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Generalizations of the supersymmetric t–J model*

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ABSTRACT

Families of integrable vertex models based on higher dimensional representations of the superalgebra $gl(2|1)$ are constructed. Their solutions by means of several algebraic Bethe Ansätze is presented and applications to problems of correlated electrons in one spatial dimension are briefly discussed.

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1 Introduction

Spawned by the recent interest in a better understanding of the properties of low dimensional correlated electronic systems, integrable vertex models build from low dimensional representations fo the graded Lie algebra $gl(2|1)$ have attracted increased attention recently [1–6]. The different grading of states in these models translates naturally to the fermionic (bosonic) signatures of singly occupied (empty or doubly occupied) states in electronic models. The most prominent electronic model of this type belongs to the class of t–J models

$$\mathcal{H}_{tJ} = -t\mathcal{P}\sum_{j=1}^{L}\sum_{\sigma=\uparrow,\downarrow}\left(c_{j\sigma}^{\dagger}c_{j+1,\sigma} + c_{j+1,\sigma}^{\dagger}c_{j\sigma}\right)\mathcal{P} + J\sum_{j=1}^{L}\left(\mathbf{S}_j\cdot\mathbf{S}_{j+1} - \frac{1}{4}n_j n_{j+1}\right) . \tag{1.1}$$

Here $c_{j\sigma}$ ($c_{j\sigma}^{\dagger}$) are annihilation (creation) operators for spin-σ at site j, $n_j = \sum_\sigma c_{j\sigma}^{\dagger}c_{j\sigma}$ is the number operator and $\mathcal{P}$ projects onto the subspace without double occupancies.

The model (1.1) is soluble by means of the Bethe Ansatz [7–9] for $J = 2t = 2$ where the Hamiltonian is supersymmetric [10, 11]. For these values of the parameters the Hamiltonian can be written in terms of graded permutation operators

$$\mathcal{H}_{stJ} = L - 2\hat{N} - \sum_{j=1}^{L}\Pi_{j,j+1} . \tag{1.2}$$

Here Π acts on the tensor product of two three dimensional spaces spanned by the states $\{|0\rangle, |\uparrow\rangle, |\downarrow\rangle\}$ with grading $[\epsilon_0, \epsilon_\uparrow, \epsilon_\downarrow] = [0, 1, 1]$ as

$$\Pi\,|a\rangle\otimes|b\rangle = (-1)^{\epsilon_a\epsilon_b}|b\rangle\otimes|a\rangle . \tag{1.3}$$

Written as in (1.2) the Hamiltonian bears an obvious similarity to that of the $S = \frac{1}{2}$ Heisenberg spin chain, hence various generalizations of this model which preserve the integrability are feasible: q-deformation of the symmetry leads to generalizations of the t–J model first studied by Bariev [12]. Systems with different symmetries show new mechanisms for superconductivity in systems of correlated electrons, see e.g. the $gl(2|2)$ supersymmetric Hubbard model [13].

Here we follow a different route, namely we study vertex models built from higher dimensional representations of the symmetry algebra of the $gl(2|1)$, similar as the approach leading to the family of $SU(2)$-symmetric higher spin XXX models studied by Takhtajan and Babujian [14–16]. In the following section we give a brief overview on the representations of the algebra $gl(2|1)$. In Section 3 we introduce some integrable vertex models and discuss their solution by means of the algebraic Bethe Ansatz. Applications to quantum systems of correlated electrons are given in Section 4.

2 Representations of $gl(2|1)$

The nine generators of $gl(2|1)$ are classified into even (1, S^z, $S^\pm$, B) and odd ($V_\pm$, $W_\pm$) ones depending on their parity w.r.t. grading. The even generators are $SU(2)$ spin operators S^α

satisfying commutation relations $[S^z, S^\pm] = \pm S^\pm$, $[S^+, S^-] = 2S^z$ and the $U(1)$ charge B which commutes with the S^α: $[B, S^\pm] = [B, S^z] = 0$. From the commutators between the even and odd generators

