**1)** $T = 4\sqrt{\frac{m}{2}} \int_0^a dx \, \frac{1}{\sqrt{-V_0 + V_0 a^2/x^2}} = 4a\sqrt{\frac{m}{2V_0}}\, J \;, \quad J = \int_0^1 dx \, \frac{x}{\sqrt{1-x^2}} = 1 \;,$

denn Substitution $x = \sin(\varphi)$ gibt $J = \int_0^{\pi/2} d\varphi \, \sin(\varphi)$.

**2)** $V_1 - V_2 = -\gamma m M \left( \frac{1+\epsilon}{p} - \frac{1-\epsilon}{p} \right) = -\gamma m M \frac{2\epsilon}{p} \;, \quad \vec{r} = p\, \frac{(c\,,\, s)}{1+\epsilon c} \;, \quad \partial_\varphi \vec{r} =: \vec{v} \;,$

$\vec{v} = p\, \frac{(-s\,,\, c)}{1+\epsilon c} + p\, \frac{\epsilon s\,(c\,,\, s)}{(1+\epsilon c)^2} = p\, \frac{(-s, \epsilon + c)}{(1+\epsilon c)^2} \;, \quad \vec{v} \cdot \frac{\vec{r}}{r^3} = \frac{1}{p}\, \epsilon s \;, \quad A = \int_{\mathcal{C}} d\vec{r} \cdot \vec{K} \;,$

$A = -\gamma m M \int_0^\pi d\varphi \;\; \vec{v} \cdot \frac{\vec{r}}{r^3} = -\gamma m M \frac{\epsilon}{p} \int_0^\pi d\varphi \; \sin(\varphi) = -\gamma m M \frac{2\,\epsilon}{p} \;.$

**3)** $I = \int_S d^2 r \; (-\vec{e}_3) \cdot \vec{\jmath}\,\Big|_{z=0} = \int_S d^2 r \; \frac{\alpha}{\rho^3} \; = \; 2\pi \int_R^\infty d\rho \; \frac{\alpha}{\rho^2} \; = \; \frac{2\pi\,\alpha}{R}$ (Mit $+\vec{e}_3$ zu rechnen, sei OK: gibt lediglich $I = -2\pi\alpha/R$.)

**4)** $\frac{1}{r} \int_0^r dr' \; r'^2 \, \rho(r') + \int_0^r dr' \; r' \, \rho(r') \;, \quad M = \int d^3 r \; \rho = \alpha\, 4\pi \int_0^R dr \; 1 \; = \; 4\pi\,\alpha\, R \;,$

$V_{\text{innen}} = -4\pi\gamma m\, \alpha \left[ 1 + \ln\left(\frac{R}{r}\right) \right] \;, \quad V_{\text{aussen}} = -4\pi\gamma m \frac{1}{r} \int_0^R dr' \; \alpha \; = \; -\gamma m M \frac{4\pi\alpha R}{r} \;,$

$\Delta\, V_{\text{innen}} = 4\pi\gamma m \alpha \; \frac{1}{r} \partial_r^2 \, r \, \ln(r) = 4\pi\gamma m\, \alpha \, \frac{1}{r}\, \partial_r \; (\ln(r) + 1) = 4\pi\gamma m \; \frac{\alpha}{r^2} = 4\pi\gamma m \, \rho_{\text{innen}} \;.$

**5)** Kein $t$ : „$y' =: p(y)$". Hier: $\dot{x} =: v(x) \;\curvearrowright\; v' = -\frac{2\,v}{x} \;, \quad v(1) = 1 \;\; ; \quad \frac{v'}{v} = \frac{-2}{x} \;,$

$\ln(v) = C + \ln\left(\frac{1}{x^2}\right) \;, \quad C = 0 \;\; ; \quad v = \frac{1}{x^2} \;, \quad \dot{x} = \frac{1}{x^2} \;, \quad x^2\, \dot{x} = (x^3/3)^\bullet = 1 \;,$

$x^3 = 3\,t + A \;, \quad x(0) = 1 \;:\; A = 1 \;, \quad x = (1 + 3\,t)^{1/3} \;.$

**6)** $t\,\dot{G} - G = \delta(t-a) \;, \quad G_{\text{hom}} = t \;, \quad G = t\,u \;, \quad \dot{u} = \frac{1}{t^2}\,\delta(t-a) = \frac{1}{a^2}\,\delta(t-a) \;,$

$u = \frac{1}{a^2}\,\theta(t-a) \;, \quad G = \frac{t}{a^2}\,\theta(t-a) \;, \quad x = \int_0^t da \; \frac{t}{a^2}\, a^2 = t \int_0^t da = t^2 \;, \quad t\,2t - t^2 = t^2 \;.$

