Magnetic Monopoles

Magnetic monopoles are hypothetical elementary particles which are isolated magnets possessing only one magnetic pole. In the classical electromagnetism such a particle (Dirac monopole) is forced to have infinite energy, but smooth solutions with finite energy exist in certain nonabelian gauge theories (including some Grand Unified Theories). They are minima of the Yang-Mills-Higgs energy and can be described by first-order partial differential equations, the Bogomolny equations (whereas the Euler-Lagrange equations have, as usual, the second order).

The space of all monopoles with a fixed magnetic charge is (after identifying gauge-equivalent monopoles) a smooth manifold equipped with a natural Riemannian metric, which is extremely interesting from both mathematical and physics’ point of view. For example, both classical and quantum dynamics of magnetic monopoles can be studied via this metric on the moduli space.

In the seminar we want to discuss several of these topics and to introduce several deep mathematical methods and physical ideas which arose around the theory of monopoles.

A possible selection of topics is as follows:

1. Vector bundles, connections, electromagnetism, Dirac monopole
2. Yang-Mills-Higgs action, Bogomolny equations, topological charges
3. ’t Hooft-Polyakov Monopole and multimonopoles
4. Monopoles in Grand Unified Theories and Cosmology
5. The moduli spaces, scattering along geodesics, spectral curves, rational maps
6. The hyperkähler metric and its physical significance
7. Examples of geodesic scattering in the moduli space
8. Dyons and the Witten effect
9. The Penrose-Ward-Hitchin correspondence
10. The Nahm transform
11. The Fourier-Mukai transform
Literature:

Place and Time: Tuesdays (beginning 21.10.), 10-12, room A310

First (organisational) meeting: Tuesday, October 21, 10-12, room A310

Related courses: Ergänzungen zur klassischen Physik: Solitonen, Monopole, Instantonen - a lecture course by Prof. Dr. Olaf Lechtenfeld, WS 2014/2015, Mo and Fr, 8-10, ITP, room 267 (beginning 22.10.).