$$\begin{aligned} [S^z, V_\pm] &= \pm\frac{1}{2}V_\pm, \quad [S^\pm, V_\pm] = 0, \quad [S^\mp, V_\pm] = V_\mp, \quad [B, V_\pm] = \frac{1}{2}V_\pm, \\ [S^z, W_\pm] &= \pm\frac{1}{2}W_\pm, \quad [S^\pm, W_\pm] = 0, \quad [S^\mp, W_\pm] = W_\mp, \quad [B, W_\pm] = -\frac{1}{2}W_\pm, \end{aligned} \tag{2.1}$$

it is clear that $W_\pm$ *adds* a spin-$\frac{1}{2}$ particle with $m = \pm\frac{1}{2}$ and $V_\pm$ *removes* a particle with $m = \mp\frac{1}{2}$. In the electronic representations of the generators introduced below we shall express them in terms of anticommuting creation and annihilation operators leading to $W_\sigma \propto c_\sigma^\dagger$ and $V_\sigma \propto c_{-\sigma}$.

The remaining *anti*commutators between odd generators are

$$\begin{aligned} &\{V_\pm, V_\pm\} = \{V_\pm, V_\mp\} = \{W_\pm, W_\pm\} = \{W_\pm, W_\mp\} = 0 \ , \\ &\{V_\pm, W_\pm\} = \pm S^\pm \ , \quad \{V_\pm, W_\mp\} = -S^z \pm B \ . \end{aligned} \tag{2.2}$$

There exists an automorphism of the algebra:

$$\begin{aligned} S^z &\to S^z \qquad S^\pm \to S^\pm \\ B &\to -B \qquad V_\pm \leftrightarrow W_\pm \end{aligned} \tag{2.3}$$

The typical representations of $gl(2|1)$ are characterized by the eigenvalues of B and $\mathbf{S}^2$ and labeled $[b, s]$. They are built up from four $SU(2)$ multiplets (see e.g. [17, 18])

$$\begin{gathered} \left(b + \tfrac{1}{2}, s - \tfrac{1}{2}, m = -s + \tfrac{1}{2}, \ldots, s - \tfrac{1}{2}\right), \\ (b, s, m = -s, \ldots, s)\,, \\ (b, s - 1, m = -s + 1, \ldots, s - 1)\,, \\ \left(b - \tfrac{1}{2}, s - \tfrac{1}{2}, m = -s + \tfrac{1}{2}, \ldots, s - \tfrac{1}{2}\right). \end{gathered} \tag{2.4}$$

Hence their dimension is $8s$.

The simplest example is the four dimensional representation obtained for $s = \frac{1}{2}$: In this case the third multiplet in (2.4) is missing and the basis states can be identified with that of a single electronic orbital as follows:

$$\begin{gathered} |b + \tfrac{1}{2}, s = 0, m = 0\rangle = |0\rangle \\ |b, s = \tfrac{1}{2}, m = \tfrac{1}{2}\rangle = |\uparrow\rangle \qquad |b, s = \tfrac{1}{2}, m = -\tfrac{1}{2}\rangle = |\downarrow\rangle \\ |b - \tfrac{1}{2}, s = 0, m = 0\rangle = |\uparrow\downarrow\rangle \end{gathered} \tag{2.5}$$

The generators in this "electronic" representation can be written in terms of fermionic creation and annihilation operators as $S^\alpha = \frac{1}{2}c_a^\dagger \sigma_{ab}^\alpha c_b$ with Pauli matrices σ^α, $B = b + \frac{1}{2} - \frac{1}{2}n$ and

$$\begin{aligned} V_+ &= \left[\sqrt{b + \frac{1}{2}}\,(1 - n_\uparrow) + \sqrt{b - \frac{1}{2}}\,n_\uparrow\right]c_\downarrow\,, \\ V_- &= -\left[\sqrt{b + \frac{1}{2}}\,(1 - n_\downarrow) + \sqrt{b - \frac{1}{2}}\,n_\downarrow\right]c_\uparrow\,, \end{aligned}$$

$$
\begin{aligned}
W_+ &= \left[\sqrt{b+\frac{1}{2}}\,(1-n_\downarrow) + \sqrt{b-\frac{1}{2}}\,n_\downarrow\right] c_\uparrow^+ \,, \\
W_- &= \left[\sqrt{b+\frac{1}{2}}\,(1-n_\uparrow) + \sqrt{b-\frac{1}{2}}\,n_\uparrow\right] c_\downarrow^+ \,.
\end{aligned}
$$

Later we will restrict ourselves to unitary representations in order to construct systems with hermitean Hamiltonian. From the above equations for the odd generators this implies $|b| \geq \frac{1}{2}$.