**7)** $\frac{1}{2}\left(\frac{1}{3}\mathrm{e}^x + \mathrm{e}^{-x}\right) \;\; ; \;\; x^2 \partial_x^2 x^3 = 6x^3 \;:\; \mathrm{e}^{-6\,\alpha}\, x^3 \;\; ; \;\; \mathrm{e}^{-tD4k^2} \, \frac{\cos(2kr)}{r} \;\; ; \;\; (\vec{r}\,\nabla)\, r^2 = 2\, r^2 \;:\; \mathrm{e}^2\, r^2 \;.$

**8)** Z.B. $\phi$ zuerst: $-\Delta_2 \phi = -\frac{1}{\rho}\, \partial_\rho \rho \partial_\rho \, \phi \overset{!}{=} \alpha \;, \quad \rho\, \partial_\rho \, \phi = -\frac{\alpha}{2}\rho^2 + A \;, \quad \phi' = -\frac{\alpha}{2}\rho + A\frac{1}{\rho} \;,$

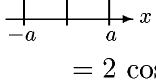
$\phi = -\frac{\alpha}{4}\,\rho^2 + A\ln(\rho) + B \;, \quad A = 0$ (kein Draht auf Achse) , $\vec{E} = -\nabla\phi = \frac{\alpha}{2}\vec{\rho} \;.$

**9)** $Mg\,\frac{a}{\sqrt{2}} = Mg\frac{a}{2} + \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \;, \quad \omega = \frac{v}{a/2} \quad (!\lightning!) \;, \quad v = \sqrt{\frac{(\sqrt{2} - 1)\, g\, a}{1 + \frac{4\,I}{M\,a^2}}} \;.$

**10)** (4) : $\vec{0} = \vec{\jmath}/\varepsilon_0 + \dot{\vec{E}}$ ; (2) : $\nabla \times \vec{E} = \vec{e}_3\, B_0\, \omega\, \mathrm{e}^{-\omega t}$. Es gibt mehrere $\vec{E}$ (und je zugehörige $\vec{\jmath}$), welche diese diese Dgl. erfüllen. Zum Beispiel „$\vec{E}$ wie $\vec{v}$ von gefrorenem Wasser":

$\vec{E} = f(t)\,(-y, x, 0) \;, \quad \nabla \times \vec{E} = f(t)\, 2\,\vec{e}_3 \;, \quad \vec{E} = \frac{\omega}{2}\, B_0\, \mathrm{e}^{-\omega t}\, (-y, x, 0) \;,$

(1) : $\rho \equiv 0$ , (4) s. oben : $\vec{\jmath} = -\varepsilon_0\, \dot{\vec{E}} = \frac{1}{2}\,\varepsilon_0\, \omega^2\, B_0\, \mathrm{e}^{-\omega t}\, (-y, x, 0) \;.$

**11)** 

$\vec{E} = \vec{e}_3\, E_0 \left[ \cos\Big(k\,[x+a] - \omega t\Big) + \cos\Big(k\,[x-a] - \omega t\Big) \right]$

$= 2\, \cos(kx - \omega t)\, \cos(ka) \overset{!}{\equiv} 0 \;, \quad ka = \frac{\pi}{2}\,(2n+1) \;, \quad n \in \mathbb{N} \;, \quad a = \frac{\pi}{2k}\,(2n+1) \;.$

**12)** $\delta_{\text{per}} = \sum_n \frac{1}{L}\, \mathrm{e}^{\mathrm{i}n\frac{2\pi}{L}x} = \lim_{\alpha\to\infty} \frac{\alpha}{L\,\mathrm{sh}(\alpha)}\, rhs. \;, \quad \lim_{\alpha\to\infty} \frac{\alpha}{L\,\mathrm{sh}(\alpha)}\, lhs. \to \frac{\alpha}{L}\, \mathrm{e}^{-\alpha}\, \mathrm{e}^{\alpha\left|1 - \frac{2x}{L}\right|} \Big|_{\substack{\text{in } (0,L) \\ \text{per. sonst}}} \;,$

was für $-L/2 < x < L/2$ und mit $\varepsilon := L/(2\alpha)$ prompt zur Delta–Darstellung $\frac{1}{2\varepsilon}\, \mathrm{e}^{-|x|/\varepsilon}$ wird .

**13)** $\widetilde{T} = 2\pi \int_0^\infty dr \; r^2\, T_0 \frac{a}{r}\, \mathrm{e}^{-r/a} \int_{-1}^1 du \; \cos(kru) = 4\pi\, T_0\, \frac{a}{k} \int_0^\infty dr \; \mathrm{e}^{-r/a} \sin(kr)$

$= \frac{4\pi T_0 a}{2\mathrm{i}k} \left[ \int_0^\infty dr \; \mathrm{e}^{(\mathrm{i}k - \frac{1}{a})\, r} - \text{dito}_{-k} \right] = \frac{4\pi T_0 a}{2\mathrm{i}k} \left[ \frac{a}{1 - \mathrm{i}ka} - \frac{a}{1 + \mathrm{i}ka} \right] = \frac{4\pi\, T_0\, a^3}{1 + a^2 k^2} \;.$