Michael Flohr · ⋄ · Department of Mathematics · ⋄ · King's College London
· ⋄ · The Strand · ⋄ · London WC2R 2LS · ⋄ · United Kindgom · ⋄ ·
· ⋄ · HomePage: http://www.mth.kcl.ac.uk/˜flohr/ · ⋄ · Email: flohr@mth.kcl.ac.uk · ⋄ ·
+44-(0)171- · ⋄ · -836 5454×1073 (Voice) · ⋄ · -873 2017 (Fax) · ⋄ · -873 2217 (Office)

Logarithmic Conformal Field Theory on Riemann Surfaces & Applications to Strings, Branes, and Seiberg-Witten Models

My current research centres around the application of logarithmic conformal field theory (LCFT) in high-energy physics. After establishing LCFT in a sufficiently rigorous way (see [1] and references therein), the goal is now to improve on mathematical aspects of modern string theory with its new family members such as branes, M-theory and the resulting exactly solvable low-energy effective field theories. Use of conformal field theory tools in string theory is, of course, not new. However, with the advent of LCFT we may have a better machinery to describe physics whose underlying structure is determined by the geometry of Riemannian manifolds, in particular Riemannian surfaces. Perhaps not entirely surprisingly, it turns out that, for example, the masses of BPS states in Seiberg-Witten low-energy effective field theories are determined by one-and-the-same mathematical structure as the gap-less topologically non-trivial bulk-excitations in the Haldane-Rezayi quantum Hall state (see [2] for some condensed matter physics applications of LCFT). Both are governed by the theory of Riemannian surfaces. More precisely, both observables are related to sections of the canonical line bundle over a certain Riemannian surface, and the physicists way of doing this is to put a conformal field theory (CFT) on the surface.

So far, physicists usually take the surface as given and let a CFT live on it whose main purpose is to account for any infinities, i.e. non-compact extensions (such as the in- and out-states in a string amplitude). These are simulated by vertex operators put on a compact surface. Here, the question should immediately arise whether we can reduce our surface even further and replace all other topologically non-trivial features by certain operators put then on a simple Riemannian sphere. In fact, it is possible to recast the common representation of an arbitrary Riemannian surface as a multi-sheeted ramified covering of a much simpler one (often $\mathbb{CP}^1$) entirely in terms of a particular CFT living on the simpler surface. However, the price paid is that this latter CFT becomes logarithmic – the logarithms in the correlation functions introduce the necessary scales which allow us to move and measure in the moduli-space of the represented more complicated surface. The operator content of the resulting LCFT features fields which precisely simulate branch points, nodes, etc.

Having identified the peculiarities of such LCFTs, one can move on and apply them to physics. As a first step, I started a project to treat Seiberg-Witten low energy effective field theories (e.g. four-dimensional N=2 super-symmetric Yang-Mills theories) with such CFT methods. The key point is that all physical relevant information is encoded in the periods on the moduli space of such a theory, the latter being a Riemannian surface. I proved in [3] that the periods of Seiberg-Witten theories with gauge groups A_n or D_n are given by the conformal blocks of correlation functions of the rational logarithmic CFT with $c = -2$, automatically including cases with (massive) hyper-multiplets.

The periods are taken with respect to a particular 1-form, the so-called meromorphic Seiberg-Witten differential $\lambda_{\rm SW}$. This form does not only describe the surface as a ramified covering map, but also induces a particular metric on it. The latter gives rise to a physical interpretation where the periods are related to integrals along strings which wrap around the surface created by intersecting branes, measuring the total tension of the wrapped string. More precisely, the geometry is higher-dimensional – the integration is over a certain 3-cycle in a fibred Calabi-Yau compactification space – but can be split into a trivial part and the Riemannian surface part. However, there is in principal no obstacle to apply CFT methods directly to complex manifolds in higher dimensions. Indeed, as a simple example, the periods of the quintic (which is a particular simple example of one-parameter families of Calabi-Yau spaces) can also be expressed in terms of LCFT correlation functions.

One may now ask what the advantage is of using the LCFT description instead of more common and established mathematical tools to deal with computations on Riemannian surfaces. Firstly, many computations are much simpler in the LCFT picture. For example, all differential equations appearing in $SU(N_c)$ Seiberg-Witten $N{=}2$ super-symmetric models are only of second degree, even when $N_f < 2N_c$ massive hyper-multiplets are present. This is to be contrasted with the third order Picard-Fuchs equations one is otherwise confronted with. Also, the evaluation of the Seiberg-Witten periods in asymptotic regions of moduli space becomes just a simple and straight-forward task in applying operator product expansions.

Secondly, and more importantly, the LCFT picture might be much more physical. As already indicated above, the intersections of (e.g. in the type II framework) $D4$-branes on $NS5$-branes can be viewed in a very similar way as flux quanta piercing through a two-dimensional quantum Hall droplet. In the low-energy picture, one is not interested in the interaction among the intersecting branes, but only in their effect on an excitation (such as a wrapped string), as one is not so much interested in the interaction among the flux quanta than in their influence on a quasi-particle state.

The similarities go further. The bulk wave-functions in the quantum Hall effect are described by Euclidean CFTs, while the edge excitations are given in terms of Minkowskian ones. In Seiberg-Witten theory, the Riemannian surface is actually the target space of a non-critical string. However, the former can be viewed as the world-sheet of a Euclidean string, and the LCFT put on it is nothing else than its spin zero/one matter content (also known as the spin zero/one ghost system). The non-critical string wrapping around the surface is a proper Minkowskian one, and corresponds to an edge excitation. In fact, the Seiberg-Witten differential can be rewritten as the exponential of a canonical potential a gas of charged particles would exert on a probe, where the charges are given by the precise nature of all singular fibres in the fibred Calabi-Yau compactification space. Hence, Laughlin's plasma-analogy supporting his quantum droplet hypothesis can be invoked in this setting, too. The precise mathematics and the physical interpretation of all of this is my current interest.

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[2] M. Flohr, *Fusion and Tensoring of Conformal Field Theory and Composite Fermion Picture of Fractional Quantum Hall Effect*, Mod. Phys. Lett. **A11** (1996) 55-68 [hep-th/9605152]; M. Flohr, *2-Dimensional Turbulence: A Novel Approach via Logarithmic Conformal Field Theory*, Nucl. Phys. **B482** (1996) 567-578 [hep-th/9606130]; M. Flohr, V. Gurarie, C. Nayak, *The Haldane-Rezayi Quantum Hall State and Conformal Field Theory*, Nucl. Phys. **B498** (1997) 513-538 [cond-mat/9701212]

[3] M. Flohr, *Logarithmic Conformal Field Theory & Seiberg-Witten Models*, KCL-MTH-98-36 (1998) 13pp., to be published in Phys. Lett. **B** [hep-th/9808169]