In addition to the typical representations (2.4) the algebra $gl(2|1)$ has various *atypical* ones. In particular the three dimensional representation $\left[\frac{1}{2}\right]_+$ spanning the local Hilbert space of the t–J model (1.1) does not correspond to any of the above ones. However, taking the limit $b \to +\frac{1}{2}$ one finds that e.g. $W_\sigma = c_\sigma^+(1-n_{-\sigma})$. As a consequence the doubly occupied state ist not accesible through repeated application of the generators on the empty one. More of such representations (denoted by $[b]_\pm$) are found in the limit $b \to \pm s$ from $[b,s]$.

3 Integrable vertex models and the Algebraic Bethe Ansatz

The basis for the construction and solution of integrable models is the Yang-Baxter equation

$$
R_{1,2}^{a,b}(\lambda-\mu) R_{1,3}^{a,c}(\lambda) R_{2,3}^{b,c}(\mu) = R_{2,3}^{b,c}(\mu) R_{1,3}^{a,c}(\lambda) R_{1,2}^{a,b}(\lambda-\mu)\,. \tag{3.1}
$$

$R_{i,j}^{a,b}(\lambda)$ is a linear operator on $V_i \otimes V_j$. a, b and c denote representations of $gl(2|1)$, $a,b,c \in \{[\frac{1}{2}]_+, [b,\frac{1}{2}]\}$ which determines the basis of V_i. Note that the tensor products to be taken in (3.1) lead to signs similar as in (1.3) when written in components with respect to this basis.

3.1 $gl(2|1)$-invariant R-matrices

3.1.1 Supersymmetric t-J model

Choosing all three spaces in (3.1) three dimensional the corresponding $gl(2|1)$-invariant R-matrix $R_{1,2}(\lambda)$ can be written as a bilinear form in the generators of the algebra in their fundamental representation $\left[\frac{1}{2}\right]_+$ resulting in Yang's solution

$$
R^{\mathrm{stJ}}(\lambda) \sim (\lambda \mathbf{1}_9 + \Pi)\,. \tag{3.2}
$$

Here Π denotes the graded permutation operator. Because R^{stJ} has a shift point, i.e. $R^{\mathrm{stJ}}(\lambda = 0) \equiv \Pi$, an integrable Hamiltonian with *local* (i.e. nearest neighbour) interactions can be obtainded from the transfer matrix

$$
t(\lambda) = \mathrm{str}(R_L^{\mathrm{stJ}}(\lambda) R_{L-1}^{\mathrm{stJ}}(\lambda) \cdots R_2^{\mathrm{stJ}}(\lambda) R_1^{\mathrm{stJ}}(\lambda))\,. \tag{3.3}
$$

Taking the logarithmic derivative of $t(\lambda)$ at the shift point we obtain the Hamiltonian (1.2).

3.1.2 $[\frac{1}{2}]_+ \otimes [b, \frac{1}{2}]$

This tensor product is the sum of the following two irreducible components:

$$[\frac{1}{2}]_+ \otimes [b, \frac{1}{2}] = [b + \frac{1}{2}, 1] \oplus [b + 1, \frac{1}{2}] \tag{3.4}$$

with dimensions 8 and 4, respectively.

