Graded Algebra $(x, \phi, +, \cdot, \partial, *)$

$$\phi^i \phi^j = (-1)^{|\phi^i|\cdot|\phi^j|}\phi^j\phi^i =: (-)^{ij}\phi^j\phi^i \tag{1}$$

$$|\phi^i| = \begin{cases} 0 \text{ if } \phi^i \text{ is bosonic} \\ 1 \text{ if } \phi^i \text{ is fermionic} \end{cases} , \quad |\phi^i\phi^j| = |\phi^i| + |\phi^j| \bmod 2 , \tag{2}$$

$$O(\lambda_1 A + \lambda_2 B) = \lambda_1 O(A) + \lambda_2 O(B) , \quad |O| = |O(A)| - |A| \bmod 2 , \tag{3}$$

$$w'_M = O_M{}^N w_N , \ X^N{}_N = X^n{}_n + X^\nu{}_\nu + X_{\dot\nu}{}^{\dot\nu} , \ X^N{}_N = (-)^{|N|} X^{\bar N}{}_{\bar N} \tag{4}$$

$$u'^M = u^L O^{-1}{}_L{}^M = (-)^{(|u|+|L|)(|O|+|L|+|M|)} O_L^{-1M} u^L , \tag{5}$$

$$v(AB) = (vA)B + (-)^{|v|\cdot|A|}A(vB) , \quad v = v^i\partial_i , \quad v^i = v(\phi^i) , \tag{6}$$

$$\partial_i\phi^j = \delta_i{}^j , \quad |\partial_i| = |\phi^i| , \quad \partial_i\partial_j = (-)^{ij}\partial_j\partial_i , \tag{7}$$

$$(\lambda_1 A + \lambda_2 B)^* = \lambda_1^* A^* + \lambda_2^* B^* , \quad (AB)^* = B^* A^* = (-)^{|A||B|} A^* B^* , \quad |\phi^*| = |\phi| , \tag{8}$$

$$O^*(A) = (-)^{|O||A|}(O(A^*))^* , \quad (O_1 O_2)^* = (-)^{|O_1||O_2|} O_1^* O_2^* , \tag{9}$$

$$\partial_\beta\theta^\alpha = \delta_\beta{}^\alpha , \quad \bar\theta^{\dot\alpha} = (\theta^\alpha)^* , \quad \partial_{\dot\beta} = (\partial_\beta)^* \quad \theta_\alpha = \varepsilon_{\alpha\beta}\theta^\beta , \quad \partial^{\dot\beta}\bar\theta_{\dot\alpha} = \delta^{\dot\beta}{}_{\dot\alpha} , \tag{10}$$

$$\{\text{Tensors}\} \subset \{\text{Polynomials}(\phi, \partial\phi, \partial\partial\phi, \dots)\} , \quad \Delta_M : \{\text{Tensors}\} \to \{\text{Tensors}\} , \tag{11}$$

$$(\Delta_M) = (D_a,\, D_\alpha,\, D^{\dot\alpha},\, \delta_i,\, l_{[ab]}) , \quad (\Delta_M)^* = \Delta_{\bar M} , \quad |\Delta_M| = |M| , \tag{12}$$

$$(w^M\Delta_M)^* = (-)^{|M|(|w|+|M|)}(w^M)^*\Delta_{\bar M} = \bar w^M \Delta_M , \ (w^M)^* = (-)^{|w||M|}\bar w^{\bar M} , \tag{13}$$

$$[\Delta_M, \Delta_N] = \Delta_M\Delta_N - (-)^{MN}\Delta_N\Delta_M = \mathcal{F}_{MN}{}^K\Delta_K , \ \mathcal{F}_{MN}{}^K = -(-)^{MN}\mathcal{F}_{NM}{}^K , \tag{14}$$

$$(\mathcal{F}_{MN}{}^K)^* = (-)^{K(M+N)+MN}\mathcal{F}_{\bar M\bar N}{}^{\bar K} , \tag{15}$$

$$\sum_{MNP}\!\!\!\!\!\!\bigcirc\ \text{sign}(MNP)[\Delta_M, [\Delta_N, \Delta_P]] = 0 , \tag{16}$$

$$\sum_{MNP}\!\!\!\!\!\!\bigcirc\ \text{sign}(MNP)(\Delta_M\mathcal{F}_{NP}{}^K - \mathcal{F}_{MN}{}^L\mathcal{F}_{LP}{}^K) , \tag{17}$$

