Consistent String in any Dimension $\mathrm{D} > 2$

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Quantum Theory and Unitarity

Inconsistent: Discrete Momentum Eigenstates

Failure of the BRST Construction

Stone and von Neumann exclude States of the Covariant String

Mackey Theorem: Induced Representations

Consistent String in any Dimension?

Who cares?

Relativistic Quantum Theory and Unitarity

Quantum theories give probability distributions for results of measurements

$$w(i, A, \Psi) = \frac{|\langle \Lambda_i | \Psi \rangle|^2}{\langle \Lambda_i | \Lambda_i \rangle \langle \Psi | \Psi \rangle}, \Psi, \Lambda_i \in \mathcal{H}, w(i, A, \Lambda_j) = \delta_{ij} .$$

Wigner: Symmetries are realized by unitary or antiunitary transformations.

Relativistic is a quantum theory, if a unitary representation (of the connected part of the cover) of the Poincaré group acts on it's Hilbert space.

Textbook string theory is no relativistic quantum theory, because it lacks a Hilbert space, e.g. $A^\dagger = A$ is undefined and string calculations yield no probabilities.

Fixing the gauge and solving the constraints before quantization yields a consistent quantum string at least in $D \leq 26$ dimension.

Discrete Scalar Product

$\langle p \,|\, p' \rangle \stackrel{?}{=} \delta_{p,p'}$

$\Rightarrow$ Lorentz transformations act completely discontinuous or not at all.

$|\Psi\rangle = |0\rangle\, \psi_0 + |\Psi'\rangle$, $|\Psi'\rangle = \sum_i |p_i\rangle\, \psi_i \neq 0$ w.l.g .

H_i little group of $|p_i\rangle$, $\dim H_i < \dim SO(1, D-1)$.

Each pair $|p_i\rangle\,, |p_j\rangle$ defines a left coset $H_{ij} = g_{ij} H_j$ of transformations, which map $|p_j\rangle$ to $|p_i\rangle$. Each H_{ij} and $\bigcup_{ij} H_{ij}$ have measure 0 in $SO(1, D-1)$.

The complement of $\bigcup_{ij} H_{ij}$ has full measure. Therefore, in each neighbourhood of the unit element, there are elements g which map each $|p_i\rangle$ to $|g p_i\rangle \neq |p_j\rangle \;\; \forall j$.

$\Rightarrow \langle \Psi' \,|\, 1 \Psi' \rangle \neq 0, \langle \Psi' \,|\, g \Psi' \rangle = 0$ discontinuous, no infinitesimal generators.

All group averages vanish, $\forall \Delta :$ $\quad \int_\Delta \mathrm{d}g \;\langle \Psi' \,|\, g \Psi' \rangle \,/\, \int_\Delta \mathrm{d}g = 0$.

The BRST Construction

Quantum theories with spin ≥ 1 have Fock spaces

$$\langle \Phi \,|\, \Psi \rangle = \int d^{D-1}p \; g_{mn} \, \Phi^{m\,*}(p) \Psi^{n}(p)$$

with indefinite (but nondegenerate) $g = \text{diag}(\underbrace{1, 1, \ldots 1}_{p}, \underbrace{-1, -1 \cdots -1}_{q})$.

$$\text{signature: } \text{sig}\, g = p - q \;,\; \text{sig}\, g_{M \otimes N} = \text{sig}\, g_M \;\text{sig}\, g_N$$

If a BRST operator Q, $Q^2 = 0$, commutes with P, then it's cohomology inherits a metric $g_{\text{coh}Q}$ with the same signature,

$$\text{sig}(g_{\text{coh}(Q)}) = \text{sig}(g) \;.$$

A BRST construction Q is successful, if the cohomology of Q defines a Hilbert space,

$$|\,\text{sig}(g_{\text{coh}(Q)})| = \dim \text{coh}(Q) \;.$$

Failure of the BRST Construction

The BRST construction Q of modern string theory is completely unsuccessful:

$$\mathrm{sig}(g_{\mathrm{coh}(Q)}) = 0 \ .$$

The string contains hermitean zero modes $\{b_0, c_0\} = 1$ and therefore as many physical states with positive norm as physical states with negative norm.

$$\chi = b_0 c_0 \chi + c_0 b_0 \chi$$

$$\mathcal{F}_1 = \{b_0 \chi : \chi \in \mathcal{F}\}$$

$$\mathcal{F}_2 = \{c_0 \chi : \chi \in \mathcal{F}\}$$

$$\langle \mathcal{F}_1 | \mathcal{F}_1 \rangle = 0 = \langle \mathcal{F}_2 | \mathcal{F}_2 \rangle$$

Choose a basis Ψ_i in $\mathcal{F}_1$ and the dual basis Φ_i in $\mathcal{F}_2$,

$$\langle \Psi_i | \Phi_j \rangle = \delta_{ij} \, , \ \langle \Psi_i + \Phi_i | \Psi_j + \Phi_j \rangle = 2\delta_{ij} = -\langle \Psi_i - \Phi_i | \Psi_j - \Phi_j \rangle \ .$$

The subsidiary condition $b_0 \Psi_{\text{phys}} = 0$ picks zero norm states from $\mathcal{F}_1$.