Again we want $R^{3,b}(\lambda)$ to be invariant under the action of $gl(2|1)$. This implies that it can be written as a sum of elementary intertwiners which act only on a single component of the tensor product:

$$R^{3,b}(\lambda) = It_1 + \frac{4\lambda - (2b+3)}{4\lambda + (2b+3)} It_2 \tag{3.5}$$

(a different representation of $R^{3,b}$ is given in [1]). The intertwiners are chosen such that $It_1 + It_2 = 1$. Note that $R^{3,b}(\lambda)$ can *never* reduce to a (graded) permutation operator, hence no lattice Hamiltonian with nearest neighbour interactions can be constructed from it.

3.1.3 $[b_1, \frac{1}{2}] \otimes [b_2, \frac{1}{2}]$

On this tensor product the $gl(2|1)$ invariant R matrix can be written as

$$R^{b_1,b_2}(\lambda) = \frac{2\lambda - (b_1 + b_2 - 1)}{2\lambda + (b_1 + b_2 - 1)} It_1 + \frac{2\lambda - (b_1 + b_2 - 1)}{2\lambda + (b_1 + b_2 - 1)} \frac{2\lambda - (b_1 + b_2 + 1)}{2\lambda + (b_1 + b_2 + 1)} It_2 + It_3 \,, \tag{3.6}$$

where It_1, It_2 and It_3 are the intertwiner on the irreducible components $[b_1+b_2, 1]$, $[b_1+b_2+\frac{1}{2}, \frac{1}{2}]$ and $[b_1 + b_2 - \frac{1}{2}, \frac{1}{2}]$, respectively.

In general $R^{b_1,b_2}(\lambda)$ does not have a shift point, i.e. for all values of λ we have $R^{b_1,b_2}(\lambda) \not\propto \Pi$. One finds, however, that the "unwanted" matrix elements are proportional to $(b_1 - b_2)$, which implies:

$$R^{b,b}(\lambda = 0) = \Pi \,. \tag{3.7}$$

This leads to a new integrable lattice model of correlated electrons which contains a free parameter, namely b [19–22]

3.2 Diagonalization of the transfer matrix $t^{b,b}(\lambda)$

From the above solutions to the Yang-Baxter equation we construct vertex models with transfer matrices defined as

$$t^{b,b}(\lambda) \equiv \mathrm{str}_4(R_L^{b,b}(\lambda) \cdots R_1^{b,b}(\lambda)) \equiv \mathrm{str}_4(T^{b,b}(\lambda)) \tag{3.8}$$

and

$$t^{3,b}(\lambda) \equiv \mathrm{str}_3(R_L^{3,b}(\lambda) \cdots R_1^{3,b}(\lambda)) \equiv \mathrm{str}_3(T^{3,b}(\lambda)) \,. \tag{3.9}$$

By construction the transfer matrices $t^{3,b}(\lambda)$ and $t^{b,b}(\lambda)$ act on the same 4^L-dimensional quantum space. For the corresponding monodromy matrices we have the Yang-Baxter equation

$$R^{3,b}(\lambda - \mu) T^{3,b}(\lambda) T^{b,b}(\mu) = T^{b,b}(\mu) T^{3,b}(\lambda) R^{3,b}(\lambda - \mu) \,. \tag{3.10}$$

This implies that the transfer matrices $t^{b,b}(\lambda)$ and $t^{3,b}(\mu)$ commute, $[t^{3,b}(\mu), t^{b,b}(\lambda)] = 0$, and thus share a common system of eigenstates.

For the diagonalization of $t^{b,b}(\lambda)$ by means of the algebraic Bethe Ansatz we shall first determine the eigenstates for $t^{3,b}(\lambda)$ and then the corresponding eigenvalues of $t^{b,b}(\lambda)$. As starting points for the Bethe Ansatz we need to find a simple eigenstate of $t^{3b}(\lambda)$. For the $gl(2|1)$ invariant models considered here there are three such "reference states", namely

i.) The completely empty lattice: $|\Omega_1\rangle = \otimes^L |b + \frac{1}{2}, S = 0, m = 0\rangle$.

ii.) The completely filled lattice: $|\Omega_2\rangle = \otimes^L |b - \frac{1}{2}, S = 0, m = 0\rangle$.

iii.) The ferromagnetic state with L electrons with spin $\uparrow$: $|\Omega_3\rangle = \otimes^L |b, S = \frac{1}{2}, m = \frac{1}{2}\rangle$.