$$\mathrm{d} = \mathrm{d}x^m\partial_m , \quad [\partial_n, \mathrm{d}x^m] = 0 , \quad \mathrm{d}^2 = 0 , \tag{18}$$

$$\mathrm{s} = \mathrm{s}^* , \quad |\,\mathrm{s}\,| = 1 , \quad \mathrm{s}^2 = 0 , \tag{19}$$

$$[\mathrm{s}, \partial_m] = 0 , \quad \mathrm{s}(x^m) = 0 , \quad \{\mathrm{s}, \mathrm{d}\} = 0 , \tag{20}$$

$$\mathrm{s}\,T = -C^N\Delta_N T , \quad \mathrm{d}\,T = \mathrm{d}x^m\partial_m T = -\mathrm{d}x^m A_m{}^N\Delta_N T , \tag{21}$$

$$\partial_m = -A_m{}^N\Delta_N = e_m{}^a D_a - A_m{}^I\delta_I , \quad D_a = E_a{}^m(\partial_m + A_m{}^I\delta_I) , \tag{22}$$

$$\mathrm{s}\,C^N = -\frac{1}{2}C^K C^L\mathcal{F}_{LK}{}^N , \quad \mathrm{s}\,A_m{}^N = \partial_m C^N - A_m{}^K C^L\mathcal{F}_{LK}{}^N , \tag{23}$$

$$\mathcal{F}_{ab}{}^N = E_a{}^m E_b{}^n\Big(\partial_m A_n{}^N - \partial_n A_m{}^N\Big) + A_a{}^I\mathcal{F}_{Ib}{}^N - A_b{}^I\mathcal{F}_{Ia}{}^N + A_a{}^I A_b{}^J\mathcal{F}_{JI}{}^N . \tag{24}$$

Bianchi-Identities

$$[D_A, D_B] = -T_{AB}{}^C D_C + F_{AB}{}^i \delta_i \ , \quad \text{torsion and field strength} \tag{25}$$
$$[\delta_i, D_A] = -G_{i\,A}{}^B D_B \ , \quad \text{representation matrices} \tag{26}$$
$$[\delta_i, \delta_j] = f_{ij}{}^k \delta_k \ , \quad \text{structure constants} \tag{27}$$

$$0 = \circlearrowright\!\!\!\!\!\!\sum f_{ij}{}^l f_{lk}{}^m \ , \quad [G_i, G_j] = f_{ij}{}^k G_k \ , \tag{28}$$
$$\delta_i T_{AB}{}^C = -G_{i\,A}{}^D T_{DB}{}^C - G_{i\,B}{}^D T_{AD}{}^C + G_{i\,D}{}^C T_{AB}{}^D \ , \tag{29}$$
$$\delta_i F_{AB}{}^j = -G_{i\,A}{}^D F_{DB}{}^j - G_{i\,B}{}^D F_{AD}{}^j - f_{i\,k}{}^j F_{AB}{}^k \ , \tag{30}$$
$$0 = \circlearrowright\!\!\!\!\!\!\sum D_A T_{BC}{}^D + T_{AB}{}^E T_{EC}{}^D - R_{ABC}{}^D \ , \quad R_{ABC}{}^D = F_{AB}{}^i G_{i\,C}{}^D \ , \tag{31}$$
$$0 = \circlearrowright\!\!\!\!\!\!\sum D_A F_{BC}{}^i + T_{AB}{}^D F_{DC}{}^i \ , \quad A \in \{a, {}_\alpha, {}^{\dot\alpha}\} \ , \quad i \in \{i, [a,b]\} \ . \tag{32}$$

