Examination, **Statistische Physik**

conducted on 26 January, 2012

Name, Vorname	Matrikelnummer	Semester	Punktzahl

Problem 1 *Ten short questions* (10 Punkte)

If not explicitly asked for, do not provide calculations or explanations, but just short answers.

a. What is the entropy of a system of N states with uniform probability distribution?

b. Two systems with partition functions Z_1 and Z_2 are combined without interaction. What is the total partition function Z ?

c. Can the fugacity be negative? Why (not)?

d. In case the Hamiltonian is spin independent, how does a fermion's spin j affect its density of states?

e. Which two types of (slow) processes make up the Carnot cycle?

f. What property is determined by the Clausius-Clapeyron equation?

g. Which mathematical operations transform the energy U to the free enthalpy G ?

h. To which value tends the Ising-model partition function for N spins as the temperature goes to infinity?

i. What is the thermodynamic limit?

j. Is the Maxwell velocity distribution symmetric about its maximum? Why (not)?

Problem 2 *Ficticious thermodynamics* (8 Punkte)

Imagine an ideal gas in the classical limit characterized for large N by the (hypothetical) free energy

$$F(V,N,\tau) \;=\; \tau N\Big(\ln\frac{N}{V} - \frac{\tau}{\tau_0} - 1\Big)\;,$$

where V is the volume accessible to the gas, N is the average number of particles, τ is the temperature, and τ_0 is some positive constant.

a. Compute the internal energy $U(V,N,\tau)$, the pressure $p(V,N,\tau)$ and the entropy $\sigma(V,N,\tau)$.

b. What is the minimal amount of work W that is needed in order to compress this gas from a volume V_1 to a volume V_2 while maintaining the temperature τ and number of particles N constant?

c. Consider a different scenario where the gas expands isentropically (i.e., with no change in entropy) from the volume V_2 back to the volume V_1. Suppose the number N of particles did not change. If the initial gas temperature (at V_2) is τ, what is its final temperature τ' (at V_1)?

Problem 3 *Canonical ensembles* (12 Punkte)

Consider a quantum system whose Hamiltonian has an eigenvalue 0 with no degeneracy, an eigenvalue $\epsilon > 0$ with degeneracy 2, and *no further* eigenvalues.

a. Suppose that this system is in thermodynamic equilibrium at temperature τ. Express the average internal energy U and the entropy σ as functions of ϵ and τ.

b. Let us call the previous system a "particle". Suppose there are two such particles in thermal equilibrium at temperature τ, that they do not interact, and that they are (indistinguishable) *bosons*. Write down the expected internal energy U.

c. Suppose the number of these bosons is allowed to vary, and the bosons are in thermal equilibrium (at temperature τ) and in diffusive equilibrium with fugacity λ. Write down the partition function (Gibbs sum) $\mathcal{Z}$ and the average internal energy U.

d. Suppose now that our particles are *fermions*. Again, assume they are in thermal equilibrium at temperature τ and in diffusive equilibrium with fugacity λ. Write down the partition function (Gibbs sum) $\mathcal{Z}$ and the average number N of particles.

Hint: Recall that $\sum_{n=0}^{\infty} x^n = 1/(1-x)$.

Duration: 180 minutes **Sum: 30 Punkte** **Good luck!**