

# Factorization Constraints in Non-Compact Non-Rational Conformal Field Theory

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# Outline

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- What is non–compact non–rational CFT?

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Motivation and Introduction

Sewing Constraints and  
D–Branes

Conclusions

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- What is non-compact non-rational CFT?
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- Rôle of sewing constraints in CFT and BCFT

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- Our results and observations on the factorization constraint  
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# Motivation and Introduction

# Non-Compact Non-Rational CFT...

Rational CFT (RCFT)

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$\hat{\mathfrak{su}}(2)_k$  WZNW model



minimal models

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minimal models

## Non-Compact Non-Rational CFT

- ▶ continuum of highest weight states
- ▶ only some specific models feasible
- ▶ lack of general structural results
- ▶ prototype examples:

$H_3^+$  model ( $(\hat{\mathfrak{sl}}(2, \mathbb{C})_k$  WZNW))



Liouville Theory

# ... and String Theory

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# ... and String Theory

- String theory: Critical dimension

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- In particular: Anti-de-Sitter ( $AdS$ ) spaces



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- benefit of  $AdS_3$ :  $SL(2, \mathbb{R})$  group manifold  $\longrightarrow$  WZNW model

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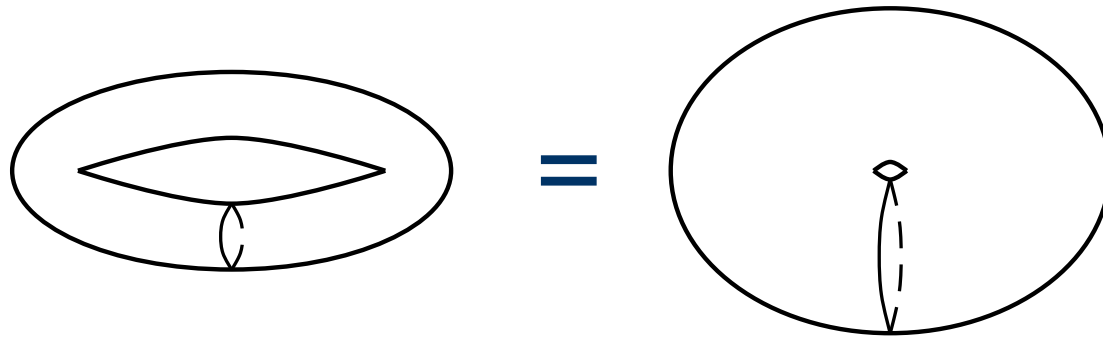
Conclusions

- String theory: Critical dimension
- Away from criticality: Liouville sector
- Need to treat non-compact curved spacetime backgrounds
- In particular: Anti-de-Sitter ( $AdS$ ) spaces
- benefit of  $AdS_3$ :  $SL(2, \mathbb{R})$  group manifold  $\longrightarrow$  WZNW model
- even nicer: euclidean rotation to  $SL(2, \mathbb{C})/SU(2) = H_3^+$