Identification and Constraints

$$D_a = E_a{}^m(\partial_m - \frac{1}{2}\psi_m{}^\alpha D_\alpha - \frac{1}{2}\bar\psi_{m\,\dot\alpha} D^{\dot\alpha} + \frac{1}{2}\omega_m{}^{ab} l_{ab} + A_m{}^i \delta_i) \ , \tag{33}$$
$$\mathcal{F}_{ab}{}^N = E_a{}^m E_b{}^n \Big(\partial_m A_n{}^N - \partial_n A_m{}^N\Big) + A_a{}^I \mathcal{F}_{Ib}{}^N - A_b{}^I \mathcal{F}_{Ia}{}^N + A_a{}^I A_b{}^J \mathcal{F}_{JI}{}^N \ , \tag{34}$$
$$\begin{aligned} R_{abc}{}^d = {} & E_a{}^m E_b{}^n(\partial_m \omega_{n\,c}{}^d - \partial_n \omega_{m\,c}{}^d - \omega_{m\,c}{}^e \omega_{n\,e}{}^d + \omega_{n\,c}{}^e \omega_{m\,e}{}^d) - \\ & - \frac{1}{2}\psi_a{}^\alpha R_{\alpha bc}{}^d - \frac{1}{2}\bar\psi_{a\,\dot\alpha} R^{\dot\alpha}{}_{bc}{}^d + \frac{1}{2}\psi_b{}^\alpha R_{\alpha ac}{}^d + \frac{1}{2}\bar\psi_{b\,\dot\alpha} R^{\dot\alpha}{}_{ac}{}^d \ , \end{aligned} \tag{35}$$
$$\begin{aligned} F_{ab}{}^i = {} & E_a{}^m E_b{}^n(\partial_m A_n{}^i - \partial_n A_m{}^i - A_m{}^j A_n{}^k f_{jk}{}^i) - \\ & - \frac{1}{2}\psi_a{}^\alpha F_{\alpha b}{}^i - \frac{1}{2}\bar\psi_{a\,\dot\alpha} F^{\dot\alpha}{}_b{}^i + \frac{1}{2}\psi_b{}^\alpha F_{\alpha a}{}^i + \frac{1}{2}\bar\psi_{b\,\dot\alpha} F^{\dot\alpha}{}_a{}^i \ , \end{aligned} \tag{36}$$

$$0 = T_{\underline{\alpha}\underline{\beta}}{}^{\underline{\gamma}} = T_{ab}{}^c = T_{\alpha\beta}{}^c = T_{\dot\alpha\dot\beta}{}^c = T_{\alpha b}{}^c = T_{\dot\alpha b}{}^c \ , \quad T_{\alpha\dot\beta}{}^c = 2\mathrm{i}\sigma_{\alpha\dot\beta}{}^c \ : \tag{37}$$
$$\begin{aligned} T_{ab}{}^\gamma = {} & \frac{1}{2} E_a{}^m E_b{}^n((\partial_m + \frac{1}{2}\omega_m{}^{cd} l_{cd})\psi_n{}^\gamma - (\partial_n + \frac{1}{2}\omega_n{}^{cd} l_{cd})\psi_m{}^\gamma) - \\ & - \frac{1}{2}\psi_a{}^{\underline{\alpha}} T_{\underline{\alpha} b}{}^\gamma + \frac{1}{2}\psi_b{}^{\underline{\alpha}} T_{\underline{\alpha} a}{}^\gamma \end{aligned} \tag{38}$$
$$\begin{aligned} T_{ab}{}^c = 0 = {} & E_a{}^m E_b{}^n(\partial_m e_n{}^c - \partial_n e_m{}^c) + \omega_{a\,b}{}^c - \omega_{b\,a}{}^c + \frac{1}{2}\psi_a{}^{\underline{\alpha}} T_{\underline{\alpha} b}{}^c - \frac{1}{2}\psi_b{}^{\underline{\alpha}} T_{\underline{\alpha} a}{}^c + \\ & + \frac{\mathrm{i}}{2}\psi_a \sigma^c \bar\psi_b - \frac{\mathrm{i}}{2}\psi_b \sigma^c \bar\psi_a \ , \end{aligned} \tag{39}$$
$$\omega_{a\,bc} = \tilde\omega_{a\,bc} + \frac{\mathrm{i}}{4}\big(\psi_a\sigma_b\bar\psi_c - \psi_a\sigma_c\bar\psi_b + \psi_b\sigma_c\bar\psi_a - \psi_c\sigma_b\bar\psi_a - \psi_c\sigma_a\bar\psi_b + \psi_b\sigma_a\bar\psi_c\big) \ , \tag{40}$$
$$\tilde\omega_{a\,bc} = \frac{1}{2}\big(\eta_{ad} E_b{}^m E_c{}^n + \eta_{bd} E_a{}^m E_c{}^n - \eta_{cd} E_a{}^m E_b{}^n\big)\big(\partial_m e_n{}^d - \partial_n e_m{}^d\big) \ . \tag{41}$$

