7. Präsenzübung, **Statistische Physik**

zu bearbeiten am Donnerstag, 24.11.2011

Aufgabe P18 *Electromagnetic modes*

a. Write down the energy eigenvalues of the Hamiltonian of the quantum electromagnetic field in a cubic cavity of side L, together with their degeneracies.

b. Compute the corresponding "density of state" function per unit energy $D(\epsilon)$ such that, when integrated from 0 to ϵ, it yields the number of field modes with frequencies smaller than $\omega = \epsilon/\hbar$, times 2 to account for the elicity degeneracy. Your formula need only be valid for frequencies large compared to their quanta.

Aufgabe P19 *The sun's radiation field*

The Planck radiation law states that the energy per unit volume per unit angular frequency of the electromagnetic field in thermal equilibrium at temperature τ is

$$u_\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/\tau} - 1}.$$

Its integral over ω yields the energy density

$$u = \int d\omega\, u_\omega = \frac{\pi^2}{15\hbar^3 c^3}\tau^4$$

We will assume that the electromagnetic near the surface of the sun is in thermal equilibrium with it, and that this energy is carried away at the speed of light by photons traveling in random directions.

a. Determine the maximum of u_ω over the angular frequency ω as a function of τ. Given that the angular frequency at which the sun's light is most intense is

$$\omega_0 \simeq 2.14 \cdot 10^{15}\text{Hz},$$

give an estimate of its surface temperature T_S in Kelvin.

Hint: The solutions to the equation $1 - e^{-x} = x/3$ are $x = 0$ and $x \simeq 2.82$.

b. Imagine that the energy density u is carried by photons traveling at the speed of light in random directions. Show that the flux of energy (power per unit area) leaving the surface of the sun as thermal radiation has the form

$$J = \frac{1}{4} c\, u.$$

Hint: imagine a gas of photon with a flat boundary, and count how many are escaping that boundary per unit time, if they travel along straight lines in random directions at the speed of light.

c. Compute the total power P_S radiated away by the sun's surface.

Hint: the sun's radius is $R_S \simeq 7 \cdot 10^8$ m.

d. Suppose that the earth absorbs the energy of all the photons that it collides with. What must be the temperature at the surface of the earth for it to evacuate all that power away as thermal radiation?

Hint: the distance between the earth and the sun is $d \simeq 1.5 \cdot 10^{11}$ m. This is far enough that you can treat the sun as a point.