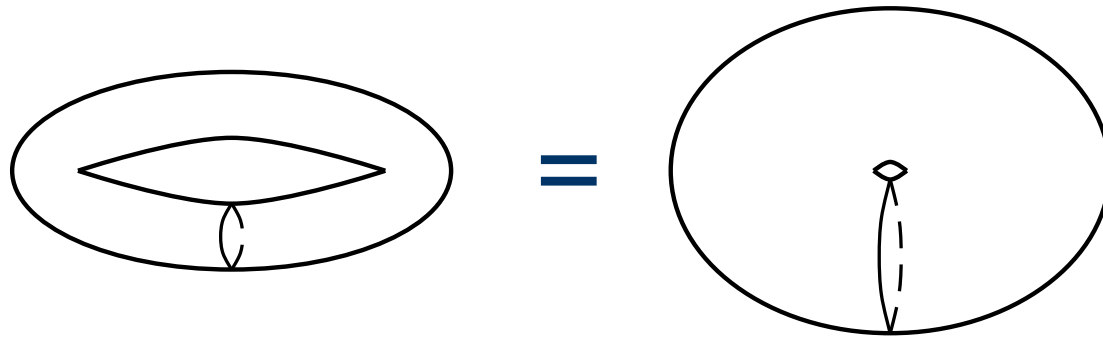


# **Sewing Constraints and D–Branes**

# Sewing Constraints in Bulk CFT

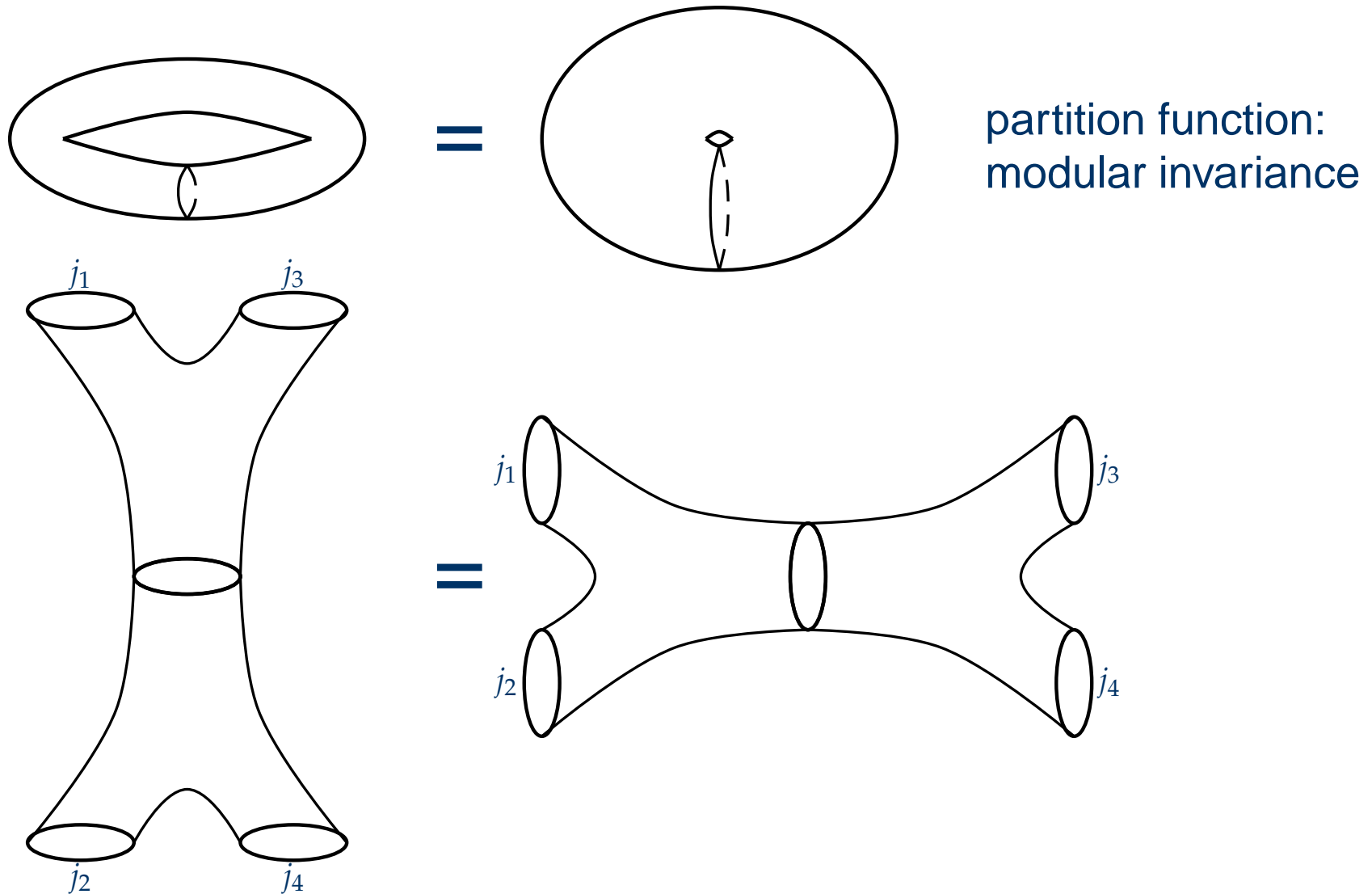


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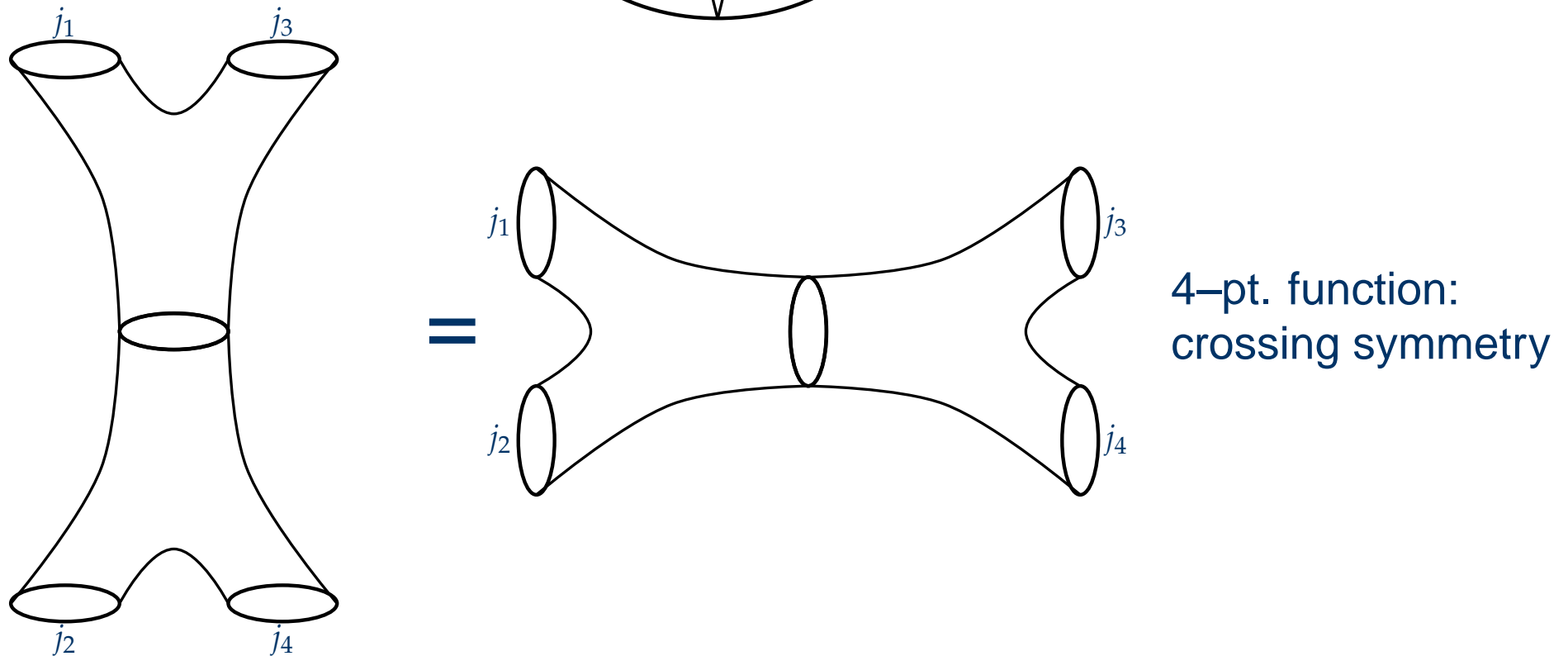
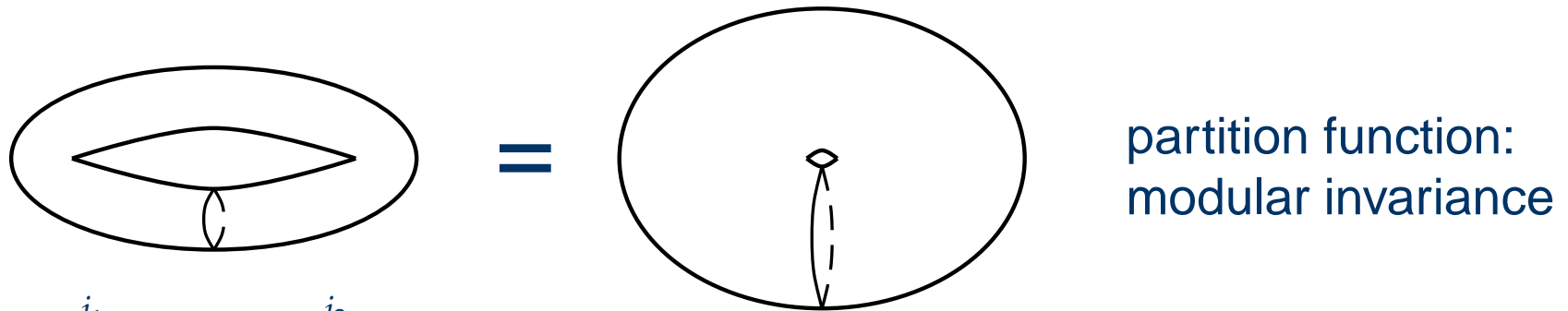