Spinors and Lorentzalgebra

$$\eta_{00}=1\ ,\ \eta_{11}=\eta_{22}=\eta_{33}=-1\ ,\ a\neq b:\ \eta_{ab}=0\ , \tag{42}$$

$$[l_{ab},D_c]=-(\eta_{ac}\delta_b{}^d-\eta_{bc}\delta_a{}^d)D_d\ ,\ [l_{ab},D_\gamma]=-\sigma_{ab\,\gamma}{}^\delta D_\delta\ ,\ [l_{ab},D^{\dot\gamma}]=-\bar\sigma_{ab}{}^{\dot\gamma}{}_{\dot\delta}D^{\dot\delta}\ , \tag{43}$$

$$[l_{ab},l_{cd}]=-\eta_{ac}l_{bd}+\eta_{bc}l_{ad}+\eta_{ad}l_{bc}-\eta_{bd}l_{ac}\ , \tag{44}$$

$$\sigma^a_{\alpha\dot\beta}:\ \sigma^0=\begin{pmatrix}1&\\&1\end{pmatrix}\ ,\ \sigma^1=\begin{pmatrix}&1\\1&\end{pmatrix}\ ,\ \sigma^2=\begin{pmatrix}&-\mathrm{i}\\\mathrm{i}&\end{pmatrix}\ ,\ \sigma^3=\begin{pmatrix}1&\\&-1\end{pmatrix}\ , \tag{45}$$

$$\varepsilon^{\alpha\beta}=-\varepsilon^{\beta\alpha}\ ,\ \varepsilon^{12}=1\ ,\ \varepsilon^{\alpha\beta}\varepsilon_{\beta\gamma}=\delta^\alpha{}_\gamma\ ,\ \varepsilon_{\dot\alpha\dot\beta}=(\varepsilon_{\alpha\beta})^*\ ,\ \varepsilon_{\dot\alpha\dot\beta}\varepsilon^{\dot\beta\dot\gamma}=\delta_{\dot\alpha}{}^{\dot\gamma}\ , \tag{46}$$

$$\chi_\alpha=\varepsilon_{\alpha\beta}\chi^\beta\ ,\ \chi^\alpha=\varepsilon^{\alpha\beta}\chi_\beta\ ,\ \chi_{\dot\alpha}=\varepsilon_{\dot\alpha\dot\beta}\chi^{\dot\beta}\ ,\ \chi^{\dot\alpha}=\varepsilon^{\dot\alpha\dot\beta}\chi_{\dot\beta}\ ,\ X_{\alpha\dot\beta}=X_a\sigma^a_{\alpha\dot\beta}\ , \tag{47}$$

$$\bar\sigma^{a\,\dot\alpha\alpha}=\varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\alpha\beta}\sigma^a_{\beta\dot\beta}\ ,\ \bar\sigma^0=\sigma^0\ ,\ \bar\sigma^1=-\sigma^1\ ,\ \bar\sigma^2=-\sigma^2\ ,\ \bar\sigma^3=-\sigma^3\ , \tag{48}$$

$$G^{ab}{}_\gamma{}^\delta=\sigma^{ab}{}_\gamma{}^\delta=\frac{1}{4}(\sigma^a\bar\sigma^b-\sigma^b\bar\sigma^a)_\gamma{}^\delta\ ,\ G^{ab\dot\gamma}{}_{\dot\delta}=\bar\sigma^{ab\dot\gamma}{}_{\dot\delta}=\frac{1}{4}(\bar\sigma^a\sigma^b-\bar\sigma^b\sigma^a)^{\dot\gamma}{}_{\dot\delta}\ , \tag{49}$$

$$\sigma_{a\,\alpha\dot\alpha}\sigma^a_{\beta\dot\beta}=2\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta}\ , \tag{50}$$

$$\frac{1}{2}R_{AB\,cd}\,G^{cd}{}_{\gamma\delta}=R_{AB\,\gamma\delta}\ ,\ \frac{1}{2}R_{AB\,cd}\,G^{cd}{}_{\dot\gamma\dot\delta}=R_{AB\,\dot\gamma\dot\delta}\ , \tag{51}$$

$$R_{A\,B\,\gamma\dot\gamma\,\delta\dot\delta}=2\varepsilon_{\gamma\delta}R_{A\,B\,\dot\gamma\dot\delta}-2\varepsilon_{\dot\gamma\dot\delta}R_{A\,B\,\gamma\delta}\ ,\ (R_{A\,B\,\gamma\delta})^*=-(-)^{AB}R_{\bar A\,\bar B\,\dot\gamma\dot\delta}\ , \tag{52}$$

