# Irrelevance of $\mathrm{D} = 26$

Norbert Dragon
Bad Honnef
March 12th, 2018

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Stone-von Neumann Theorem

Covariant String

Timelike String

Lightcone String

Representations of Lie-Algebras

Blessing in Disguise

www.itp.uni-hannover.de/~dragon

# Unitary Representations

Quantum theories give probability distributions for results of measurements

$$w(i, A, \Psi) = \frac{|\langle \Lambda_i | \Psi \rangle|^2}{\langle \Lambda_i | \Lambda_i \rangle \langle \Psi | \Psi \rangle} \ , \Psi, \Lambda_i \in \mathcal{H} \ , w(i, A, \Lambda_j) = \delta_{ij}$$

Wigner: Symmetries are realized by unitary or antiunitary transformations.

Relativistic is a quantum theory, if a unitary representation (of the connected part of the cover) of the Poincaré group acts on its Hilbert space.

String theory is no relativistic quantum theory, because it lacks a Hilbert space with a unitary representation of the Poincaré group

Recognize problems not as fans but with scientific impartiality and curiosity

# Stone-von Neumann Theorem

All unitary, irreducible representations of

$$e^{itP}e^{isX} = e^{it\cdot s}e^{isX}e^{itP} \ , \ [X_m, P^n] = i\hbar\,\delta_m{}^n \ , \ t, s \in \mathbb{R}^D$$

are unitarily equivalent to the multiplicative and differential operations

$$(P^n\Psi)(p) = p^n\Psi(p) \ , \ (X_m\Psi)(p) = -i\frac{\partial\Psi}{\partial p^m}(p)$$

Algebra of $X^m, P_n$ acts on the Schwartz space $\mathcal{S}(\mathbb{R}^D)$ with scalar product

$$\langle\Phi|\Psi\rangle = \int d^Dp\,\Phi^*(p)\,\Psi(p)$$

representation is irreducible

no compatible restriction to functions which vanish in some fixed domain

# The Covariant String

The covariant string (flag ship) employs

$$[X_m, P^n] = i\,\delta_m{}^n\ ,\ \ m, n \in \{0, 1, \ldots D - 1\}$$

and restricts physical states to mass shells (D dimensional volume = 0)

$$\Big(\big((P^0)^2 - \vec{P}^2\big) - 2(N - 1)\Big)\Psi_{\text{phys}} = 0$$

where N has a discrete spectrum.

$\Rightarrow \langle\Phi|\Psi_{\text{phys}}\rangle = 0\ \forall\Phi \Rightarrow \Psi_{\text{phys}} = 0$

Theorem: Covariant String has no physical state.

To compare: acceptable Quantum theories contain $\leq (D - 1)$ Heisenberg pairs

# The Timelike Gauge

employs excitation operators $\alpha^{i}_{-n}$ which commute with $\vec{P}$

Incompatible with a unitary representation of the Lorentz group

# The Lightcone String

related problem: continuous helicity $h$

$$D = \begin{pmatrix} \partial_{p_x} \\ \partial_{p_y} \\ \partial_{p_z} \end{pmatrix} - \frac{ih}{|\vec{p}|(|\vec{p}| + p_z)} \begin{pmatrix} p_y \\ -p_x \\ 0 \end{pmatrix}$$

noncommutative (monopolian) geometry

$$[D_i, D_j] = F_{ij} = i\, h\, \epsilon_{ijk} \frac{p^k}{|\vec{p}|^3}$$

$$-iM_{ij} = -(p^i\, D_j - p^j\, D_i) - i\, h\, \epsilon_{ijk} \frac{p^k}{|\vec{p}|}\ ,\ \ -iM_{0i} = |\vec{p}|\, D_i$$

satisfy the Lorentz algebra for all $h \in \mathbb{R}$! Possible?

# Representation of a Lie-Algebra Lie$(G)$

$\mathcal{D} \subset \mathcal{H}$ dense subspace

Representation $\pi$ : Lie(G) $\rightarrow L(\mathcal{D}) : \pi(x)\mathcal{D} \subset \mathcal{D}$

$$\pi(\alpha x + \beta y) = \alpha\pi(x) + \beta\pi(y)$$
$$\pi([x, y]) = \pi(x)\pi(y) - \pi(y)\pi(x)$$
$$\langle \pi(x)\Phi|\Psi\rangle = -\langle \Phi|\pi(x)\Psi\rangle$$

$\forall x, y \in$ Lie$(G)$ , $\forall \alpha, \beta \in \mathbb{R}$ , $\forall \Phi, \Psi \in \mathcal{D}$

Schmüdgen: $U_g : \mathcal{H} \rightarrow \mathcal{H}$, $\langle \Phi|U_g\Psi\rangle$ measurable

$\mathcal{D} =$ Gårding space $\{\Psi_f : \Psi_f = \int_G d\mu_g\, f(g)\, U_g\Psi \}$

f smooth, compact support, $U_g\Psi_f = \Psi_{f\circ g^{-1}}$, $U_g\Psi$ smooth function of g

$U_{e^x} = e^{\pi(x)}$, $\pi(x)\Psi = \lim_{t\rightarrow 0}(e^{t\pi(x)} - 1)/t\,\Psi$ defined $\forall\Psi \in \mathcal{D}$,

$\mathcal{D} \sim \mathcal{S}(\mathbb{R}^{D-1})$ (massive) or $\mathcal{D} \sim \mathcal{S}(S^{D-2} \times \mathbb{R})$ (massless, tachyonic)

# The Lightcone String

The Lightcone string employs operators $(p_+)^{-1}$ and $\alpha^i_n$

which do not map $\mathcal{D}$ to $\mathcal{D}$

whatever Lie algebra they formally satisfy

they cannot generate a unitary representation of G=Poincaré group

$(p_+)^{-n}$ diverges on tachyonic and massless states

excitations $\alpha^i_m$ cannot relate smoothly massive and massless shells

$\alpha^i_m(P + k(P)) = P\alpha^i_m$ , $q + k(q) = p(q)$

$(\alpha\Psi)(p(q)) = U(q) \left(\det \frac{\partial p}{\partial q} \frac{N(q)}{N(p)}\right)^{\frac{1}{2}} \Psi(q)$

$p$ massive, $q$ massless $\exists \underline{q} : \frac{\partial p}{\partial q} = 0$ or $\infty$

$\alpha^i_m$ generate negative energy shells and states with diverging $\langle\Psi|(P)^n\Psi\rangle$

## Blessing in Disguise

If the canonical generators of string theory generated the $D = 26$ Poincaré group:

25 continuous, unbounded spatial momenta would be predicted irrevocably!

Canonical generators of the lightcone gauged string do not generate Poincaré

Does not rule out successful other constructions (unknown however)

Compactification to $D = 4$ has to occur *before* the quantization

and requires a check of the $D = 4$ Poincaré representation afterwards