partition function:  
modular invariance

# Sewing Constraints in Bulk CFT



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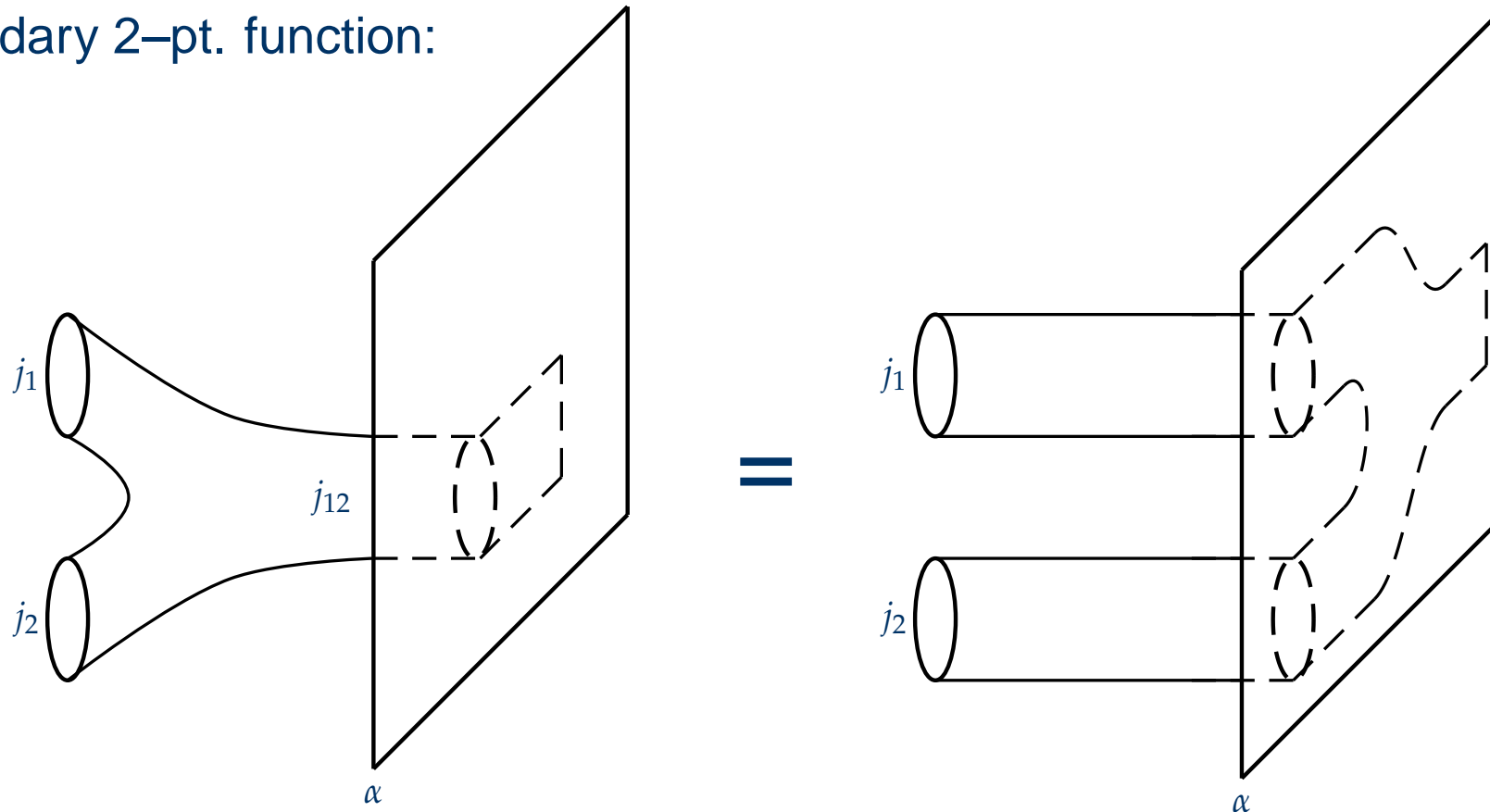
# Sewing Constraints in Boundary CFT





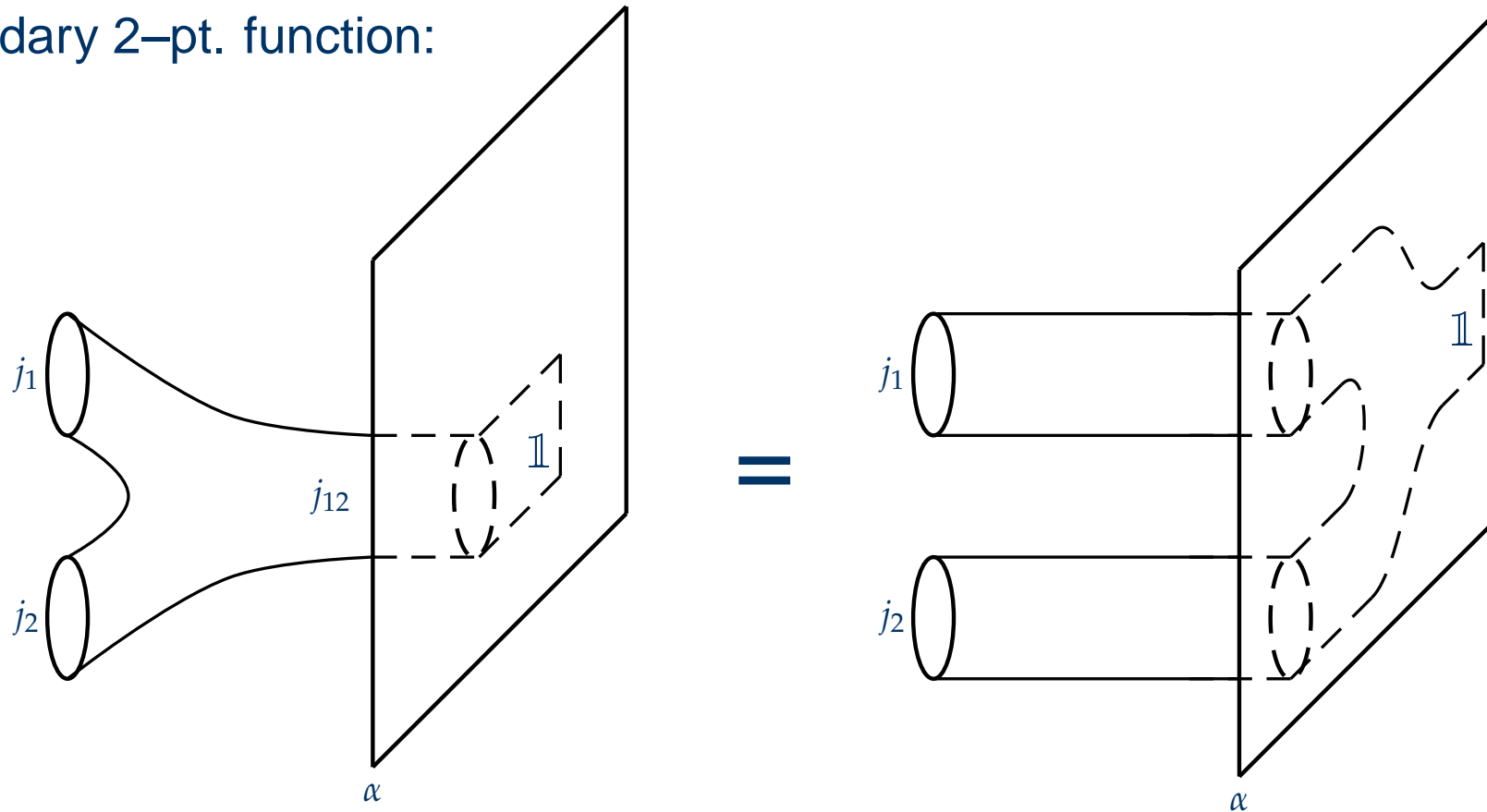
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boundary 2-pt. function:



