“Modular Chaos”

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Abstract

Since the invention of two dimensional conformal quantum field theory by BPZ [1], one of the great outstanding problems is the classification of all rational conformal field theories (RCFTs). BPZ have found the discrete series of minimal models, later all unitary RCFTs with \( c = 1 \) could be classified [2,3,4]. The study of \( W \)-algebras proved to be fruitful for the search of new rational models [5,6]. In particular \( W \)-algebras were found which exist only for some special values of the central charge. Recently two discrete series of new rational models (coming from certain \( W \)-algebras) have been found [7]. These theories exist for \( c = 1 - 8k, k \in \mathbb{Z} \) and complete the classification of all theories with \( c_{\text{eff}} = c - 24h_{\text{min}} = 1 \) which is the non unitary generalization of the \( c = 1 \) theories. \( h_{\text{min}} \) denotes the energy of the minimal highest weight state.)

The set of all theories with \( c = 1 \) is a well known manifold \( M \subset \mathbb{R}^2 \). Here we want to discuss the set of all models with \( c_{\text{eff}} = 1 \). Of course, this set \( X \subset \mathbb{R}^3 \) contains \( M \), but also, in a new direction, the set \( M' \) of non unitary theories. But the most surprising fact is the structure of the subset \( \mathcal{R}(M') \) of non unitary RCFTs in \( M' \). It is highly “chaotic” and seems to be a self similar fractal, nonetheless dense in \( \mathbb{R}^2 \). Moreover, its points are in one-to-one correspondence to the elements of the modular group \( \text{PSL}(2, \mathbb{Z}) \).

Besides nice images of \( \mathcal{R}(M') \), we present first explanations of its structure and point out the connection to the modular group. Moreover, there might be a deep relationship to the theory of the fractional quantum hall effect.

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The moduli space of non-unitary RCFTs with effective central charge $c_{\text{eff}} = 1$. The partition function can be expressed in terms of two Gaussian models partition functions with different compactification radii, $Z = (Z(R) + Z(R'))/2$. Rationality of the CFT imposes the following condition on the radii: $2R^2 = p/q, 2R'^2 = p'/q'$ with $p, q, p', q'$ positive integers such that $p'q' - pq = \pm 1$. The central charge is then $c = 1 - 24pq$. We plotted all points in the $(x = 2R^2, y = R'^2)$ plane, for which $pq \leq 10^6$. Shown is the region $[0, 1] \times [0, 1]$, the other regions of the plane are identified via $R \leftrightarrow 1/2R$ duality. Also, the image is symmetric under $R \leftrightarrow R'$. 