Stone-von Neumann Theorem

All unitary, irreducible representations of

$$[X_m, P^n] = i\hbar\, \delta_m{}^n \ , \ [X_m, i\hbar] = 0 = [P^n, i\hbar] \ , \ m, n \in \{0, 1, \dots D-1\}$$

are unitarily equivalent to the multiplicative and differential operations

$$(P^n \Psi)(p) = p^n \Psi(p) \ , \ (X_m \Psi)(p) = -i \frac{\partial \Psi}{\partial p^m}(p)$$

acting on (a dense subset of) wavefunctions $\Psi : \mathbb{R}^D \to \mathbb{C}$ with scalar product

$$\langle \Phi \,|\, \Psi \rangle = \int d^D p \, \Phi^*(p)\, \Psi(p) \ .$$

Hilbert space $L_2(\mathbb{R}^D)$: Set of equivalence classes of square integrable functions.

$\Phi \sim \Psi \Leftrightarrow \Phi(p) = \Psi(p)$ almost everywhere.

Stone-von Neumann Theorem

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acting on (a dense subset of) wavefunctions $\Psi : \mathbb{R}^D \to \mathbb{C}$ with scalar product

$$\langle\Phi\,|\,\Psi\rangle = \int d^D p\,\Phi^*(p)\,\Psi(p) \ .$$

$U_a = \exp(-i a^m X_m)$, $(U_a\Psi)(p) = \Psi(p-a) \Rightarrow$

No compatible restriction to functions which vanish in some domain (representation is irreducible).

No-state Theorem for the Covariant String

The covariant string and the toy relativistic quantum particle employ

$$[X_m, P^n] = i\,\delta_m{}^n\ ,\ \ m, n \in \{0, 1, \dots D-1\} \tag{1}$$

and restrict physical states to mass shells (D dimensional volume = 0)

$$\Big(\big((P^0)^2 - \vec{P}^2\big) - 2(N-1)\Big)\Psi_{\text{phys}} = 0$$

where N has a discrete spectrum.

In D-dimensions $\Psi_{\text{phys}}(p)$ vanishes almost everywhere, $\Psi_{\text{phys}} \sim 0$.

Each Ψ_{phys} vanishes in a Hilbert space with a unitary representation of (1).

Acceptable quantum theories with a vacuum do not contain any Heisenberg pair $[X_m, P^n] = i\delta_m{}^n$, but only momentum operators P^m such as $(P^0\Psi)(p) = \sqrt{m^2+p^2}\,\Psi(p)$ acting on $\Psi \in L_2(\mathbb{R}^{D-1})$.

Negative Energy States of the Lightcone String

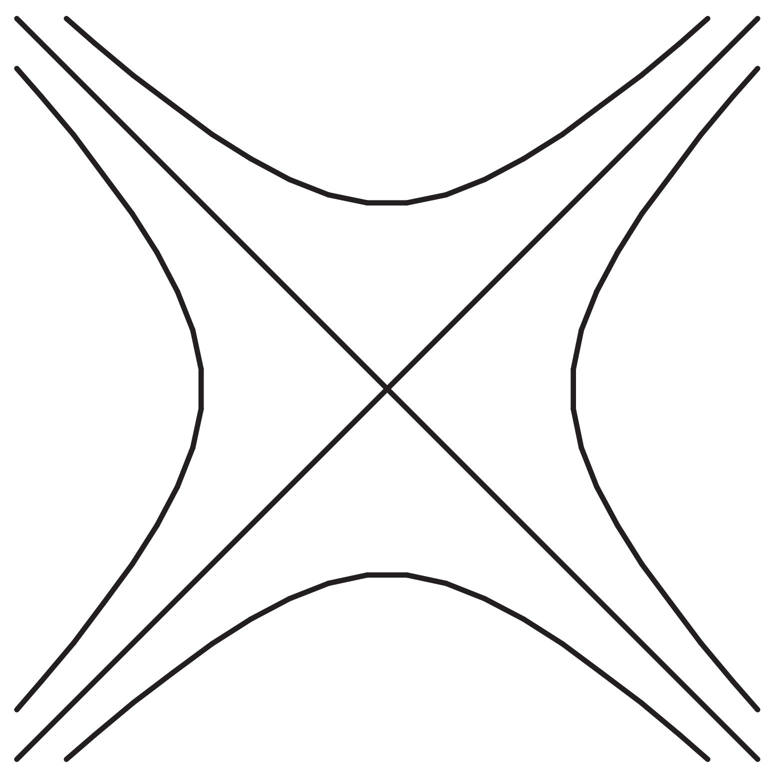
Lightcone String contains $[X^-, P^+] = i\hbar$ acting on

$$\Psi \in L_2(\mathbb{R}^{D-1})\ ,\ (p^+, \vec{p}) \mapsto \Psi(p^+, \vec{p})$$

multiplicative operator $P^- = \frac{1}{P^+}\big(\vec{P}^2 + 2(N-1)\big)$ cannot coexist with X^-