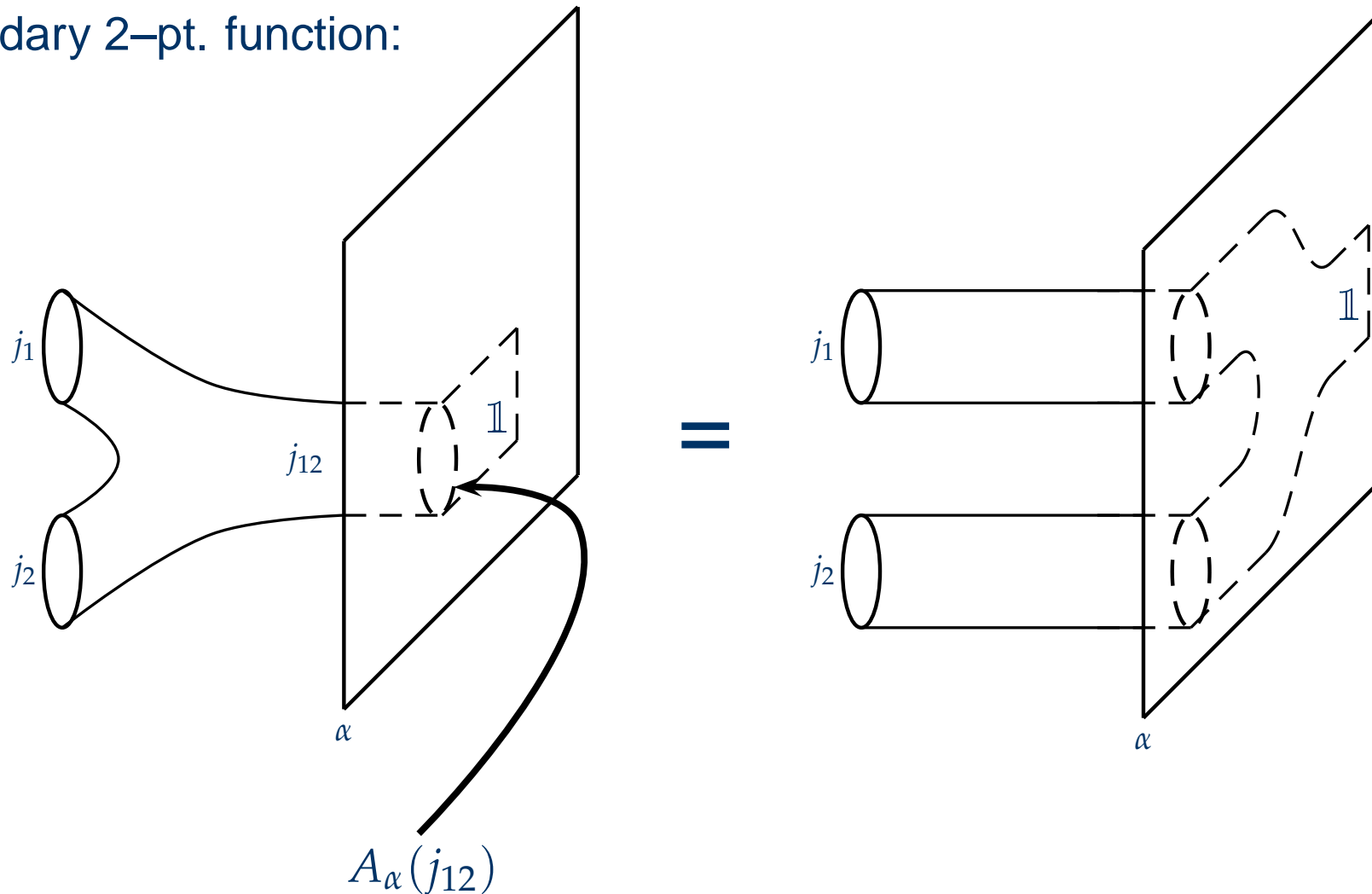
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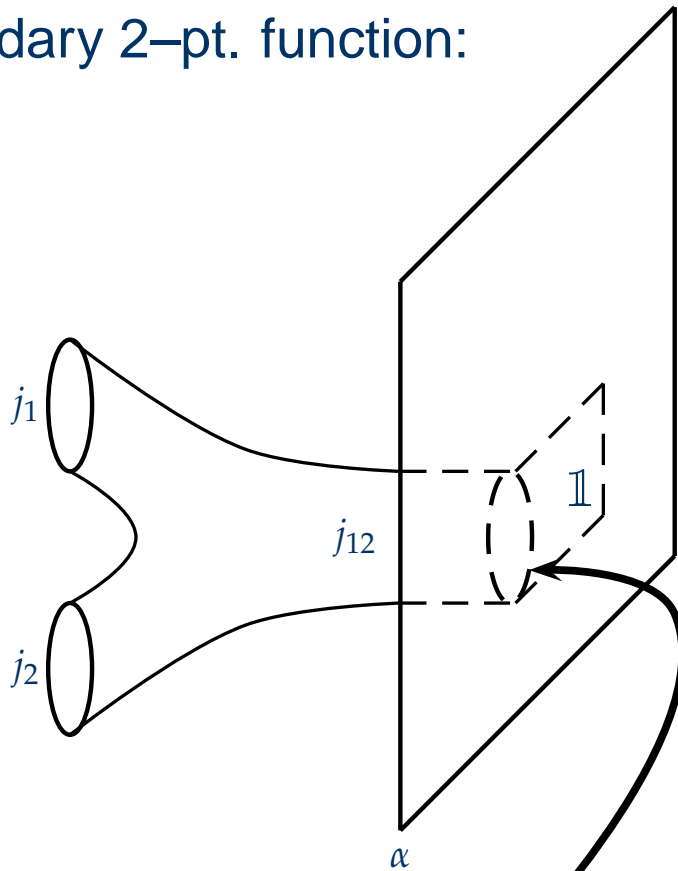
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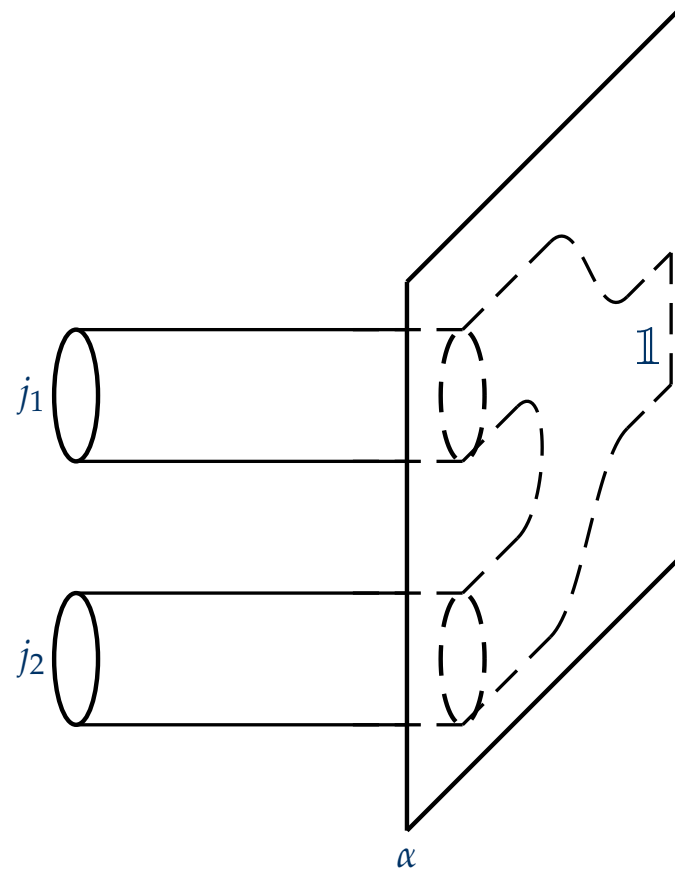
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boundary 2-pt. function:



$$\sum_{j_{12}} A_{\alpha}(j_{12})$$

=

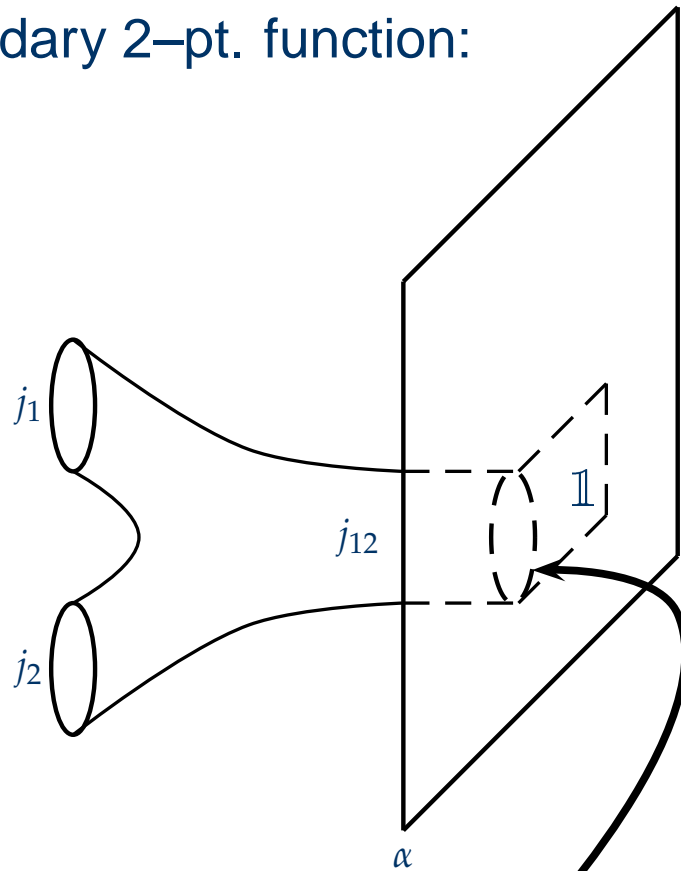


$\propto$

$$\sum_{q, \bar{q}} C_{\alpha}(j_1, q) C_{\alpha}(j_2, \bar{q})$$

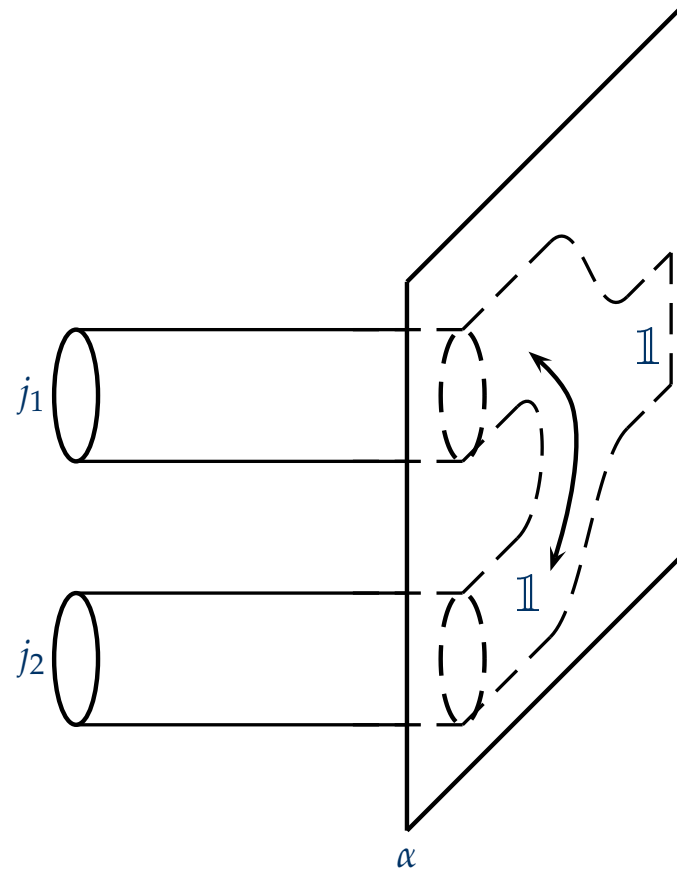
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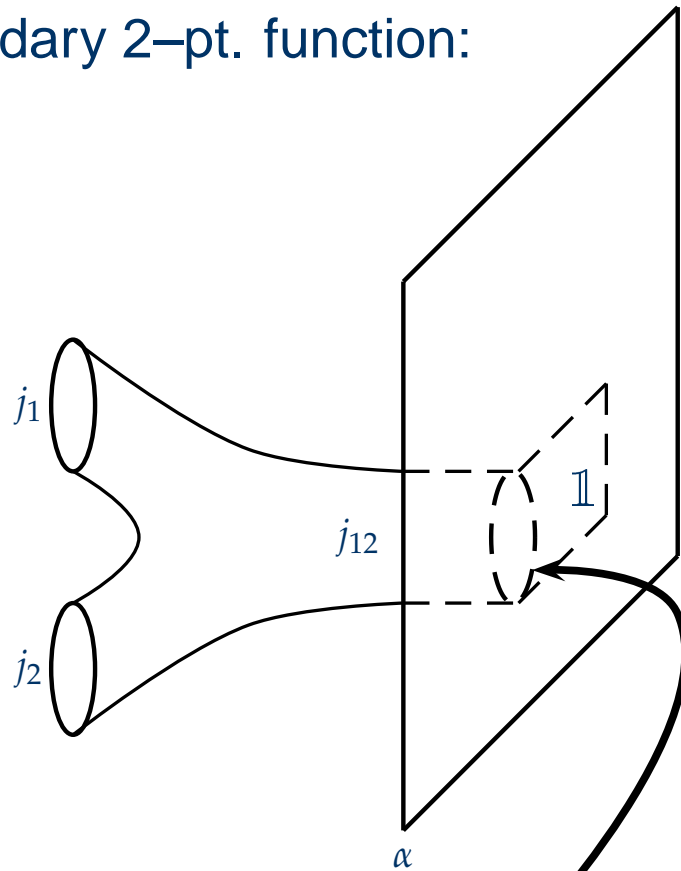


