LITTLE EXAM

You may consider this test as passed if you can collect about one third of the total number of points.

Name: ________________________________

Matrikelnummer: ________________________________

Email: ________________________________

[1] What is a symmetry? Why are symmetries useful properties of physical systems? (1+1 P)

[2] Symmetries naturally have the mathematical structure of groups. What are the group axioms? (3 P)


[4] Let a Lie group element $g = \exp(X)$ be given in terms of a generator $X$ of the corresponding Lie algebra. Express $\det(g)$ in terms of $\text{tr}(X)$. (1 P)

[5] Name the four types of the classical Lie groups. (4 P)

[6] What is the Jacobi identity? What is its meaning? (1+1 P)


[8] What is the Killing form? Which are the characterizing properties of the Killing form for a semi-simple Lie algebra and for a compact Lie algebra? (1+1+1 P)

[9] What is the Cartan sub-algebra of a Lie algebra? Describe briefly, what the weights of a representation of a Lie algebra are. What are the roots of a Lie algebra? (1+2+1 P)

[10] What is the master formula for a weight $\mu$ with respect to a root $\alpha$? What is its meaning? Argue, why two weights must always differ by integer valued linear combinations of roots. (2+2+2 P)

[11] Say in words, why the angles between two roots $\alpha, \beta$ are restricted such that $4 \cos^2 \angle(\alpha, \beta) \in \{0, 1, 2, 3\}$. (3 P)

[12] What are the simple roots of a Lie algebra? (2 P)

[13] A Lie algebra is completely characterized by the set of its simple roots. Explain briefly, what the Dynkin diagram of a Lie algebra is. (3 P)

[14] What are the fundamental representations of a Lie algebra? How many fundamental representations does a rank $r$ Lie algebra have? (3 P)

[15] Name up to four physical systems or physical theories together with their symmetry groups. (4 P)

(45 P)