A Note on Four-Point Functions in Logarithmic Conformal Field Theory

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Abstract: The generic structure of 4-point functions of fields residing in indecomposable representations of arbitrary rank is given. The presented algorithm is illustrated with some non-trivial examples and permutation symmetries are exploited to reduce the number of free structure-functions, which cannot be fixed by global conformal invariance alone.

1 Introduction

During the last few years, logarithmic conformal field theory (LCFT) has been established as a well-defined variety of conformal field theories in two dimensions. The concept was considered in its own right first by Gurarie [12], Since then, a large amount of work has appeared, see the reviews [8, 11] and references therein. The defining feature of a LCFT is the occurrence of indecomposable representations which, in turn, may lead to logarithmically diverging correlation functions. Thus, in the standard example of a LCFT a primary field $\phi(z)$ of conformal weight h has a so-called logarithmic partner field ψ with the characteristic properties

$$\langle\phi(z)\phi(0)\rangle = 0\,, \quad \langle\phi(z)\psi(0)\rangle = Az^{-2h}\,, \quad \langle\psi(z)\psi(0)\rangle = z^{-2h}\left(B - 2A\log(z)\right)\,. \tag{1}$$

To this corresponds the fact that the highest weight state $|h\rangle$ associated to the primary field ϕ is the ground state of an irreducible representation which, however, is part of a larger, indecomposable, representation created from $|\tilde{h}\rangle$, the state associated to ψ. The conformal weight is the eigenvalue under the action of L_0, the zero mode of the Virasoro algebra, which in such LCFTs cannot be diagonalized. Instead, we have

$$L_0\,|h\rangle = h\,|h\rangle\ , \quad L_0|\tilde{h}\rangle = h|\tilde{h}\rangle + |h\rangle\ . \tag{2}$$

Thus, the two states $|h\rangle$ and $|\tilde{h}\rangle$ span a Jordan cell of rank two with respect to L_0. As can be guessed from eq. (1), there must exist a zero mode which is responsible for the vanishing of the 2-pt function of the primary field. Another characteristic fact in LCFT is the existence of at least one field, which is a perfect primary field, but whose operator product expansion (OPE) with itself produces a logarithmic field. Such fields

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μ are called pre-logarithmic fields [15]. This is important, since in many cases, the pre-logarithmic fields arise naturally forcing us then to include the logarithmic fiels as well into the operator algebra. Note that this implies that the fusion product of two irreducible representations is not necessarily completely reducible into irreducible representations. In fact, we know today quite a few LCFTs, where precisely this is the case, such as ghost systems [17], WZW models at level zero or at fractional level such as $\widehat{SU(2)}_{-4/3}$ [10, 16], WZW models of supergroups such as $GL(1,1)$ [23] or certain supersymmetric $c = 0$ theories such as $OSP(2n|2n)$ or $CP(n|n)$ [13, 21]. Finally, many LCFTs are generated from free anticommuting fields such as the symplectic fermions [14]. LCFT enjoys numerous applications in condensed matter physics, but it is important in string theory as well, e.g. for the understanding of decaying D-branes [19].

In these notes, we generalize LCFT to the case of Jordan cells of arbitrary rank, but we will restrict ourselves to the Virasoro algebra as the chiral symmetry algebra to keep things simple. As has been shown in [7, 9], the generic form of 1-, 2- and 3-pt functions can be fixed up to structure constants under mild assumption on the structure of the indecomposable representations. From this, the general structure of the OPE can then easily be obtained. However, in order to be able to compute arbitrary correlation functions in LCFT, one at least needs the 4-point functions such that crossing symmetry can be exploited. Unfortunately, this turns out to be more complicated [5, 6]. In the following, we present an algorithm with which the generic form of 4-point functions can be fixed up to functions, which only depend on the globally conformal invariant crossing ratio. In contrast to ordinary CFT, the number of these free functions grows heavily with the total rank r of the involved Jordan cells and the number of logarithmic partner fields. However, there exist certain permutation symmetries which relate many of these functions to each other. The full derivation and further generalizations of our results will appear elsewhere [18].