$$\varepsilon^{0123}=1=-\varepsilon_{0123}\ . \tag{53}$$

Index Pictures

$$0 = \sum D_A T_{BC}{}^D + T_{AB}{}^E T_{EC}{}^D - R_{ABC}{}^D \tag{54}$$

$$0 = T_{\underline{\alpha\beta}}{}^{\underline{\gamma}} = T_{ab}{}^c = T_{\alpha\beta}{}^c = T_{\dot\alpha\dot\beta}{}^c = T_{\alpha b}{}^c = T_{\dot\alpha b}{}^c \ , \ T_{\alpha\dot\beta}{}^c = 2\mathrm{i}\sigma_{\alpha\dot\beta}{}^c \ : \tag{55}$$

$_{\alpha\beta\gamma}{}^d$: $$0 = 0 \ , \tag{56}$$

$_{\alpha\beta\dot\gamma}{}^d$: $$0 = 0 \ , \tag{57}$$

$_{\alpha\beta c}{}^d$: $$R_{\alpha\,\beta\,\dot\gamma\,\dot\delta} = 0 \ , \ T_{\delta\,\gamma\dot\gamma\,\dot\alpha} = -2\mathrm{i}\varepsilon_{\delta\gamma}\varepsilon_{\dot\gamma\dot\alpha}\bar R \ , \ R_{\alpha\,\beta\,\gamma\,\delta} = 4(\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta} + \varepsilon_{\alpha\delta}\varepsilon_{\beta\gamma})\bar R \ , \tag{58}$$

$_{\alpha\beta\gamma}{}^\delta$: $$\text{o.k.} \tag{59}$$

$_{\alpha\dot\beta\dot\gamma}{}^\delta$: $$\text{o.k.} \tag{60}$$

$_{\alpha\beta\dot\gamma}{}^\delta$: $$T_{\beta\,\dot\gamma\,\alpha\,\delta} = T_{(\alpha\beta\delta)\dot\gamma} + \varepsilon_{\beta\alpha}T_{\delta\dot\gamma} + \varepsilon_{\delta\beta}U_{\alpha\dot\gamma} + \varepsilon_{\delta\alpha}U_{\beta\dot\gamma} \tag{61}$$

$$R_{\beta\,\dot\gamma\,\alpha\,\delta} = -2\mathrm{i}T_{(\alpha\beta\delta)\dot\gamma} + 4\mathrm{i}\varepsilon_{\beta\alpha}U_{\delta\dot\gamma} + 4\mathrm{i}\varepsilon_{\beta\delta}U_{\alpha\dot\gamma} \tag{62}$$

$_{\alpha\dot\beta c}{}^d$: $$R_{\beta\,\dot\delta\,\gamma\,\alpha} = -\varepsilon_{\beta\gamma}G_{\alpha\dot\delta} - \varepsilon_{\beta\alpha}G_{\gamma\dot\delta} \ , \tag{63}$$

$$T_{\beta\,\gamma\dot\gamma\,\delta} = \frac{\mathrm{i}}{4}(\varepsilon_{\gamma\delta}G_{\beta\dot\gamma} - 3\varepsilon_{\beta\gamma}G_{\delta\dot\gamma} - 3\varepsilon_{\beta\delta}G_{\gamma\dot\gamma}) \ , \ (G_{\alpha\dot\beta})^* = (G_{\beta\dot\alpha}) \ , \tag{64}$$

$$R_{\delta\,\dot\beta\,\dot\gamma\,\dot\alpha} = -\varepsilon_{\dot\beta\dot\gamma}G_{\delta\dot\alpha} - \varepsilon_{\dot\beta\dot\alpha}G_{\delta\dot\gamma} \ , \tag{65}$$

$_{\dot\alpha\dot\beta\dot\gamma}{}^\delta$: $$0 = 0 \ , \tag{66}$$

$_{\alpha bc}{}^d$: $$R_{\alpha\,b\,c\,d} = \mathrm{i}\big((\sigma_b\bar T_{cd})_\alpha + (\sigma_c\bar T_{bd})_\alpha - (\sigma_d\bar T_{bc})_\alpha\big) \ , \tag{67}$$

