Dirac- Born- Infeld - Elektrodynamik

1) Es gilt

$$\sqrt{-\det(\eta + 2\pi\alpha' F)} = \sqrt{-\det\left(\eta\left(\eta^{-1}\eta + 2\pi\alpha'\,\eta^{-1}F\right)\right)}$$
$$= \sqrt{-\det\eta}\,\sqrt{\det(1+M)}$$

2)
$$\sqrt{\det(1+M)} = e^{\frac{1}{2}\operatorname{tr}\ln(1+M)}$$
$$= \exp\left[\frac{1}{2}\operatorname{tr}\left(-\sum_{n=1}^{\infty}\frac{(-1)^n}{n}M^n\right)\right]$$
$$= \exp\left(-\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\operatorname{tr}(M^n)\right)$$
$$\overset{*}{=} \exp\left(-\frac{1}{2}\sum_{m=1}^{\infty}\frac{1}{2m}\operatorname{tr}(M^{2m})\right)$$

* ungerade Potenzen von M tragen unter der Spur nicht bei (s.u.)

Entwicklung bis zur Ordnung F^4:

$$\sqrt{\det(1+M)} \sim \exp\left[-\frac{1}{2}\left(\frac{1}{2}\operatorname{tr}(M^2) + \frac{1}{4}\operatorname{tr}(M^4) + \frac{1}{6}\operatorname{tr}(M^6) + \dots\right)\right]$$
$$= \exp\left(-\frac{1}{4}\operatorname{tr}(M^2) - \frac{1}{8}\operatorname{tr}(M^4) - \frac{1}{12}\operatorname{tr}(M^6) + \dots\right)$$

$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} \longrightarrow$

$$= 1 - \frac{1}{4}\operatorname{tr}(M^2) - \frac{1}{8}\operatorname{tr}(M^4) - \frac{1}{12}\operatorname{tr}(M^6)$$
$$+ \frac{1}{2!}\left(\frac{1}{4}\operatorname{tr}(M^2) + \frac{1}{8}\operatorname{tr}(M^4) + \frac{1}{12}\operatorname{tr}(M^6)\right)^2$$
$$- \frac{1}{3!}\left(\frac{1}{4}\operatorname{tr}(M^2) + \dots\right)^3$$

$$= 1 - \frac{1}{4}\operatorname{tr}(M^2) - \frac{1}{8}\operatorname{tr}(M^4) - \frac{1}{12}\operatorname{tr}(M^6)$$
$$+ \frac{1}{2!}\,\frac{1}{16}\left(\operatorname{tr}(M^2)\right)^2 + \dots \operatorname{tr}(M^2)\operatorname{tr}(M^4) + \dots$$

$$\boxed{\operatorname{tr}(M^2) \sim \eta^{\mu\sigma}F_{\sigma\lambda}\eta^{\lambda\nu}F_{\nu\mu} = F^{\mu\nu}F_{\nu\mu}}$$

$$= 1 - \frac{1}{4}\,4\pi^2\alpha'^2 F_{\mu\nu}F^{\nu\mu} - \frac{1}{8}\,16\pi^4\alpha'^4 F_{\mu\nu}F^{\nu\sigma}F_{\sigma\lambda}F^{\lambda\mu}$$
$$+ \frac{1}{32}\left(4\pi^2\alpha'^2\right)^2 F_{\mu\nu}F^{\nu\mu}F_{\sigma\lambda}F^{\lambda\sigma} + \dots$$

$$= 1 - \pi^2\alpha'^2 F_{\mu\nu}F^{\nu\mu} - 2\pi^4\alpha'^4 F_{\mu\nu}F^{\nu\sigma}F_{\sigma\lambda}F^{\lambda\mu}$$
$$+ \frac{1}{2}\pi^4\alpha'^4 F_{\mu\nu}F^{\nu\mu}F_{\sigma\lambda}F^{\lambda\sigma} + \dots$$