Hilbert space contains with each Ψ also $(\exp(-\mathrm{i}aX^-)\Psi)(p_+, \vec{p}) = \Psi(p_+ - a, \vec{p})$

i.e. also the negative energy shells



Mackey Theorem

$$L^{mn} \stackrel{?}{=} X^m P^n - X^n P^m - i \sum_{k=1}^{\infty} a_k^{\dagger m} a_k^n - a_k^{\dagger n} a_k^m$$

How to construct the unitary representations of the Poincaré group without $X^0, X^1, \ldots X^{D-1}$?

Mackey: Each unitary, irreducible representation of the Poincaré group is unitarily equivalent to the representation, which is induced by an irreducible, unitary representation R of the little group of a vector $\underline{p}$ on its mass shell $M_{m^2} \subset \mathbb{R}^D$.

$\{p : p^0 = \sqrt{m^2 + \vec{p}^2}\} \sim \mathbb{R}^{D-1}$, $\{p : |\vec{p}| = \sqrt{|m^2| + (p^0)^2}\} \sim \mathbb{R} \times S^{D-2}$, $\{p = 0\}$

$$\Psi : M_{m^2} \to \mathbb{C}^d \ , \ \langle \Phi \,|\, \Psi \rangle = \int \frac{d^{D-1}p}{p^0}\, \Phi^{*T}(\vec{p})\, \Psi(\vec{p}) \ , \ m^2 \geq 0$$

$$(U_\Lambda \Psi)(p) = R(W(\Lambda, p))\Psi(\Lambda^{-1}p)$$

Wigner Rotation

$$W(\Lambda_2\Lambda_1, p) = W(\Lambda_2, \Lambda_1 p)\, W(\Lambda_1, p)$$

$$W(\Lambda, p) = L^{-1}_{\Lambda p}\Lambda L_p \ , L_p \underline{p} = p$$

$m^2 > 0$, $\underline{p} = (m, 0, 0, \ldots 0)$ little group $SO(D-1)$

$$L_p = \begin{pmatrix} \frac{p^0}{m} & \frac{\vec{p}^T}{m} \\ \frac{\vec{p}}{m} & 1 + \frac{\vec{p}\vec{p}^T}{m(p^0+m)} \end{pmatrix}$$

infinitesimal transformation (antihermitean)

$$l_{ij} = -(p^i \frac{\partial}{\partial p^j} - p^j \frac{\partial}{\partial p^i}) + \Gamma_{ij} \ , \ \Gamma_{ij} = -\Gamma_{ji} = -(\Gamma_{ij})^\dagger$$

$$[\Gamma_{ij}, \Gamma_{kl}] = \delta_{ik}\Gamma_{jl} - \delta_{il}\Gamma_{jk} - \delta_{jk}\Gamma_{il} + \delta_{jk}\Gamma_{il}$$

$$l_{0i} = p^0 \frac{\partial}{\partial p^i} + \Gamma_{ik}\frac{p^k}{p^0 + m} \text{ unique up to equivalence}$$

unlike George Jorjadze, Jan Plefka, Jonas Pollok: Bosonic String Quantization in the Static Gauge

Massless Representations

$$W(\Lambda, p) = L^{-1}_{\Lambda p} \Lambda L_p \ , L_p \underline{p} = p$$

$$p^0 = \sqrt{(p_z)^2 + \vec{p}^2} \ , \ \underline{p} = (1, 1, 0, \ldots 0) \text{ little group } \mathrm{ISO}(D-2)$$

$$L_p = D_p B_p$$

$$B_p = \begin{pmatrix} \frac{1}{2}(p^0 + \frac{1}{p^0}) & \frac{1}{2}(p^0 - \frac{1}{p^0}) & \\ \frac{1}{2}(p^0 - \frac{1}{p^0}) & \frac{1}{2}(p^0 + \frac{1}{p^0}) & \\ & & 1_{(D-2)\times(D-2)} \end{pmatrix}$$

$$D_p = \begin{pmatrix} 1 & \\ & \hat{D}_p \end{pmatrix} \ , \ \hat{D}_p = \begin{pmatrix} \frac{p_z}{p^0} & -\frac{\vec{p}^T}{p^0} \\ \frac{\vec{p}}{p^0} & 1 - \frac{\vec{p}\vec{p}^T}{p^0(p^0+p_z)} \end{pmatrix}$$