2 Ansatz for 4-point functions

Let r denote the rank of the Jordan cells we consider. One can show, that in LCFTs with Jordan cells with respect to (at least) the L_0 mode, the $h = 0$ sector necessarily must carry such a Jordan cell structure. Furthermore, its rank defines the maximal possible rank of all Jordan cells. Thus, without loss of generality, we can assume that the rank of all Jordan cells is equal to r, other cases can easily be obtained by setting certain structure constants to zero. Each Jordan cell contains one proper highest weight state giving rise to one proper irreducible subrepresentation. We will label this state for a Jordan cell with conformal weight h by $|h;0\rangle$. We choose a basis in the Jordan cell with states $|h;k\rangle$, $k = 0, \ldots, r-1$, such that eq. (2) is replaced by

$$L_0\,|h;k\rangle = h\,|h;k\rangle + |h;k-1\rangle \quad \text{for} \quad k = 1, \ldots, r-1\,, \qquad L_0\,|h;0\rangle = h\,|h;0\rangle\ . \tag{3}$$

The corresponding fields will be denoted $\Psi_{(h;k)}$. Although the OPE of two primary fields might produce logarithmic fields, we will further assume, that primary fields *which are members of Jordan cells* are proper primaries in the sense that OPEs among them only yield again primaries.

As discussed by Rohsiepe [22], the possible structures of indecomposable representations with respect to the Virasoro algebra are surprisingly rich. Besides the defining condition eq. (3), further conditions have to be employed to fix the structure. The simplest case is defined via the additional requirement

$$L_1 \, |h;k\rangle = 0 \,, \qquad 0 \leq k < r \,. \tag{4}$$

This condition means that all fields spanning the Jordan cell are quasi-primary. It will be our starting point in the following. This condition can be relaxed, but this will not concern us here.

Under these assumptions, as shown in [4], the action of the Virasoro modes receives an additional non-diagonal term. The off-diagonal action is defined via $\hat{\delta}_{h_i}\Psi_{(h_j;k_j)}(z) = \delta_{ij}\Psi_{(h_j;k_j-1)}(z)$ for $k_j > 0$ and $\hat{\delta}_{h_i}\Psi_{(h_j;0)}(z) = 0$. Thus,

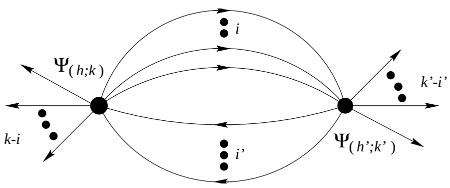
$$L_n \left\langle \Psi_{(h_1;k_1)}(z_1) \ldots \Psi_{(h_n;k_n)}(z_n) \right\rangle = \sum_i z_i^n \left[z_i \partial_i + (n+1)(h_i + \hat{\delta}_{h_i}) \right] \left\langle \Psi_{(h_1;k_1)}(z_1) \ldots \Psi_{(h_n;k_n)}(z_n) \right\rangle \tag{5}$$

for $n \in \mathbb{Z}$. Only the generators L_{-1}, L_0, and L_1 of the Möbius group admit globally valid conservation laws, which usually are expressed in terms of the so-called conformal Ward identities

$$0 = \left\{ \begin{array}{rcl} L_{-1}G(z_1, \ldots z_n) & = & \sum_i \partial_i G(z_1, \ldots z_n) \,, \\ L_0 G(z_1, \ldots z_n) & = & \sum_i (z_i \partial_i + h_i + \hat{\delta}_{h_i}) G(z_1, \ldots z_n) \,, \\ L_1 G(z_1, \ldots z_n) & = & \sum_i (z_i^2 \partial_i + 2z_i[h_i + \hat{\delta}_{h_i}]) G(z_1, \ldots z_n) \,, \end{array} \right. \tag{6}$$

where $G(z_1, \ldots z_n)$ denotes an arbitrary n-point function $\left\langle \Psi_{(h_1;k_1)}(z_1) \ldots \Psi_{(h_n;k_n)}(z_n) \right\rangle \equiv \langle k_1 k_2 \ldots k_n \rangle$ of primary fields and/or their logarithmic partner fields. Here, we already have written down the Ward identities in the form valid for proper Jordan cells in logarithmic conformal field theories. Note that these are now inhomogeneous equations. In principle, we thus obtain a hierarchical scheme of solutions, starting with correlators of total Jordan-level $K = \sum_i k_i = r - 1$, which fix the generic form of all n-pt functions. In particular, correlators of solely proper primary fields vanish identically.

