only up to a multiplicative constant, since with any spinor, which solves the Dirac equation, any multiple of it also does. Show, however, that this constant is fixed by the condition $(\psi_c)_c = \psi$, i.e. by the condition that charge conjugation is an involution.

[H3] Charge Conjugation and Chirality [3 pts]
Show that the charge conjugate field of a left-chiral field is right-chiral and vice versa.

[H4] An Interesting Lorentz scalar [3 pts]
Show that $\psi C \psi$ is a Lorentz scalar.

[H5] Dirac Equation in low-dimensional space-times [5+10 pts]
It is often useful, to consider the Dirac equation in low space-dimensions.

i. Find the Dirac equation in a $(1+1)$-dimensional space-time. To do so, ask yourself, how many components a spinor in a space-time with just one space dimension can possess.

ii. Find the Dirac equation in a $(1+2)$-dimensional space-time. Show that the apparently so innocent mass term breaks invariance under both, parity as well as time reversal. Hint: The three gamma matrices are in this case simply the three Pauli matrices multiplied with suitable chosen factors of the imaginary unit $i$. 