D_p shortest rotation from $(p^0, p^0, 0 \ldots)$ to $(p^0, p_z, \vec{p})$,

not defined for $(p^0, p_z, \vec{p}) = (p^0, -|p_z|, 0)$

Infinitesimal Transformations (antihermitean)

$$l_{ij} = -(p^i \frac{\partial}{\partial p^j} - p^j \frac{\partial}{\partial p^i}) + \gamma_{ij} \ , \ i,j \in \{2, \dots D-1\}$$

$$[\gamma_{ij}, \gamma_{kl}] = \delta_{ik}\gamma_{jl} - \delta_{il}\gamma_{jk} - \delta_{jk}\gamma_{il} + \delta_{jk}\gamma_{il}$$

$$l_{0i} = p^0 \frac{\partial}{\partial p^i} + \gamma_{ik} \frac{p^k}{p^0 + p_z} \ , \ p^0 + p_z = p_+$$

$$l_{zi} = -(p_z \frac{\partial}{\partial p^j} - p^j \frac{\partial}{\partial p_z}) + \gamma_{ik} \frac{p^k}{p^0 + p_z}$$

$$l_{oz} = p^0 \frac{\partial}{\partial p_z}$$

Singular on the negative z-axis.

Spin bundle with northern and southern hemisphere

Southern Chart

$L_p^S = D_p^S B_p^S D_{zx}$

D_{zx} rotation by π in zx-plane, $D_{zx}(1,1,0\ldots)^T = (1,-1,0\ldots)^T$

$$B_p^S = \begin{pmatrix} \frac{1}{2}(p^0 + \frac{1}{p^0}) & -\frac{1}{2}(p^0 - \frac{1}{p^0}) & \\ -\frac{1}{2}(p^0 - \frac{1}{p^0}) & \frac{1}{2}(p^0 + \frac{1}{p^0}) & \\ & & 1_{(D-2)\times(D-2)} \end{pmatrix}$$

$$D_p^S = \begin{pmatrix} 1 & \\ & \hat{D}_p^S \end{pmatrix} \ , \ \hat{D}_p^S = \begin{pmatrix} -\frac{p_z}{p^0} & \frac{\vec{p}^T}{p^0} \\ -\frac{\vec{p}}{p^0} & 1 - \frac{\vec{p}\vec{p}^T}{p^0(p^0 - p_z)} \end{pmatrix}$$

D_p^S shortest rotation from $(p^0, -p^0, 0\ldots)$ to $(p^0, p_z, \vec{p})$,

not defined for $(p^0, p_z, \vec{p}) = (p^0, |p_z|, 0)$.

Southern Infinitesimal Transformations

$$l_{ij} = -(p^i \frac{\partial}{\partial p^j} - p^j \frac{\partial}{\partial p^i}) + \gamma_{ij}\ ,\ \ i, j \in \{2, \ldots D-1\}$$

$$l_{0i} = p^0 \frac{\partial}{\partial p^i} + \gamma_{ik} \frac{p^k}{p^0 - p_z}$$

$$l_{zi} = -(p_z \frac{\partial}{\partial p^j} - p^j \frac{\partial}{\partial p_z}) - \gamma_{ik} \frac{p^k}{p^0 - p_z}$$

$$l_{oz} = p^0 \frac{\partial}{\partial p_z}$$

Singular on the positive z-axis .

Transition Function

$$T_p = (L_p^S)^{-1} L_p^N = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 + 2\frac{p_x^2}{p^2} & 2\frac{p_x \vec{p}^T}{p^2} \\ & & -2\frac{p_x \vec{p}}{p^2} & 1 - 2\frac{\vec{p}\vec{p}^T}{p^2} \end{pmatrix} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & C & S\vec{n}^T \\ & & -S\vec{n} & 1 + (C-1)\vec{n}\vec{n}^T \end{pmatrix}$$

$p^2 = p_x^2 + \vec{p}^2$ (no p_z^2) , $C = \cos 2\varphi$, $S = \sin 2\varphi$, $p \neq (p^0, p_z, 0, \ldots, 0)$

Transition well defined also for half-integer Spin(D-2).

The improved Lightcone string is a relativistic quantum theory, if the massive $SO(D-2)$ modes constitute $SO(D-1)$ multiplets.

Known for $D = 26$, difficulties with Young diagrams with more than 24 rows at $N > 324$?