We will discuss all these possibilities in the sequel.

3.2.1 Empty lattice

The action of the monodromy matrix on the reference state is

$$T^{3,b}(\lambda)|\Omega_1\rangle = \begin{pmatrix} \alpha(\lambda)^L & 0 & C_1(\lambda) \\ 0 & \alpha(\lambda)^L & C_2(\lambda) \\ 0 & 0 & \delta(\lambda)^L \end{pmatrix} |\Omega_1\rangle \ . \tag{3.11}$$

Thus we have found an eigenstate of $t^{3,b}$ (and $t^{b,b}$). More eigenstates are constructed using the Ansatz

$$|\lambda_1, \ldots, \lambda_n\rangle = F^{a_1,\ldots,a_n} C_{a_1}(\lambda_1) \cdots C_{a_n}(\lambda_n) |\Omega_1\rangle \tag{3.12}$$

where the spectral parameters $\{\lambda_i\}$ have to satisfy the Bethe equations [4]

$$\begin{aligned} \left(\frac{\lambda_j + \frac{i}{2}(b - \frac{1}{2})}{\lambda_j - \frac{i}{2}(b - \frac{1}{2})} \right)^L &= \prod_{\alpha=1}^{n_1} \frac{\lambda_j - \nu_\alpha - \frac{i}{2}}{\lambda_j - \nu_\alpha + \frac{i}{2}} , \qquad j = 1, \ldots, n \\ \prod_{j=1}^{n} \frac{\lambda_j - \nu_\alpha + \frac{i}{2}}{\lambda_j - \nu_\alpha + \frac{i}{2}} &= - \prod_{\beta=1}^{n_1} \frac{\nu_\beta - \nu_\alpha + i}{\nu_\beta - \nu_\alpha - i} , \qquad \alpha = 1, \ldots, n_1 \ . \end{aligned} \tag{3.13}$$

3.2.2 Completely Filled lattice

Constructing Bethe vectors from the full lattice $|\Omega_2\rangle$ as a reference state leads to the same Bethe equations as before up to the replacement $b \to -b$. The two possibilities are related by the automorphism of $gl(2|1)$.

3.2.3 Ferromagnetic pseudovacuum

To construct Bethe vectors from the ferromagnetic pseudovacuum we consider

$$T^{3,b}(\lambda)|\alpha_1, \ldots, \alpha_L\rangle = \begin{pmatrix} \tilde{A}_{1,1}(\lambda) & \tilde{A}_{1,2}(\lambda) & 0 \\ \tilde{A}_{2,1}(\lambda) & \tilde{A}_{2,2}(\lambda) & 0 \\ C_1(\lambda) & C_2(\lambda) & D(\lambda) \end{pmatrix} |\alpha_1, \ldots, \alpha_L\rangle \ , \tag{3.14}$$

where $|\alpha\rangle \in \{|b,\frac{1}{2},\frac{1}{2}\rangle, |b-\frac{1}{2},0,0\rangle\}$. Notice that

- $V = \mathrm{span}\,[|\alpha_1\rangle \otimes \cdots \otimes |\alpha_L\rangle]$ is eigenspace of $D(\lambda)$ with eigenvalue 1.
- $\tilde{A}$ leaves the subspace V invariant.

Therefore we consider the generalized Bethe Ansatz

$$|\lambda_1,\ldots,\lambda_L\rangle = F^{a_1,\ldots,a_n}_{\alpha_1,\ldots,\alpha_L} C_{a_1}(\lambda_1)\cdots C_{a_n}(\lambda_n)|\alpha_1,\ldots,\alpha_L\rangle\,. \tag{3.15}$$