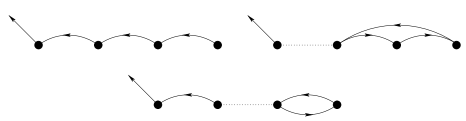
It is helpful to use a graphical representation where each field $\Psi_{(h;k)}(z)$ in a Jordan cell is depicted by a vertex with k outgoing lines. Contractions of logarithmic fields give rise to logarithms in the correlators, where the possible powers with which $\log(z_{ij})$ may occur are determined by graph combinatorics.



Essentially, the terms of the generic of an n-pt function are given by a sum over all admissible graphs subject to the rules

- Each vertex with $k_{\text{out}} > 0$ legs may at most receive $k'_{\text{in}} i \leq r - 1$ legs.
- Each vertex i may only receive legs from vertices $j \neq i$.
- A vertex for a proper primary field, $k_{\text{out}} = 0$, does never receive legs.
- A total of exactly $r - 1$ legs remains open, i.e. are not linked to other vertices.

Let us look at a small example. All admissible graphs, up to permutations, for a 4-pt functions of a $r = 2$ LCFT, where all fields are logarithmic, are given by

graphs($\langle 1111 \rangle$) =  .

Here, and in the following, we suppress all dependencies on the conformal weights and coordinates and denote correlations functions simply by the set of Jordan-levels k_i of the fields $\Psi_{(h_i;k_i)}(z_i)$.

The linking numbers $A_{ij}(g)$ of a graph g yield upper bounds on the powers, with which $\log(z_{ij})$ may occur. One needs a recursive procedure to find all possible terms. One starts with the set of all different choices f_i of $r-1$ legs which remain unlinked. Then, for all levels K' and all choices f_i one has to find all graphs which connect the remaining $K-K'-(r-1)$ legs to vertices. Since the recurrence runs over K', one can now immediately write down the monomials in the $\log(z_{ij})$ as given by the graphs g, multiplied with as yet undetermined constants $C(g)$. Imposing global conformal invariance via eq. (6) fixes some of these constants. Further constraints stem from certain permutation symmetries, since much of the generic structure of the correlators only depends on the Jordan-levels, but not on which fields has what Jordan-levels. Thus, the general ansatz is of the form

$$\langle k_1k_2k_3k_4\rangle = \prod_{i<j}(z_{ij})^{\mu_{ij}} \sum_{(k'_1,k'_2,k'_3,k'_4)} \left[\sum_{g\in G_{K-K'}} C(g)\left(\prod_{i<j}\log^{A_{ij}(g)}(z_{ij})\right)\right] F_{k'_1k'_2k'_3k'_4}(x)\,, \quad (7)$$

where $G_{K-K'}$ is set of graphs for $(k_1-k'_1,\ldots,k_4-k'_4)$, $x=\frac{z_{12}z_{34}}{z_{14}z_{23}}$ is the crossing ratio, and $\mu_{ij}=\frac{1}{3}\left(\sum_k h_k\right)-h_i-h_j$. Since the only explicit dependence on the conformal weights is through the μ_{ij}, one may put $h_1=\ldots=h_4=0$ for simplicity. A further convenient abbreviation is $\ell_{ij}\equiv\log(z_{ij})$. Then, for $r=2$, one easily finds

$$\begin{aligned}
\langle 1000\rangle &= F_0\,,\\
\langle 1100\rangle &= F_{1100}-2\ell_{12}F_0\,,\\
\langle 1110\rangle &= F_{1110}+(\ell_{12}-\ell_{13}-\ell_{23})F_{1100}+(\ell_{13}-\ell_{12}-\ell_{23})F_{1010}-(\ell_{23}-\ell_{12}-\ell_{13})F_{0110}\\
&\quad+(-\ell_{12}^2-\ell_{13}^2-\ell_{23}^2+2\ell_{12}\ell_{23}+2\ell_{12}\ell_{13}+2\ell_{23}\ell_{13})F_0\\
&= F_{1110}+\mathcal{P}_{(123)}\left\{(\ell_{12}-\ell_{23}-\ell_{13})F_{1100}\right\}+\mathcal{P}_{(123)}\left\{\ell_{12}(\ell_{12}-\ell_{23}-\ell_{13})F_0\right\}\,.
\end{aligned}$$