Young Tables and Frames

Inhaltssumme 2

$$\begin{array}{|c|c|}\hline 1 & 1 \\ \hline\end{array}_{v1} \qquad \begin{array}{|c|}\hline 2 \\ \hline\end{array}_{t1}$$

$$\begin{array}{|c|c|}\hline & \\ \hline\end{array}_{1,1}$$

$$\begin{array}{|c|}\hline \\ \hline\end{array}_{1,0}$$

Inhaltssumme 3

$$\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline\end{array}_{v1} \qquad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline\end{array}_{t1} \qquad \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline\end{array}_{v2} \qquad \begin{array}{|c|}\hline 3 \\ \hline\end{array}_{t2}$$

$$\begin{array}{|c|c|c|}\hline & & \\ \hline\end{array}_{1,1}$$

$$\begin{array}{|c|c|}\hline & \\ \hline\end{array}_{1,0} \qquad \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}_{1,1}$$

$$\begin{array}{|c|}\hline \\ \hline\end{array}_{1,0}$$

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Inhaltssumme 4

$\begin{array}{|c|c|c|c|}\hline 1&1&1&1\\\hline\end{array}_{v1}$ $\begin{array}{|c|c|c|}\hline 1&1&2\\\hline\end{array}_{t1}$ $\begin{array}{|c|c|}\hline 1&1\\\hline 2\\\cline{1-1}\end{array}_{v2}$ $\begin{array}{|c|c|}\hline 1&3\\\hline\end{array}_{t2}$ $\begin{array}{|c|c|}\hline 2&2\\\hline\end{array}_{v3}$ $\begin{array}{|c|}\hline 1\\\hline 3\\\hline\end{array}_{t2}$ $\begin{array}{|c|}\hline 4\\\hline\end{array}_{t3}$

$\begin{array}{|c|c|c|c|}\hline \ &\ &\ &\ \\\hline\end{array}_{1,1}$

$\begin{array}{|c|c|c|}\hline \ &\ &\ \\\hline\end{array}_{1,0}$ $\begin{array}{|c|c|}\hline \ &\ \\\hline \ \\\cline{1-1}\end{array}_{1,1}$

$\begin{array}{|c|c|}\hline \ &\ \\\hline\end{array}_{2,1}$ $\begin{array}{|c|}\hline \ \\\hline \ \\\hline\end{array}_{1,0}$

$\begin{array}{|c|}\hline \ \\\hline\end{array}_{1,0}$

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Inhaltssumme 5

$\ytableaushort{11111}_{\nu 1}$ $\ytableaushort{1112}_{t1}$ $\ytableaushort{111,2}_{\nu 2}$

$\ytableaushort{113}_{t2}$ $\ytableaushort{122}_{\nu 3}$ $\ytableaushort{11,3}_{t2}$ $\ytableaushort{12,2}_{\nu 4}$

$\ytableaushort{14}_{t3}$ $\ytableaushort{23}_{t4}$ $\ytableaushort{1,4}_{t4}$ $\ytableaushort{2,3}_{\nu 5}$ $\ytableaushort{5}_{t5}$

$\ydiagram{5}_{1,1}$

$\ydiagram{4}_{1,0}$ $\ydiagram{3,1}_{1,1}$

$\ydiagram{3}_{2,1}$ $\ydiagram{2,1}_{2,1}$

$\ydiagram{2}_{2,0}$ $\ydiagram{1,1}_{2,1}$

$\ydiagram{1}_{1,0}$

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Inhaltssumme 6

$\begin{ytableau} 1 & 1 & 1 & 1 & 1 & 1 \end{ytableau}_{\nu 1}$ $\begin{ytableau} 1 & 1 & 1 & 1 & 2 \end{ytableau}_{t1}$ $\begin{ytableau} 1 & 1 & 1 & 1 \\ 2 \end{ytableau}_{\nu 2}$

$\begin{ytableau} 1 & 1 & 1 & 3 \end{ytableau}_{t2}$ $\begin{ytableau} 1 & 1 & 2 & 2 \end{ytableau}_{\nu 3}$ $\begin{ytableau} 1 & 1 & 1 \\ 3 \end{ytableau}_{t2}$ $\begin{ytableau} 1 & 1 & 2 \\ 2 \end{ytableau}_{\nu 4}$ $\begin{ytableau} 1 & 1 \\ 2 & 2 \end{ytableau}_{\nu 5}$

$\begin{ytableau} 1 & 1 & 4 \end{ytableau}_{t3}$ $\begin{ytableau} 1 & 2 & 3 \end{ytableau}_{t4}$ $\begin{ytableau} 2 & 2 & 2 \end{ytableau}_{\nu 6}$ $\begin{ytableau} 1 & 1 \\ 4 \end{ytableau}_{t4}$ $\begin{ytableau} 1 & 2 \\ 3 \end{ytableau}_{t5}$ $\begin{ytableau} 1 & 3 \\ 2 \end{ytableau}_{\nu 7}$ $\begin{ytableau} 1 \\ 2 \\ 3 \end{ytableau}_{\nu 8}$

$\begin{ytableau} 1 & 5 \end{ytableau}_{t6}$ $\begin{ytableau} 2 & 4 \end{ytableau}_{t7}$ $\begin{ytableau} 3 & 3 \end{ytableau}_{\nu 9}$ $\begin{ytableau} 1 \\ 5 \end{ytableau}_{t7}$ $\begin{ytableau} 2 \\ 4 \end{ytableau}_{t8}$ $\begin{ytableau} 6 \end{ytableau}_{t9}$