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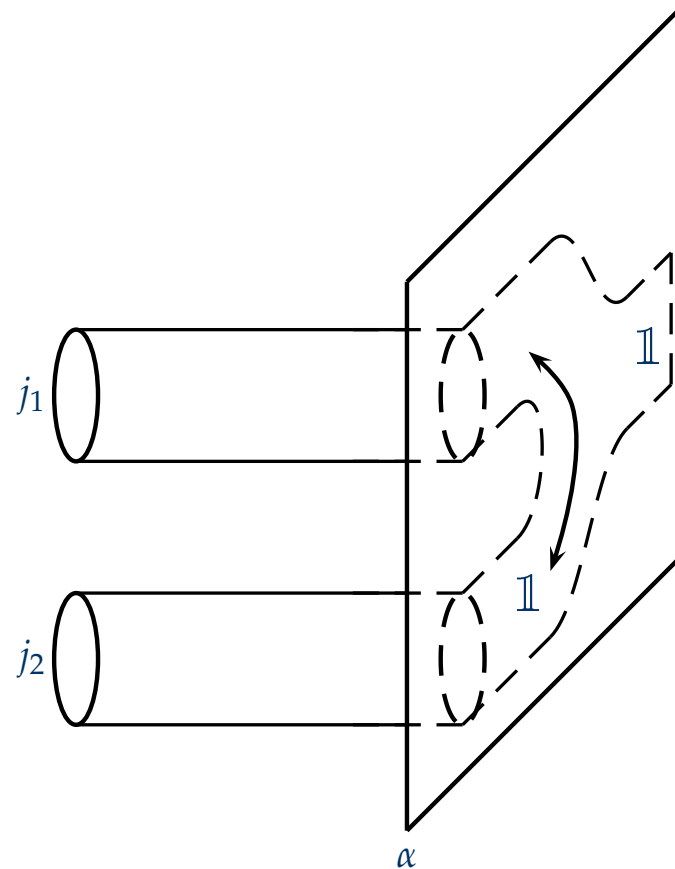
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boundary 2-pt. function:



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=



$\propto$

$$A_\alpha(j_1) A_\alpha(j_2)$$

# $H_3^+$ model

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- » Sewing Constraints in Bulk

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# $H_3^+$ model

## ▶ $\hat{sl}(2, \mathbb{C})_k$ symmetry

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- ▶  $\hat{sl}(2, \mathbb{C})_k$  symmetry  $\longrightarrow$  highest weight states fall into  $sl(2, \mathbb{C})$  representations

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- ▶ relevant: principal continuous series with "spins"  
 $j \in -\frac{1}{2} + i\mathbb{R}$

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 $j \in -\frac{1}{2} + i\mathbb{R} \longrightarrow$  fields  $\Theta_j(u|z)$

# Factorization Constraint in the $H_3^+$ model I

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# Factorization Constraint in the $H_3^+$ model I

- analyze boundary 2–point function

$$G_\alpha^{(2)}(u, z) = \left\langle \Theta_j(u_1, z_2) \Theta_{j'}(u_2, z_2) \right\rangle_\alpha$$

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- ▶ e.g.  $j' = 1/2$  constraint:

$$A_\alpha(1/2)A_\alpha(j) \propto \sum_{\pm} A_\alpha(j \pm 1/2)$$

[Giveon, Kutasov, Schwimmer'01], [Lee, Ooguri, Park'02], [Ponsot, Schomerus, Teschner'02]

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- ▶ ... does not fix the solution  $A_\alpha(j)$  uniquely
- ▶ goal: another constraint from next simple reducible representation  $j' = b^{-2}/2$

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- ▶ Recall: boundary 2–point function  $G_\alpha^{(2)}(u, z)$

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## Conclusions

- ▶ Recall: boundary 2–point function  $G_\alpha^{(2)}(u, z)$
- ▶ Technically, need to take factorization limit  $z \rightarrow 1$

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## Conclusions

- ▶ Recall: boundary 2–point function  $G_\alpha^{(2)}(u, z)$
- ▶ Technically, need to take factorization limit  $z \rightarrow 1$
- ▶ but for  $j' = b^{-2}/2$ : boundary 2–point function is only defined for  $z < u < 1$ .

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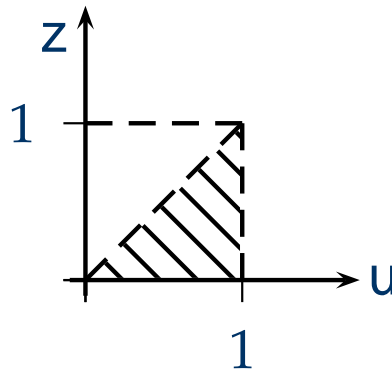
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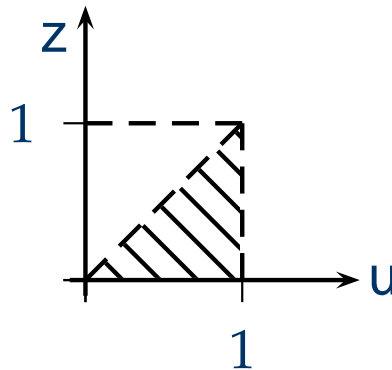
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- ▶ Technically, need to take factorization limit  $z \rightarrow 1$
- ▶ but for  $j' = b^{-2}/2$ : boundary 2-point function is only defined for  $z < u < 1$ .



- ▶ need a prescription how to reach the upper patch  $z > u$

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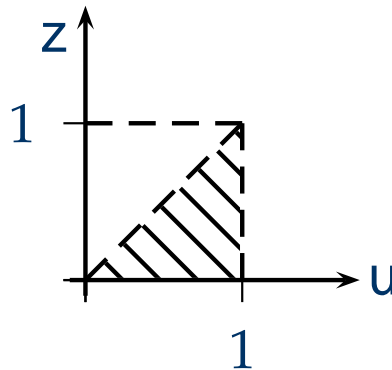
##### » Factorization Constraint in the $H_3^+$ model I

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##### » Comparison

## Conclusions

- ▶ Recall: boundary 2–point function  $G_\alpha^{(2)}(u, z)$
- ▶ Technically, need to take factorization limit  $z \rightarrow 1$
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- ▶ need a prescription how to reach the upper patch  $z > u$
- ▶ good news: this can be done

# Factorization Constraint in the $H_3^+$ model II

## Outline

### Motivation and Introduction

#### Sewing Constraints and D-Branes

##### » Sewing Constraints in Bulk CFT

##### » Sewing Constraints in Boundary CFT

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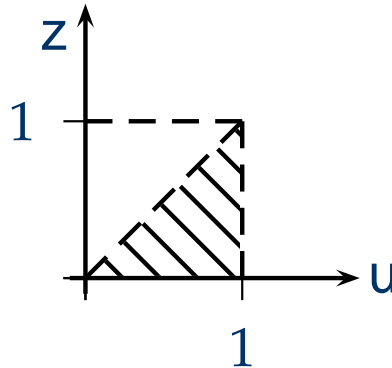
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# Factorization Constraint in the $H_3^+$ model II

Outline

Motivation and Introduction

Sewing Constraints and D-Branes

» Sewing Constraints in Bulk CFT

» Sewing Constraints in Boundary CFT

»  $H_3^+$  model

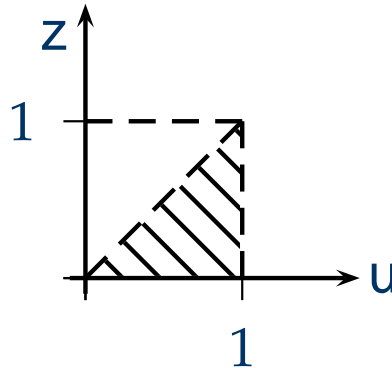
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# Factorization Constraint in the $H_3^+$ model II