In the last case, we used the fact that the resulting form of the correlator must obviously be invariant under permutation symmetry of the first three Jordan-levels (not the first three fields!), which leads to identifications between many of the $F_{k_1k_2k_3k_4}(x)$. The only caveat is that one has to respect the ordering $i<j$ in all z_{ij} or ℓ_{ij}. Thus, in the above example, $\mathcal{P}_{(123)}=(123)+(231)+(312)$ subject to the above rule. However, since the full correlators, i.e. the correct single-valued combinations of holomorphic and anti-holomorphic part, will only involve monomials in $\log|z_{ij}|^2$, the ordering can be neglected for the ℓ_{ij}. In case that one considers the subset of correlators where all four fields have the same conformal weight, $h_i=h_j$, one finds that

$$F_{k_1k_2k_3k_4}(x)=F_{k_{\sigma(1)}k_{\sigma(2)}k_{\sigma(3)}k_{\sigma(4)}}(x)\quad\forall\ \sigma\in S_4\,.$$

In the more general case with arbitrary conformal weights, σ is restricted to the subgroup of S_4, under which the original Jordan-levels on the left hand side remain invariant. It is important to note that this still implies identifications of structure functions with non-trivial exchanges of their Jordan-level labels on the right hand side. Furthermore, we found cases where an even higher symmetry can be implemented, identifying structure functions with each other, which are not related at all by permutation symmetries. We leave a full discussion to our forthcoming publication [18].

Unfortunately, even when all permutation symmetries are used to relate different $F_{k_1k_2k_3k_4}$ with each other, explicit formulæ easily become very cumbersome, in particular for $r>2$. Alread the remaining case for $r=2$, the correlator with four logarithmic fields, explodes to the following monstrum, despite the fact that permutation symmetry has been fully exploited:

$$
\begin{aligned}
\langle 1111\rangle \;=\;& F_{1111} + \mathcal{P}_{(1234)}\big\{\big[(-\ell_{12} - \ell_{34} + \ell_{23} + \ell_{14})C_1 + (\ell_{13} + \ell_{24} - \ell_{12} - \ell_{34})C_2 \\
& \qquad - \ell_{14} + \ell_{34} - \ell_{13})\big]F_{0111}\big\} \\
+\;& \mathcal{P}_{(12)(34)}\big\{\big[(\ell_{13}^2 + \ell_{24}^2 - \ell_{14}^2 - \ell_{23}^2 + 2(-\ell_{34}\ell_{24} - \ell_{12}\ell_{24} + \ell_{34}\ell_{14} + \ell_{13}\ell_{24} \\
& \quad - \ell_{13}\ell_{34} + \ell_{23}\ell_{34} + \ell_{12}\ell_{23} - \ell_{12}\ell_{13} - \ell_{23}\ell_{14} + \ell_{12}\ell_{14}))C_3 \\
& + (-(\ell_{23} + \ell_{14})^2 + \ell_{23}\ell_{34} + \ell_{12}\ell_{14} - \ell_{13}\ell_{34} + \ell_{34}\ell_{14} + \ell_{13}\ell_{14} \\
& \quad - \ell_{34}\ell_{24} - \ell_{12}\ell_{13} - \ell_{12}\ell_{24} + \ell_{23}\ell_{24} + \ell_{23}\ell_{13} + \ell_{12}\ell_{23} + \ell_{24}\ell_{14}))C_4 \\
& - \ell_{34}^2 - \ell_{23}^2 - \ell_{14}^2 + 2\ell_{23}\ell_{34} + 2\ell_{34}\ell_{14} - 2\ell_{12}\ell_{34} - \ell_{23}\ell_{14} + \ell_{23}\ell_{24} \\
& \quad - \ell_{12}\ell_{13} + \ell_{12}\ell_{14} + \ell_{12}\ell_{23} - \ell_{12}\ell_{24} + \ell_{13}\ell_{14} + \ell_{13}\ell_{24})\big]F_{1100}\big\} \\
+\;& \big[2(\ell_{12}\ell_{24}\ell_{14} - \ell_{23}\ell_{13}\ell_{14} + \ell_{23}\ell_{34}\ell_{24} - \ell_{24}\ell_{13}\ell_{34} - \ell_{23}\ell_{34}\ell_{14} \\
& \quad - \ell_{12}\ell_{23}\ell_{34} - \ell_{12}\ell_{34}\ell_{24} - \ell_{23}\ell_{13}\ell_{24} + \ell_{12}\ell_{23}\ell_{13} + \ell_{13}\ell_{34}\ell_{14} \\
& \quad - \ell_{13}\ell_{14}\ell_{24} - \ell_{23}\ell_{24}\ell_{14} - \ell_{12}\ell_{13}\ell_{24} - \ell_{12}\ell_{23}\ell_{14} - \ell_{12}\ell_{13}\ell_{34} - \ell_{12}\ell_{34}\ell_{14}) \\
& + 2(\ell_{13}^2\ell_{24} + \ell_{12}^2\ell_{34} + \ell_{14}^2\ell_{23} + \ell_{23}^2\ell_{14} + \ell_{34}^2\ell_{12} + \ell_{24}^2\ell_{13})\big]F_0\,.
\end{aligned}
$$