$\ydiagram{6}_{1,1}$

$\ydiagram{5}_{1,0}$ $\ydiagram{4,1}_{1,1}$

$\ydiagram{4}_{2,1}$ $\ydiagram{3,1}_{2,1}$ $\ydiagram{2,2}_{1,1}$

$\ydiagram{3}_{3,1}$ $\ydiagram{2,1}_{3,1}$ $\ydiagram{1,1,1}_{1,1}$

$\ydiagram{2}_{3,1}$ $\ydiagram{1,1}_{2,0}$

$\ydiagram{1}_{1,0}$

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Zerlegungen mit Inhaltssumme 7

Erstzerlegungen

$$\begin{array}{|c|c|c|c|c|c|c|}\hline 1&1&1&1&1&1&1\\\hline\end{array}_{\nu 1}\quad \begin{array}{|c|c|c|c|c|}\hline 1&1&1&1&1\\\hline \multicolumn{1}{|c|}{2}\\\cline{1-1}\end{array}_{\nu 2}\quad \begin{array}{|c|c|c|}\hline 1&1&1\\\hline 2&2\\\cline{1-2}\end{array}_{\nu 5}\quad \begin{array}{|c|c|}\hline 1&1\\\hline \multicolumn{1}{|c|}{2}\\\cline{1-1} \multicolumn{1}{|c|}{3}\\\cline{1-1}\end{array}_{\nu 9}$$

Unterdiagramme

$$\begin{array}{|c|c|c|c|c|c|}\hline 1&1&1&1&1&2\\\hline\end{array}_{t1}\quad \begin{array}{|c|c|c|c|c|}\hline 1&1&1&1&3\\\hline\end{array}_{t2}\quad \begin{array}{|c|c|c|c|c|}\hline 1&1&1&2&2\\\hline\end{array}_{\nu 3}\quad \begin{array}{|c|c|c|c|}\hline 1&1&1&1\\\hline \multicolumn{1}{|c|}{3}\\\cline{1-1}\end{array}_{t2}\quad \begin{array}{|c|c|c|c|}\hline 1&1&1&2\\\hline \multicolumn{1}{|c|}{2}\\\cline{1-1}\end{array}_{\nu 4}$$

$$\begin{array}{|c|c|c|c|}\hline 1&1&1&4\\\hline\end{array}_{t3}\quad \begin{array}{|c|c|c|c|}\hline 1&1&2&3\\\hline\end{array}_{t4}\quad \begin{array}{|c|c|c|c|}\hline 1&2&2&2\\\hline\end{array}_{\nu 6}$$

$$\begin{array}{|c|c|c|}\hline 1&1&1\\\hline \multicolumn{1}{|c|}{4}\\\cline{1-1}\end{array}_{t4}\quad \begin{array}{|c|c|c|}\hline 1&1&2\\\hline \multicolumn{1}{|c|}{3}\\\cline{1-1}\end{array}_{t5}\quad \begin{array}{|c|c|c|}\hline 1&1&3\\\hline \multicolumn{1}{|c|}{2}\\\cline{1-1}\end{array}_{\nu 7}\quad \begin{array}{|c|c|c|}\hline 1&2&2\\\hline \multicolumn{1}{|c|}{2}\\\cline{1-1}\end{array}_{\nu 8}\quad \begin{array}{|c|c|}\hline 1&1\\\hline 2&3\\\hline\end{array}_{t5}$$

$$\begin{array}{|c|c|c|}\hline 1&1&5\\\hline\end{array}_{t6}\quad \begin{array}{|c|c|c|}\hline 1&2&4\\\hline\end{array}_{t7}\quad \begin{array}{|c|c|c|}\hline 1&3&3\\\hline\end{array}_{t8}\quad \begin{array}{|c|c|c|}\hline 2&2&3\\\hline\end{array}_{\nu 10}$$

$$\begin{array}{|c|c|}\hline 1&1\\\hline \multicolumn{1}{|c|}{5}\\\cline{1-1}\end{array}_{t7}\quad \begin{array}{|c|c|}\hline 1&2\\\hline \multicolumn{1}{|c|}{4}\\\cline{1-1}\end{array}_{t8}\quad \begin{array}{|c|c|}\hline 1&3\\\hline \multicolumn{1}{|c|}{3}\\\cline{1-1}\end{array}_{t9}\quad \begin{array}{|c|c|}\hline 1&4\\\hline \multicolumn{1}{|c|}{2}\\\cline{1-1}\end{array}_{\nu 11}\quad \begin{array}{|c|c|}\hline 2&2\\\hline \multicolumn{1}{|c|}{3}\\\cline{1-1}\end{array}_{\nu 12}\quad \begin{array}{|c|}\hline 1\\\hline 2\\\hline 4\\\hline\end{array}_{t9}$$

