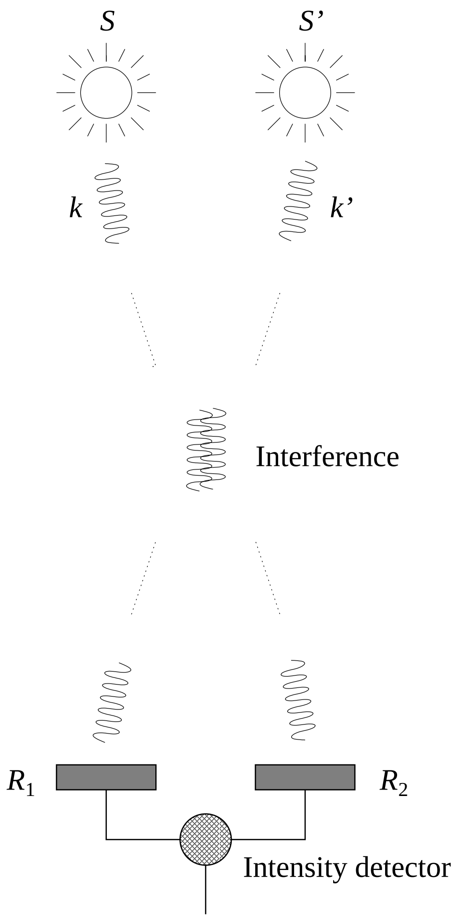
HomeWork 6

1 Hanbury-Brown and Twiss interference

A double star is sending light with wave vectors k and k'. Those two vectors are close to each other and we want to determine their angle ϕ. The method proposed by Hanbury-Brown and Twiss to do so relies on intensity correlation of the emitted light. We will examine this method, first in terms of classical electrodynamics, then in terms of photon statistics.



1.1 Suppose we have two detectors in r_1 and r_2. We measure the intensity correlation defined as :

$$C = \langle I(r_1)I(r_2)\rangle = \langle |E(r_1)|^2 |E(r_2)|^2\rangle$$

where $E(r)$ is the electric field of the wave collected at point r. Write down $E(r_1)$ and $E(r_2)$ as the sum of two plane waves (one for k one for k') and calculate C. Once done, give an expression for ϕ.

1.2 Photon interpretation

Suppose that the state of the field emitted by the double star is a state with two photons, one in k the other in k' :

$$|\psi\rangle = |1 : k, 1 : k'\rangle$$

The photon k has been emitted by the star S and the photon k' by the star S'. We want to calculate the probability Π to detect one photon at point r_1 and the other at point r_2. But now, we consider the electric field to be a quantum operator that acts on the basis of photon states.

$$\hat{E}(r) = \hat{E}^{(+)}(r) + \hat{E}^{(-)}(r)$$

with

$$\hat{E}^{(+)}(r) = i \sum_{k,\varepsilon} \sqrt{\frac{\hbar\omega}{2\epsilon_0 L^3}} \varepsilon e^{ikr} a_{k,\varepsilon}$$

and

$$\hat{E}^{(-)}(r) = (\hat{E}^{(+)}(r))^\dagger$$

ε is the field polarisation, $a_{k,\varepsilon}$ is the usual annihilation operator that destroys a photon in state $\{k, \varepsilon\}$. Using the following result of quantum optics,

$$P(r_1, r_2) = \langle \Psi | \varepsilon_1.\hat{E}^{(-)}(r_1) \varepsilon_2.\hat{E}^{(-)}(r_2) \varepsilon_2.\hat{E}^{(+)}(r_2) \varepsilon_1.\hat{E}^{(+)}(r_1) | \Psi \rangle$$

calculate Π (forget about the polarisation ...). Comment.

2 Two-particle boson state

$$|2\rangle = \int d^3x_1 \int d^3x_2 \varphi(x_1, x_2) \Psi^\dagger(x_1) \Psi^\dagger(x_2) |0\rangle$$

2.1 Show that $|2\rangle$ is normalized.

2.2 Calculate the density $\langle 2|n(x)|2\rangle$ under the assumption that $\varphi(x_1, x_2) \propto \varphi_1(x_1)\varphi_2(x_2)$.

3 The Bogoliubov transformation

The Bogoliubov transformation reads :

$$\begin{aligned} a_k &= u_k \alpha_k + v_k \alpha^\dagger_{-k} \\ a^\dagger_k &= u_k \alpha^\dagger_k + v_k \alpha_{-k}. \end{aligned}$$

We demand that the operators α satisfy Bose commutation.

3.1 What condition have to fulfil the coefficients u and v to preserve the commutation relations of the a's ?

3.2 Write the inverse of the Bogoliubov transformation.