3 Permutation symmetries

Note that certain constants still remain free. The reason for this becomes apparent, when we use the graphical notation introduced earlier, which makes the structure of the formulæ much more transparent and compact. In essence, the polynomials in the ℓ_{ij} are completely symmetrized with respect to their generating graphs. If we assume that all conformal weights are equal, $h_i = h_j$, then the correlators enjoy even more symmetries, since this means that it must be invariant under permutations of the fields of the same Jordan-level. Due to limitation of space, we will restrict ourselves here to this nice case and refer the reader to [18] for the general case. Hence, we obtain, in our $r = 2$ example,

$$
\begin{aligned}
\langle 1110\rangle &= F_{1110} - \mathcal{P}_{S_3}(\ell_{12})F_{0011} + \mathcal{P}_{S_3}(2\ell_{12}\ell_{23} - \ell_{12}^2)F_0 \\
&= F_{1110} - \mathcal{P}_{S_3}(\;)F_{0011} + \mathcal{P}_{S_3}(2\; - \;)F_0\,, \qquad (8)\\
\langle 1111\rangle &= F_{1111} - \tfrac{1}{6}\mathcal{P}_{S_4}(\;)F_{0111} + \tfrac{1}{4}\mathcal{P}_{S_4}(\; + K_{S_4}^{(2)})F_{0011} \\
&\quad + \mathcal{P}_{S_4}(\tfrac{1}{2}\; + \tfrac{1}{3}\; - \;)F_0\,. \qquad (9)
\end{aligned}
$$

Note that no free constants remained. On the other hand, to make the epxressions as symmetric as possible, we encounter additional terms $K \in \ker L_m^{\text{offdiag}}$ in the kernel of the nilpotent part of the Virasoro generators:

$$
\begin{aligned}
\ker(L_m - L'_m) &= \langle K_1 \equiv \log(x),\; K_2 \equiv -\log(1 - 1/x)\rangle \\
&= \langle \ell_{12} + \ell_{34} - \ell_{14} - \ell_{23},\; \ell_{12} + \ell_{34} - \ell_{13} - \ell_{24}\rangle\,, \qquad (10)\\
K_{S_4}^{(2)} &= K_1^2 - K_1K_2 + K_2^2\,. \qquad (11)
\end{aligned}
$$

Here, L'_m denotes the ordinary part of the Virasoro mode without the off-diagonal action $\hat{\delta}$ from eq. (5). In principle, all 4-pt functions for arbitrary rank r LCFTs can be computed in this way. We conclude these notes with a few examples for rank $r = 3$ and, for the sake of simplicity, all conformal weights identical. Each individual Jordan level k_i may now vary in the range $0 \leq k_i \leq 2$. We find

$$\langle 2000 \rangle = \langle 1100 \rangle = F_0 \,, \tag{12}$$

$$\langle 2100 \rangle = F_{2100} - 2\ell_{12} F_0 \,, \tag{13}$$

$$\langle 1110 \rangle = F_{1110} - (\ell_{12} + \ell_{23} + \ell_{13}) F_0 \tag{14}$$

$$= F_{1110} - \mathcal{P}_{(123)} \left\{ \ell_{12} F_0 \right\} \,, \tag{15}$$

where the first line is due to the general result that the lowest total Jordan level with non-vanishing correlator is $K = r - 1$ and that this correlator looks like a correlator of four primary fields in ordinray non-logarithmic CFT. A more involved correlator for $r = 3$ is the following, where again a kernel term shows up:

$$\langle 1111 \rangle = F_{1111} - \frac{1}{6} \mathcal{P}_{S_4}(\ell_{12}) F_{0111} + \Big\{ \mathcal{P}_{S_4} \Big[-\frac{1}{4} \ell_{34}^2 + \frac{1}{2} \ell_{34} \ell_{24} \Big] + K_{S_4}^{(2)} \Big\} F_0 \,. \tag{16}$$