$$\begin{array}{|c|c|}\hline 1&6\\\hline\end{array}_{t10}\quad \begin{array}{|c|c|}\hline 2&5\\\hline\end{array}_{t11}\quad \begin{array}{|c|c|}\hline 3&4\\\hline\end{array}_{t12}\quad \begin{array}{|c|}\hline 1\\\hline 6\\\hline\end{array}_{t11}\quad \begin{array}{|c|}\hline 2\\\hline 5\\\hline\end{array}_{t12}\quad \begin{array}{|c|}\hline 3\\\hline 4\\\hline\end{array}_{\nu 13}\quad \begin{array}{|c|}\hline 7\\\hline\end{array}_{t13}$$

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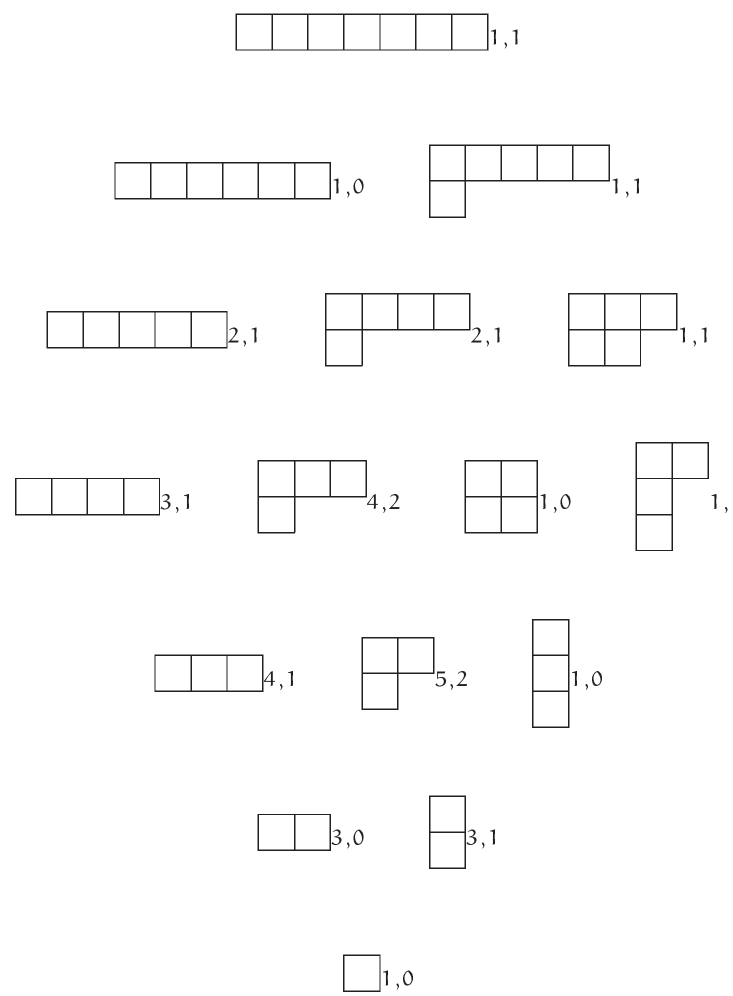
Rahmen