In the nesting we have to solve a second Bethe Ansatz on a quantum space $[\mathbb{C}^2]^n \otimes V$. This means we have to solve a model with $n+L$ lattice sites and a two dimensional quantum space attached to each site which leads to following set of Bethe equations [6]

$$\begin{aligned}
\left(\frac{\lambda_j + i\frac{c+1}{2c}}{\lambda_j - i\frac{c+1}{2c}}\right)^L &= \prod_{\alpha=1}^{n_1} \frac{\lambda_j - \nu_\alpha + \frac{i}{2}}{\lambda_j - \nu_\alpha - \frac{i}{2}}\,, \qquad j = 1,\ldots,n\,,\\
\left(\frac{\nu_\alpha - i\frac{1}{2c}}{\nu_\alpha + i\frac{1}{2c}}\right)^L &= \prod_{j=1}^{n} \frac{\nu_\alpha - \lambda_j + \frac{i}{2}}{\nu_\alpha - \lambda_j - \frac{i}{2}}\,, \qquad \alpha = 1,\ldots,n_1\,,
\end{aligned} \tag{3.16}$$

where we have introduced $c = 1/\left(b-\frac{1}{2}\right)$. Note the symmetry of the Bethe equations under exchange of $\lambda \leftrightarrow \nu$ which again is a consequence of the $gl(2|1)$ automorphism $b \leftrightarrow -b$. For $b \searrow \frac{1}{2}$ ($c \nearrow \infty$) Eqs. (3.16) become those of the supersymmetric t-J model given by Eßler and Korepin [2].

A physical interpretation of the excitations described by these equations is obtained by considering limiting cases: for $n_1 = 0$ they describe n free holes moving in front of the ferromagnetic background. Letting $n = 0$ the states have n_1 spin $\downarrow$-electrons moving (freely) and a completely filled band of spin $\uparrow$-electrons.

3.3 Eigenvalues of $t^{b,b}(\lambda)$

To compute the spectrum of t^{bb} we consider [14]

$$R_{12,3}(\lambda) \equiv R^{3,b}_{1,3}(\lambda+\lambda_0) R^{3,b}_{2,3}(\lambda-\lambda_0) \tag{3.17}$$

The tensor product $V_1 \otimes V_2$ is decomposed as $[\frac{1}{2}]_+ \otimes [\frac{1}{2}]_+ = [\frac{3}{2},\frac{1}{2}] \oplus [1]_+$. Adjusting $\lambda_0 = \frac{1}{2}$ we find

$$R_{12,3}(\lambda) = \begin{pmatrix} R^{b'=\frac{3}{2},b} & * \\ 0 & R^{5,b} \end{pmatrix}\,. \tag{3.18}$$

This gives the following relations for the transfer matrices and thus for the eigenvalues:

$$t^{3,b}(\lambda+\frac{1}{2}) t^{3,b}(\lambda-\frac{1}{2}) = t^{b'=\frac{3}{2},b}(\lambda) + t^{5,b}(\lambda) \tag{3.19}$$

The eigenvalues of the operators on the left hand side have been determined in the previous section. In addition we know the action of the transfer matrix $t^{b'=\frac{3}{2},b}$ on the reference states

$|\Omega_i\rangle$. Using this together with the analyticity of the eigenvalues we obtain the eigenvalues $t^{b'=\frac{3}{2},b}(\lambda)|\{\lambda_j\}\rangle = \Lambda^{b'=\frac{3}{2},b}(\lambda)|\{\lambda_j\}\rangle$ from (3.19) [4,23]. Since the representation $[\frac{3}{2},\frac{1}{2}]$ is the only four dimensional one which can be obtained from the tensor product of the three dimensional fundamental one the fusion method as sketched above cannot be used to determine the general eigenvalues $\Lambda^{b'b}$. However, using the known action of $t^{b,b}|\Omega_i\rangle$ together with the analyticity of the eigenvalues once again, we finally obtain the spectrum of $t^{b,b}(\lambda)$.