Note the similarity to the $r = 2$ case. With $K_\pm = K_1 \pm K_2$ and $\mathcal{P}_4 = 1 + P_{12} + P_{34} + P_{12}P_{34}$, a really non-trivial example, where a non-graphical expansion would fill several pages, is

$$\begin{aligned}
\langle 2211 \rangle = {} & F_{2211} \\
& + \mathcal{P}_4 \Big\{ -\tfrac{1}{2} P_{(13)(24)} + K_+ \Big\} F_{0122} + \mathcal{P}_4 \Big\{ -\tfrac{1}{2} + K_+ \Big\} F_{1112} \\
& + \mathcal{P}_4 \Big\{ \big[\tfrac{1}{2} P_{(24)} - \tfrac{1}{6} \big] + \big[\tfrac{1}{3} - \tfrac{1}{2} P_{(14)} \big] + \tfrac{1}{12} + K_{S_4}^{(2)} \Big\} F_{1111} \\
& + \mathcal{P}_4 \Big\{ \big[\tfrac{1}{2} - \tfrac{1}{2} P_{(24)} \big] + \big[P_{(12)} + \tfrac{1}{2} P_{(14)} \big] - \tfrac{1}{4} + K_{S_4}^{(2)} + K_1 K_2 \Big\} F_{0112} \\
& + \mathcal{P}_4 \Big\{ \big[\tfrac{1}{2} P_{(23)} - \tfrac{1}{2} \big] + \big[P_{(24)} - \tfrac{1}{2} P_{(243)} \big] - \tfrac{1}{4} P_{(13)(24)} + K_-^2 \Big\} F_{0022} \\
& + \mathcal{P}_4 \Big\{ \big[-\tfrac{1}{2} - 2P_{(14)} \big] + \big[2P_{(243)} + P_{(24)} - P_{(13)} \big] \\
& \qquad - P_{(14)} + 2 - - \tfrac{1}{2} + K_-^2 K_+ \Big\} F_{0012} \\
& + \mathcal{P}_4 \Big\{ \big[\tfrac{1}{6} P_{134} - \tfrac{7}{6} - \tfrac{1}{6} P_{(13)} \big] - \big[2P_{(23)} + \tfrac{5}{6} P_{(234)} + \tfrac{1}{2} P_{(132)} \big] \\
& \qquad + \big[\tfrac{1}{12} + \tfrac{11}{6} P_{(23)} \big] + \big[\tfrac{1}{6} + \tfrac{5}{6} P_{(14)} \big] + \tfrac{1}{2} + \tfrac{1}{3} + K_-^2 K_+ \Big\} F_{0111} \\
& + \mathcal{P}_4 \Big\{ \big[\tfrac{1}{2} P_{(12)(34)} + \tfrac{1}{2} P_{(1243)} - 2P_{(124)} - \tfrac{1}{2} P_{(142)} - 2P_{(143)} \big] + \tfrac{3}{16} \\
& \qquad + \big[\tfrac{1}{2} P_{(34)} + \tfrac{1}{2} P_{(123)} - \tfrac{1}{2} P_{(134)} \big] + \big[\tfrac{3}{4} P_{(234)} - P_{(1243)} + \tfrac{5}{4} P_{(132)} \big] \\
& \qquad + \big[\tfrac{1}{4} P_{(12)(34)} - \tfrac{1}{2} P_{(23)} + \tfrac{1}{2} P_{(1234)} - \tfrac{1}{8} P_{(14)} \big] + \big[\tfrac{5}{8} P_{(24)} - \tfrac{1}{8} \big] \\
& \qquad + \big[P_{(234)} - \tfrac{1}{2} P_{(243)} - \tfrac{3}{4} P_{(14)} \big] + \big[2P_{(23)} + P_{(14)} - P_{(1324)} \big] \\
& \qquad + \big[\tfrac{1}{2} P_{(123)} - P_{(12)(34)} - \tfrac{1}{2} P_{(1234)} \big] + \tfrac{1}{2} P_{(23)} + + K_-^2 K_1 K_2 \Big\} F_0 \,.
\end{aligned} \tag{17}$$

To summarize, the computational complexity grows heavily with the rank r and total Jordan level K. Already the generic solution for $r = 2$ and $K_{\max} = 4(r-1) = 4$ needs a computer program. The form of 4-pt functions, as determined by global conformal invariance, is much more complicated than in the ordinary case and crossing symmetry must explicitly be taken into account to fix it. There exist additional degrees of freedom $\ker(L_m - L'_m)$ not present in ordinary CFT.

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