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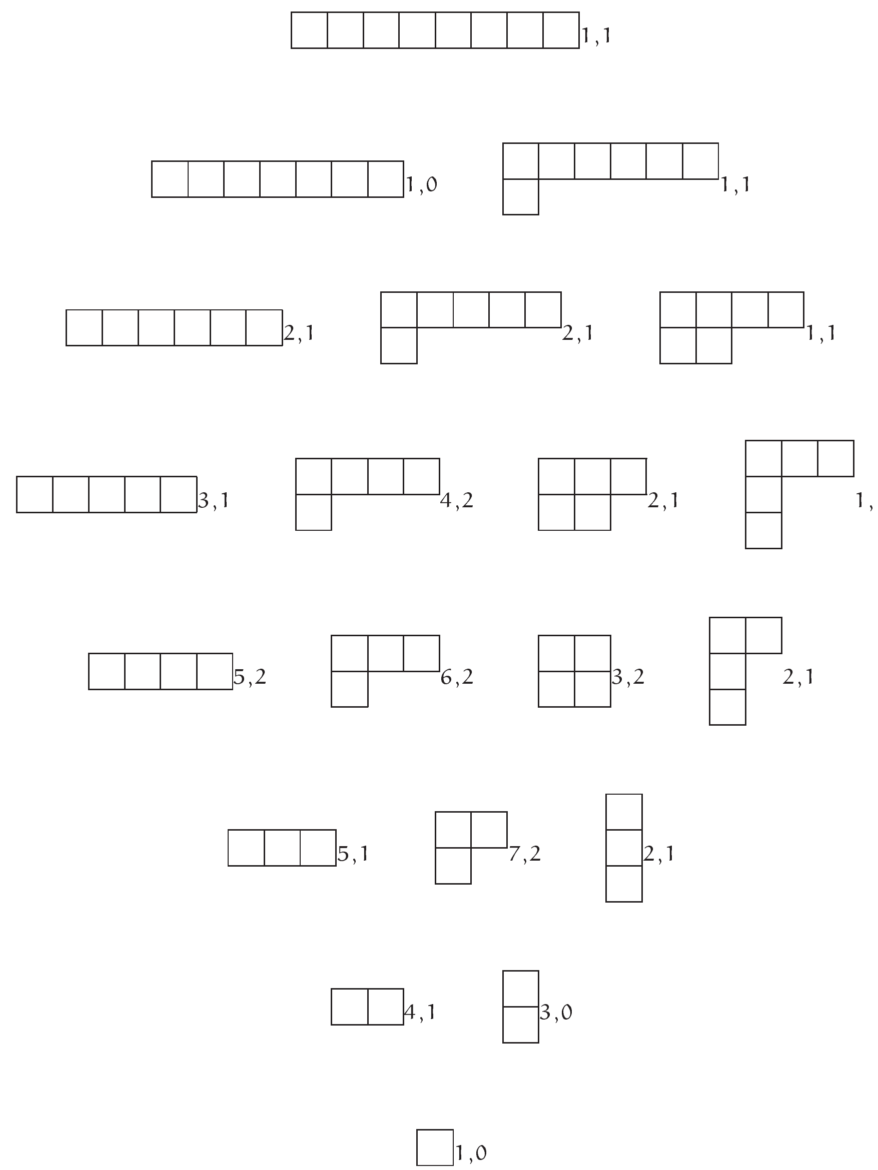
Zerlegungen mit Inhaltssumme 8

Rahmen



Proof for all Levels

Conjecture: At each level n, the polynomials $P(a^\dagger) = c^{k_1 \dots k_l}_{m_1 \dots m_l} {a^\dagger}^{m_1}_{k_1} \dots {a^\dagger}^{m_l}_{k_l}$ with $NP = nP$, $N = \sum_{k=1}^{\infty} \sum_{i=1}^{D-2} k \left({a^\dagger}^i_k \frac{\partial}{\partial {a^\dagger}^i_k} - a^i_k \frac{\partial}{\partial a^i_k} \right)$ transform under a representation of $H =$SO$(D-2)$, which is the restriction of a representation of $G =$SO$(D-1)$.

Proof: Unknown

Classical Realization

$$l_{zi} = \sum_{n \neq 0} \frac{\alpha^i_n}{n} l_{-n|_{\alpha_0=0}} \, , \; \{l_{zi}, l_{zj}\} = i l_0 \sum_{n=1}^{\infty} \frac{\alpha^i_{-n}\alpha^j_n - \alpha^j_{-n}\alpha^i_n}{n} \, , \; l_{0|_{\alpha_0=0}} = \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n$$

i.e. $\frac{l_{zi}}{\sqrt{l_0}}$ and l_{ij} generate SO$(D-1)$.

Covariant String

$$l_{ij} = -(p^i \frac{\partial}{\partial p^j} - p^j \frac{\partial}{\partial p^i}) + \sum_k (a_k^{\dagger\, i} a_k^j - a_k^{\dagger\, j} a_k^i)$$

$$l_{0i} = p^0 \frac{\partial}{\partial p^i} - \sum_k (a_k^{\dagger\, 0} a_k^i - a_k^{\dagger\, i} a_k^0)$$

defines a Fockspace representation of the Lorentz algebra.

The theorem of Del Guidice, Di Vecchia and Fubini applies, that physical states in $D = 26$ have positive norm, though their proof assumes momentum eigenstates with discrete scalar product.

Their operators $:\exp(\mathrm{i}kQ(z)):$ are defined only on states with $k \cdot p \in \mathbb{Z}$ and therefore not on string state (wavefunctions) but only on auxiliary states, which are incompatible with the Poincaré symmetry.

Nevertheless, the DDF-proof of the positivity of the integrand in the scalar product is correct.

Tachyonic Representation

$(U_\Lambda \Psi)(p) = R(W(\Lambda, p))\Psi(\Lambda^{-1}p)$

$\underline{p} = (0, \ldots, 0, 1)$, little group $SO(1, D-2)$ noncompact

either $R = 1$ or R infinite dimensional

$R = 1 \Rightarrow$ tachyon is a scalar, no supersymmetry, rep. can be worked out

No common construction of the Poincaré transformations of the different mass shells.

Who cares?

String amplitudes are not calculated as matrix elements of string states. In the BRST formulation these states do not even constitute a Hilbert space.

Transition amplitudes of the S-matrix are derived from operators, which cannot be applied to string states but act on a vacuum. It is a no-string-state and decorates a subalgebra of vertex operators.