

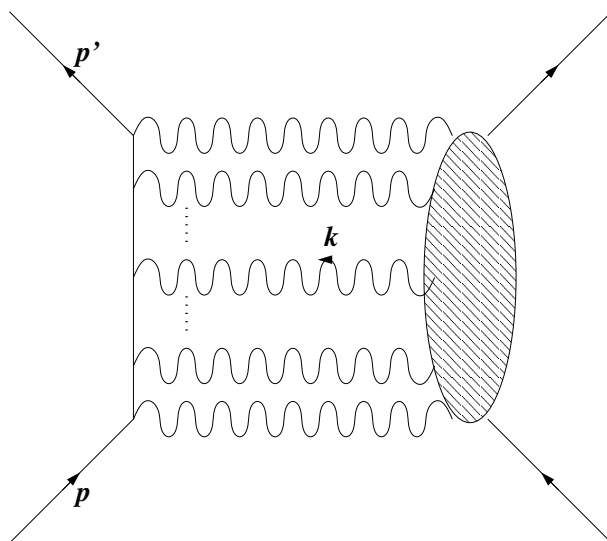
DIAGRAMMATIC PROOF OF GAUGE INVARIANCE

In the lecture, we sketched the proof of gauge invariance. Here, we attempt to close some of the gaps we left open in the lecture. The true gluttons are strongly encouraged to go through the explicit calculations for some specific diagrams.

[H1] *Spectator Photons*

[10 pts]

It was argued in the lecture that it does not matter how many additional photons watch the one photon we consider. Perform the proof in the manner sketched in the lecture explicitly for diagrams of the form



with a total of  $n + 1$  Photon lines. *Hint:* In order to avoid that you lose track of things: We did the case  $n = 1$  explicitly in the lecture.

[H2] *Divergent terms*

[10 pts]

In the lecture, I claimed that one may simply shift the variable of integration in the integral over the momentum in the internal Fermion loop. Of course, this is only true if the integral does exist at all. Check for the case considered in the lecture whether such a shift of the variable of integration is allowed. To do so, look at the integral of the type

$$\int \frac{d^4p}{(2\pi)^4} \text{tr} \left( \gamma^\mu \frac{1}{\not{p}_2 - m} \gamma^\sigma \frac{1}{\not{p}_1 - m} \gamma^\lambda \frac{1}{\not{p}' - m} \right),$$

make the denominator rational and show then, under the assumption that one takes the trace, that the integral does diverge at most logarithmically. It can be shown that for logarithmically divergent integrals a shift of the variable of integration is allowed.