4 Some applications

Consider the following Hamiltonian for a lattice models of correlated electrons with generalized hopping terms between nearest neighbour sites:

$$\begin{aligned}
\mathcal{H} &= \sum_{j=1}^{L}\sum_{\sigma=\uparrow,\downarrow}\left(c_{j,\sigma}^{+}c_{j+1,\sigma}+h.c\right)\times \\
&\qquad \times\left\{t_0(1-n_{j,-\sigma})(1-n_{j+1,-\sigma})+t_1(n_{j,-\sigma}-n_{j+1,-\sigma})^2+t_2(n_{j,-\sigma}n_{j+1,-\sigma})\right\} \\
&\quad -t_3\sum_{j=1}^{L}(c_{j+1,\uparrow}^{+}c_{j+1,\downarrow}^{+}c_{j,\downarrow}c_{j,\uparrow}+h.c)+U\sum_{j=1}^{L}n_{j,\uparrow}n_{j,\downarrow} \\
&\quad -\mu\sum_{j=1}^{L}(n_{j,\uparrow}+n_{j,\downarrow})-\frac{h}{2}\sum_{j=1}^{L}(n_{j,\uparrow}-n_{j,\downarrow})\,.
\end{aligned} \tag{4.1}$$

$t_{0,1,2}$ parametrize the single electron hopping amplitudes depending on the occupation of the affected sites with electrons of the opposite spin projection (bond-charge repulsion), t_3 is a pair hopping term, U is the local Coulomb interaction, and μ and h are chemical potential and magnetic field, respectively.

As a necessary condition to integrability, the two particle scattering matrix has to satisfy a Yang-Baxter equation. This requirement restricts the parameters in (4.1) to one of the two following sets [19,20]:

a.) $t_0 = t_1 = t_2 = t$, $t_3 = 0$

b.) $\frac{U}{2} = -t_3 = \pm(t_0 - t_2) \neq 0$ and $t_1^2 = t_0 t_2 > 0$

Case a.) is the standard Hubbard model, while b.) is found to be a $gl(2|1)$ invariant model of electrons with correlated electrons. Its integrability can be established by identifying $\mathcal{H}$ with the logarithmic derivative of the transfer matrix $t^{b,b}(\lambda)$ with [21]

$$b = \frac{1}{2}\frac{t_0+t_2}{t_0-t_2}\,.$$

For its solution two regimes have to be distinguished [22]: depending on the sign of the parameter b (or, equivalently, the Hubbard interaction U in (4.1)) the solutions of the Bethe equations (3.13) in the thermodynamic limit can be classified by different configurations of 'strings' — i.e.

complexes of complex λ and ν. As usual, the resulting Bethe states are $gl(2|1)$-highest weight states. The physical properties of the model are found to be similar to those of the repulsive and attractive Hubbard model, respectively.

Another interesting application of the vertex models introduced above to problems of correlated electrons is obtained by considering *inhomogeneous* models: as an example consider the transfer matrix of the supersymmetric t–J model (3.3) with one vertex $R^{3,b}(\lambda)$ carrying the representation $[b, \frac{1}{2}]$ added. Taking the logarithmic derivative of this object at $\lambda = 0$ one obtains an integrable model of a (nonmagnetic) impurity in a system of strongly correlated electrons [24, 25].

5 Summary

We have shown that representations of the superalgebra $gl(2|1)$ allow for a natural interpretation in terms of electronic states. In the framework of the Quantum Inverse Scattering Method integrable lattice models with this symmetry can be constructed which are interesting realizations of correlated electron systems: besides the well-known supersymmetric t-J model from the $[\frac{1}{2}]_+$–represenation we have constructed a family of model of electrons with correlated hopping starting from the $[b, \frac{1}{2}]$-reprensations and an integrable $[b, \frac{1}{2}]$–"impurity" in a t–J chain (a similar problem using the $[s]_+$ representation for the impurity has been studied in Ref. [26]).

Higher dimensional representations open additional possibilities for interesting models: in particular it should be possible to construct an integrable Kondo-lattice model from a homogeneous $[b, s]$-chain when one interprets the $[b, s]$ representation as the Hilbert space of a spin $(s - \frac{1}{2})$ tensored with that of $S = \frac{1}{2}$-electron $\{|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle\}$. The corresponding Bethe equations are easy to find from the transfer matrix $t^{3,[b,s]}$. However, due to the subtleties of the $gl(2|1)$-representations the fusion procedure to determine the eigenvalues is much more complicated than for $[b, \frac{1}{2}]$.

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