Outline

Motivation and Introduction

Sewing Constraints and  
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» Sewing Constraints in Bulk  
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» Sewing Constraints in  
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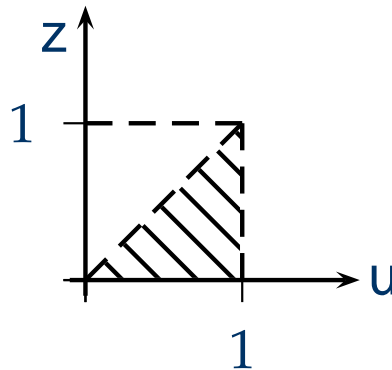
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# Comparison

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# Comparison

analytic continuation

continuity proposal

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▶ discrete *AdS* branes:  $(m, n) \in \mathbb{Z}^2$

continuity proposal



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- ▶ Brane spectra coincide

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- ▶ 1-parameter ambiguity in boundary 2-pt. function (suggested from Liouville/ $H_3^+$  relation)
- ▶ we could show: factorization constraint remains unambiguous

- ▶ Both approaches: meaningful factorization constraint
- ▶ Brane spectra coincide
- ▶ Analytic continuation slightly more restrictive



# Conclusions

# Summary

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► Non-compact non-rational CFT



# Summary

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general structure of CFT



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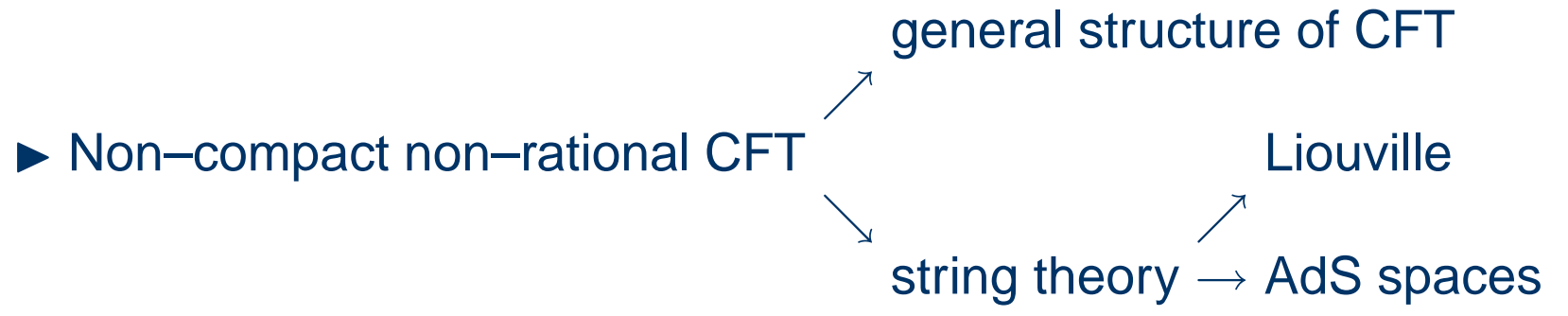
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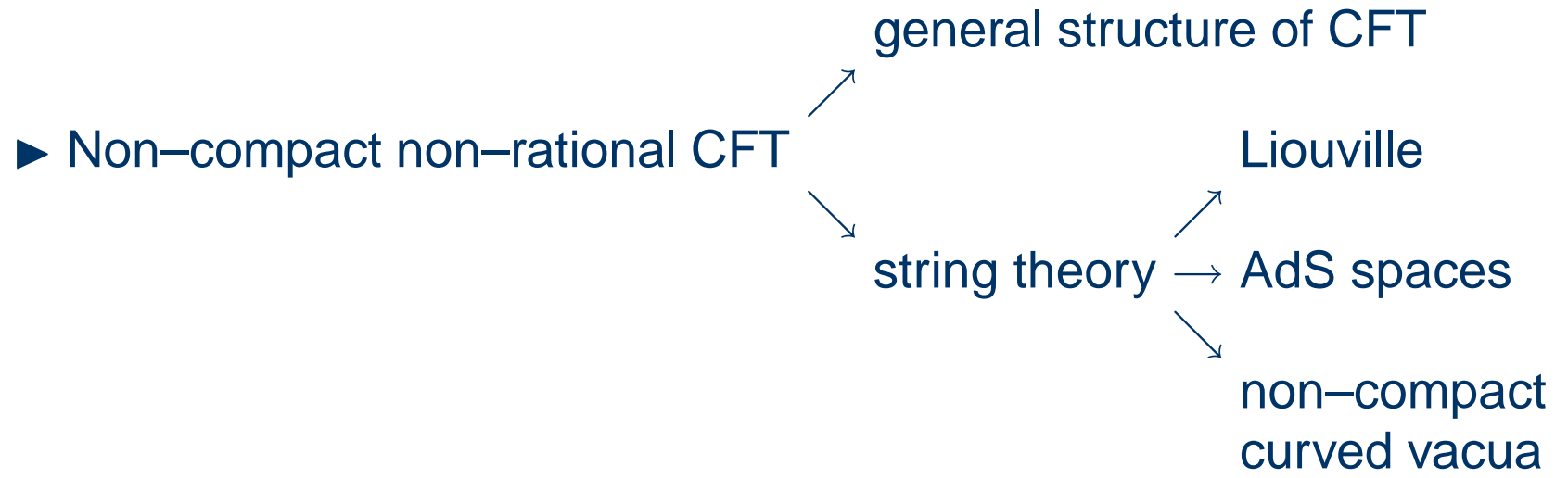
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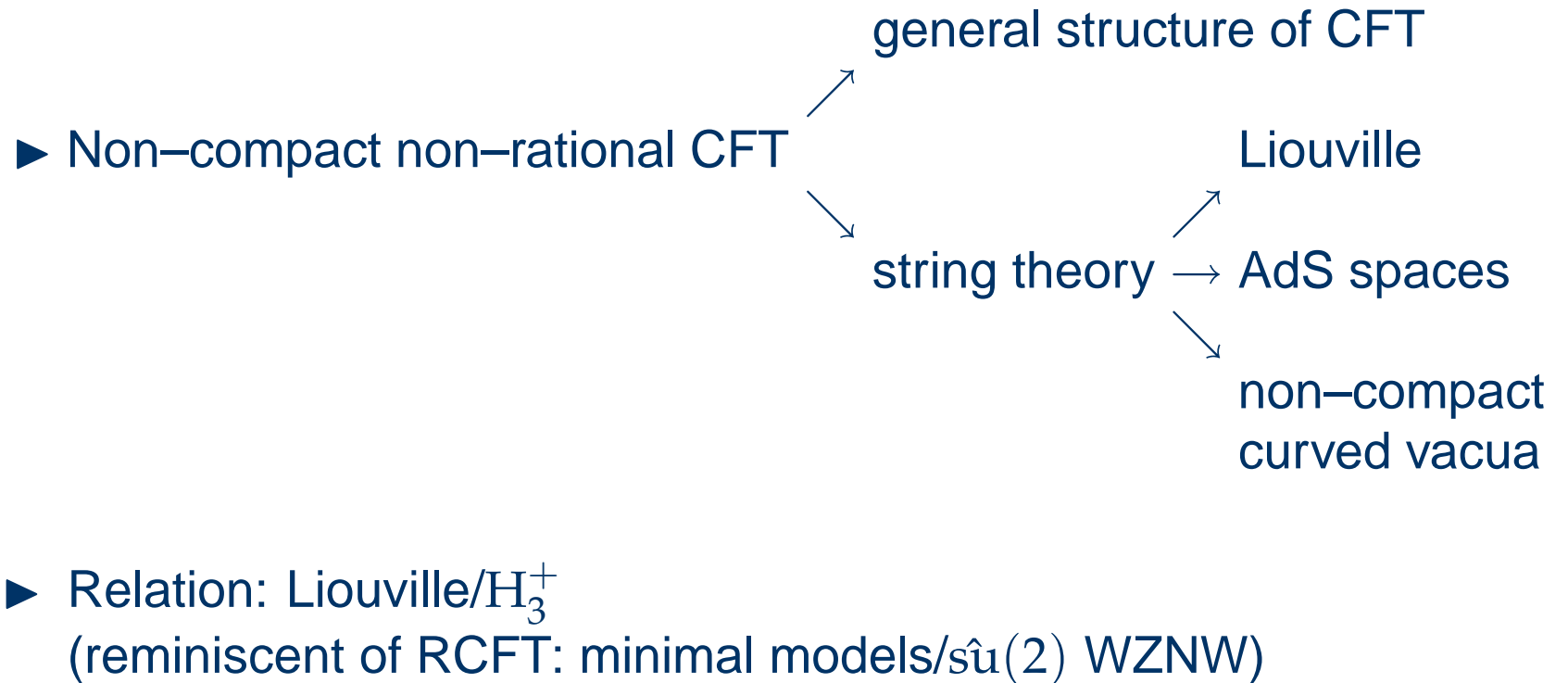
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- ▶ Non-compact non-rational CFT
  - general structure of CFT
  - string theory
    - Liouville
    - AdS spaces
    - non-compact curved vacua
- ▶ Relation: Liouville/ $H_3^+$   
(reminiscent of RCFT: minimal models/ $\hat{su}(2)$  WZNW)
- ▶ Our work: Factorization Constraint in Boundary  $H_3^+$