$_{\dot\alpha\dot\beta c}{}^\delta$: $$\bar D_{\dot\gamma}R = 0 \ , \tag{68}$$

$_{\alpha\beta c}{}^\delta$: $$\begin{aligned} T_{\delta\dot\delta\,\gamma\dot\gamma\,\dot\alpha} &= -2\,\varepsilon_{\delta\gamma}\bar W_{(\dot\delta\dot\gamma\dot\alpha)} - \\ &\quad - \frac{1}{2}\,\varepsilon_{\delta\gamma}(\varepsilon_{\dot\alpha\dot\gamma}D^\beta G_{\beta\dot\delta} + \varepsilon_{\dot\alpha\dot\delta}D^\beta G_{\beta\dot\gamma}) + \frac{1}{2}\varepsilon_{\dot\delta\dot\gamma}(D_\delta G_{\gamma\dot\alpha} + D_\gamma G_{\delta\dot\alpha}) \ , \end{aligned} \tag{69}$$

$_{\alpha\dot\beta c}{}^\delta$: $$D_\alpha R = \bar D^{\dot\beta}G_{\alpha\dot\beta} \ , \tag{70}$$

$_{\dot\alpha bc}{}^\delta$: $$D_\alpha\bar W_{(\dot\beta\dot\gamma\dot\delta)} = 0 \tag{71}$$

$_{\alpha bc}{}^\delta$: (72)

$_{\dot\alpha bc}{}^\delta$: (73)

$_{abc}{}^d$: (74)

$_{abc}{}^\delta$: (75)

$$R = -\frac{1}{6}\,M \ , \ G_a = -\frac{1}{3}\,b_a \ , \ \mathcal{R} = R_{abc}{}^b\eta^{ac} \ , \tag{76}$$

$$\begin{aligned} D^2M &= -2\mathcal{R} + 4\mathrm{i}\psi_{ab}\sigma^a\bar\psi^b + \frac{1}{2}\varepsilon^{abcd}(\bar\psi_a\bar\sigma_b\psi_{cd} + \psi_a\sigma_b\bar\psi_{cd}) + \\ &\quad + 4\mathrm{i}\,\mathcal{D}_a b^a - \frac{8}{3}\,M^*M + \frac{4}{3}\,b_a b^a + 2\,\bar\psi_a\bar\psi^a\,M - 2\,\psi_a\sigma^a\bar\psi_c\,b^c \ . \end{aligned} \tag{77}$$

Multiplets

$$\mathcal{D}_a = E_a{}^m(\partial_m + \frac{1}{2}\omega_m{}^{ab}l_{ab} + A_m{}^i\delta_i) \ , \tag{78}$$