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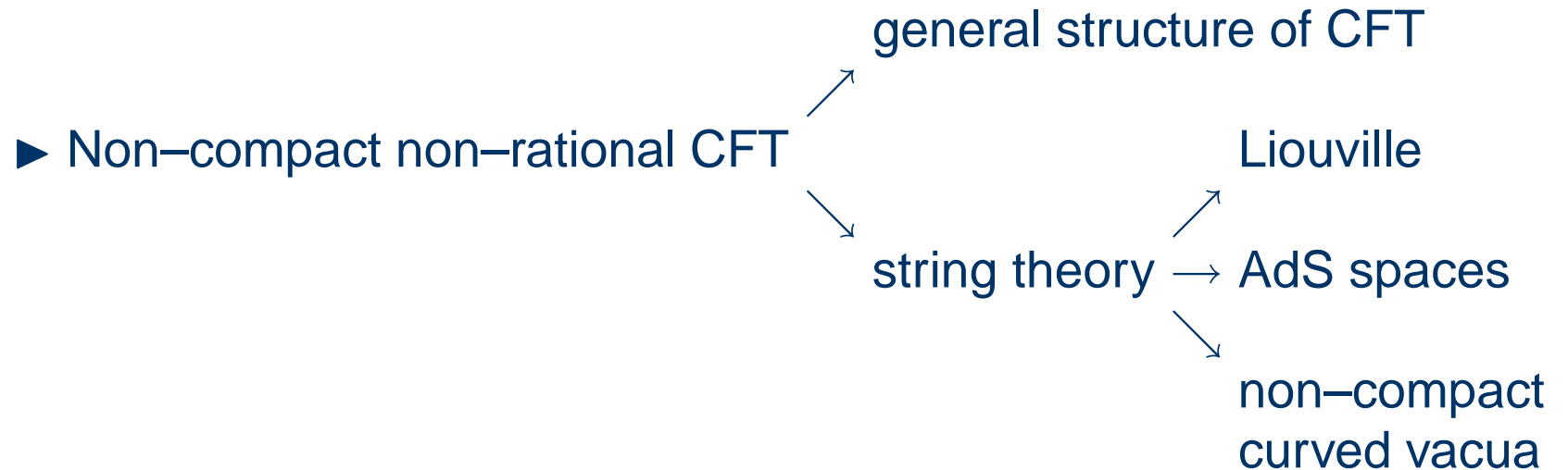
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  - "Weak" form (continuity proposal from  $\text{Liouville}/H_3^+$ )

[Adorf, Flohr'07]



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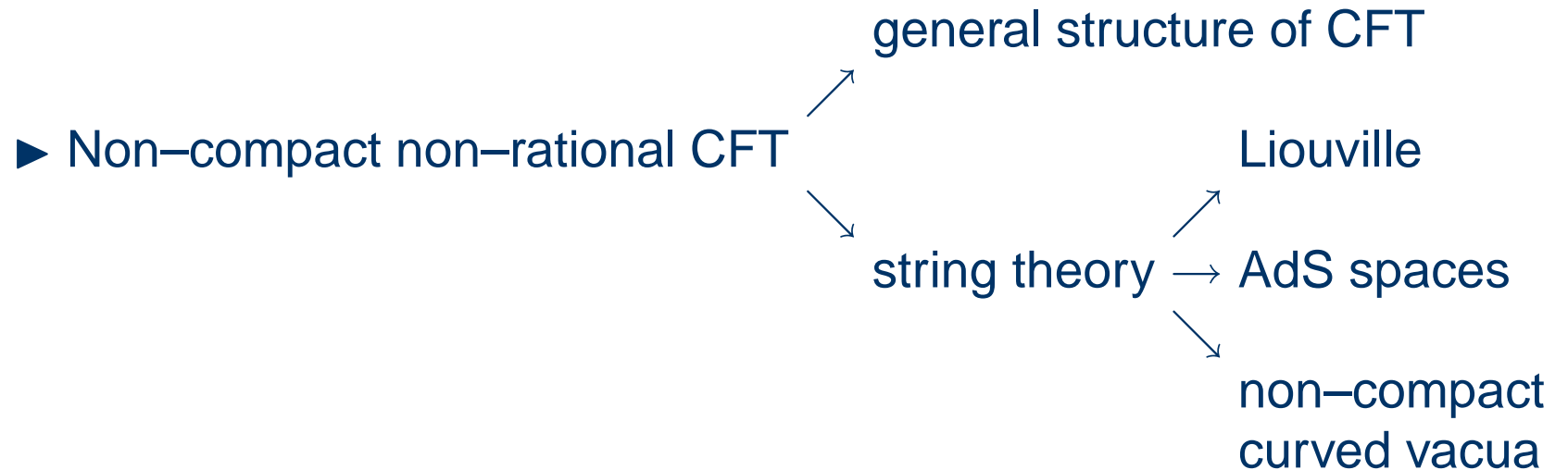
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- Relation:  $\text{Liouville}/\mathbb{H}_3^+$   
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- Our work: Factorization Constraint in Boundary  $\mathbb{H}_3^+$ 
  - "Weak" form (continuity proposal from  $\text{Liouville}/\mathbb{H}_3^+$ )
  - "Strong" form (analytic continuation)

[Adorf,Flohr'07]

[Adorf,Flohr'08]

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- ▶ Immediate Qs from our work:

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- ▶ Immediate Qs from our work:
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... based on Hendrik Adorf and Michael Flohr:

arXiv:0707.1463

arXiv:0801.2711