$$\begin{aligned}
\mathrm{s}\, e_a{}^m &= \mathrm{i}e_c{}^m(\xi\sigma^c\bar{\psi}_a - \psi_a\sigma^c\bar{\xi}) \ , &(79)\\
\mathrm{s}\,\psi_{m\,\gamma} &= -2\mathcal{D}_m\xi_\gamma + \frac{\mathrm{i}}{3}e_m{}^a(-\sigma_a\bar{\xi}M + 2b_a\xi - 2\sigma_{ac}\xi b^c)_\gamma \ , &(80)\\
\mathrm{s}\, M &= -\xi(2\sigma^{ab}\psi_{ab} - \mathrm{i}b^a\psi_a - \mathrm{i}\sigma^a\bar{\psi}_a M) \ , &(81)\\
\mathrm{s}\, b_a &= -\xi\big(\frac{\mathrm{i}}{2}\psi_a M^* - \sigma^b(\bar{\psi}_{ab} + \frac{\mathrm{i}}{4}\varepsilon_{abcd}\bar{\psi}^{cd} + \frac{\mathrm{i}}{2}(b_a\bar{\psi}_b + \frac{\mathrm{i}}{2}\varepsilon_{abcd}b^c\bar{\psi}^d))\big) + \text{c.c.} \ , &(82)\\
\psi_{ab} &= E_a{}^m E_b{}^n(\mathcal{D}_m\psi_n - \mathcal{D}_n\psi_m) \ , \bar{\psi}_{ab} = E_a{}^m E_b{}^n(\mathcal{D}_m\bar{\psi}_n - \mathcal{D}_n\bar{\psi}_m) \ , &(83)\\
\mathrm{s}\, A_m{}^i &= -\mathrm{i}\, e_m{}^a\,(\xi\sigma_a\bar{\lambda}^i - \lambda^i\sigma_a\bar{\xi}) \ , &(84)\\
\mathrm{s}\,\lambda_\beta{}^i &= -\mathrm{i}\xi_\beta D^i + (\sigma^{ab}\xi)_\beta F_{ab}{}^i + \frac{\mathrm{i}}{2}\xi\psi_a(\sigma^a\bar{\lambda}^i)_\beta - \frac{\mathrm{i}}{2}\xi\sigma^a\bar{\lambda}^i\psi_{a\,\beta} - \frac{\mathrm{i}}{2}\xi\lambda^i(\sigma^a\bar{\psi}_a)_\beta + \\
&\quad + \frac{\mathrm{i}}{2}(\xi\sigma^a\bar{\psi}_a)\lambda_\beta{}^i \ , &(85)\\
\mathrm{s}\, D^i &= \xi\sigma^a\mathcal{D}_a\bar{\lambda}^i + \xi\lambda^i\big(-\frac{3}{8}\mathrm{i}\bar{\psi}_a\bar{\psi}^a + \frac{\mathrm{i}}{4}\bar{\psi}_a\sigma^{ab}\bar{\psi}_b\big) - \\
&\quad - \frac{1}{2}\xi\sigma^a\bar{\psi}^b\big(\eta_{ab}D^i - F_{ab}{}^i + \frac{\mathrm{i}}{2}\varepsilon_{abcd}F^{cd\,i}\big) + &(86)\\
&\quad + \frac{\mathrm{i}}{8}\xi\sigma^a\bar{\lambda}^i\big(4b_a + \psi^b\sigma_a\bar{\psi}_b - \psi_a\sigma_b\bar{\psi}_b - \psi_b\sigma^b\bar{\psi}_a + \mathrm{i}\varepsilon_{abcd}\psi^b\sigma^c\bar{\psi}^d\big) + \text{c.c.}\\
\mathrm{s}\,\phi &= \sqrt{2}\xi\chi \ , &(87)\\
\mathrm{s}\,\chi_\alpha &= -\sqrt{2}\xi_\alpha F - \sqrt{2}\mathrm{i}(\sigma^a\bar{\xi})_\alpha(\mathcal{D}_a\phi - \frac{1}{\sqrt{2}}\psi_a\chi) \ , &(88)\\
\mathrm{s}\, F &= -\sqrt{2}\mathrm{i}\bar{\xi}\bar{\sigma}^a\big(\mathcal{D}_a\chi - \frac{1}{\sqrt{2}}\psi_a F - \frac{\mathrm{i}}{2}\sigma^b\bar{\psi}_a(\mathcal{D}_b\phi - \frac{1}{\sqrt{2}}\psi_b\chi)\big) + \\
&\quad + 2\bar{\xi}\bar{\lambda}^i\delta_i\phi - \frac{\sqrt{2}}{6}\chi\sigma^a\bar{\xi}b_a - \frac{\sqrt{2}}{3}M^*\xi\chi \ . &(89)
\end{aligned}$$

Density Formula

$$D_\alpha\,(D^2 + \frac{4}{3}M^*)\,K = 0 \ , \ \text{chiral projector.} \tag{90}$$

$$\begin{aligned}
\mathcal{E}(L) &= \sqrt{\mathfrak{g}}\big(-\frac{1}{4}D^2 + \frac{\mathrm{i}}{2}\,\bar{\psi}_a\bar{\sigma}^a D - M^* - \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b\big)\,L \ , \ \bar{D}_{\dot{\alpha}}L = 0 \ , &(91)\\
\delta\mathcal{E}(L) &= \partial_m\big(\sqrt{\mathfrak{g}}E_a{}^m(-2\mathrm{i}\,\bar{\xi}\bar{\sigma}^a D + 4\,\bar{\xi}\bar{\sigma}^{ab}\bar{\psi}_b)\,L\big) \ , &(92)\\
L &= \frac{3}{8}\big(\bar{D}^2 + \frac{4}{3}M\big)\,K(\bar{\phi},\phi) + g(\phi) - \frac{1}{4}\,\lambda^i\lambda^j f_{ij}(\phi) \ , &(93)\\
\mathcal{L} &= \mathcal{E}(L) + \text{c.c.} \ , &(94)
\end{aligned}$$

$$\mathcal{L}_{\text{supergravity}} = \sqrt{\mathfrak{g}}\big(2\mathcal{R} - \varepsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\psi_{cd} - \psi_a\sigma_b\bar{\psi}_{cd}) - \frac{4}{3}\,M^*M - \frac{4}{3}b_a b^a\big) \ . \tag{95}$$