

- 1) $A = mgh$. $\vec{r} = (v_0 t, h - \frac{1}{2}gt^2)$, $\vec{v} = (v_0, -gt)$, $t_1 = 0$, $z(t_2) \stackrel{!}{=} 0 \leadsto t_2 = \sqrt{2h/g}$
 $A = \int_0^{t_2} dt (v_0, -gt) \cdot (0, -mg) = mg^2 \int_0^{t_2} dt t = mg^2 \frac{1}{2} t_2^2 = mgh$.
- 2) $I_{\text{mantel}} = \int_{\text{Mantel}(R)} df \vec{e}_\rho \cdot \alpha (-z \vec{r}) = \int_{\dots} df (-\alpha z \rho) \Big|_{\rho=R} = -\alpha R 2\pi R \int_0^h dz z = -\alpha \pi R^2 h^2$, $I_{\text{deckel}} = \int_{\text{Deckel}} df \vec{e}_3 \cdot \alpha (2r^2 \vec{e}_3 - z \vec{r}) \Big|_{z=h} = 2\alpha \int_{\dots} df \rho^2 + \alpha h^2 \pi R^2 = 2\alpha 2\pi \int_0^R d\rho \rho^3 + \dots = \alpha \pi R^2 (R^2 + h^2)$.
- 3) $(\partial_t + 2t)G = \delta(t-a)$, $G_{\text{hom}} = e^{-t^2}$, $G = e^{-t^2} u$, $u' = e^{t^2} \delta(t-a) = e^{a^2} \delta(t-a)$, $u = C + e^{a^2} \theta(t-a) \leadsto G = C e^{-t^2} + e^{a^2} e^{-t^2} \theta(t-a)$ (Mit PQ-Formel ging es auch sehr gut.)
- 4) $E = \frac{M}{2}(R\dot{\varphi})^2 + \frac{I}{2}\dot{\varphi}^2 + \frac{\kappa}{2}(2R\varphi)^2$, $0 = (MR^2 + I)\dot{\varphi}\ddot{\varphi} + 4\kappa R^2 \varphi \dot{\varphi}$, $\ddot{\varphi} = -\frac{4\kappa R^2}{(\dots)} \varphi$, $\omega = \sqrt{\frac{4\kappa R^2}{MR^2 + I}}$.
- 5) $V_{\text{eff}} = \alpha r^2 + \frac{L^2}{2mr^2}$, $V'_{\text{eff}} = 2\alpha r - \frac{L^2}{mr^3} \stackrel{!}{=} 0$, $r_0^4 = \frac{L^2}{2\alpha m}$, $m\ddot{r} = -V'_{\text{eff}} = -2\alpha(r_0 + \eta) + \frac{L^2}{m(r_0 + \eta)^3}$
 $m\ddot{r} = -2\alpha r_0 - 2\alpha\eta + \frac{L^2}{mr_0^3} \left(1 - 3\frac{\eta}{r_0}\right) + \mathcal{O}(\eta^2) = -\left(2\alpha + \frac{3L^2}{mr_0^4}\right)\eta + \mathcal{O} = -8\alpha\eta + \mathcal{O}$, $\omega^2 = 8\alpha/m$.
- 6) $\frac{m}{2}v^2 = \frac{m}{2}u^2 + U$, $v_2 = u_2 \leadsto \frac{m}{2}v_1^2 = \frac{m}{2}u_1^2 + U$, und bei $v_1 < \sqrt{\frac{2}{m}U}$ gibt es kein u_1 mehr.
- 7) $\nabla \vec{B} = \vec{e}_3 \nabla 2r^2 - \vec{e}_3 \vec{r} - z \nabla \vec{r} = 0$, $(\vec{r} \nabla) \vec{B} = \vec{e}_3 (\vec{r} \nabla) 2r^2 - \vec{r} (\vec{r} \nabla) z - z (\vec{r} \nabla) \vec{r} = 2 \vec{B} \leadsto \text{Bruch} = \frac{1}{4}$
 $\vec{A} = \frac{1}{4} \vec{B} \times \vec{r}$, $4 \nabla \times \vec{A} = \nabla \times (\vec{B} \times \vec{r}) = \vec{B} (\nabla \vec{r}) + (\vec{r} \nabla) \vec{B} - \vec{r} (\nabla \vec{B}) - (\vec{B} \nabla) \vec{r}$, $3+2-0-1=4$.
- 8) $\rho = \frac{Q}{4\pi R^2} \delta(r-R)$, $\vec{v} = \omega \vec{e}_3 \times \vec{r}$, $\vec{j} = \vec{v} \rho$, $\frac{1}{\varepsilon_0 c^2} \frac{\omega Q}{4\pi R^2} =: \gamma$, $\vec{J} = \gamma \int d^3 r' \frac{\vec{r}' \delta(r'-R)}{|\vec{r} - \vec{r}'|}$
 $f(r) = \frac{\gamma}{r^2} \int d^3 r' \frac{\vec{r} \vec{r}' \delta(r'-R)}{\sqrt{r^2 + r'^2 - 2\vec{r} \vec{r}'}}$, $\cos(\vartheta) = u$, $f(r) = \frac{\gamma}{r^2} 2\pi R^2 \int_{-1}^1 du \frac{r R u}{\sqrt{r^2 + R^2 - 2r R u}}$.
- 9) $T(\vec{0}, 0) = T_0$, $T(\vec{r}, 0) = \frac{T_0}{2\kappa} \frac{\text{sh}(2\kappa r)}{r}$, $\Delta \frac{\text{sh}(2\kappa r)}{r} = \frac{1}{r} \partial_r^2 r \frac{\text{sh}(2\kappa r)}{r} = (2\kappa)^2 \frac{\text{sh}(2\kappa r)}{r}$ (was man auch im Kopf schnallen durfte), $T(\vec{r}, t) = \frac{T_0}{2\kappa} e^{tD} 4\kappa^2 \frac{\text{sh}(2\kappa r)}{r}$, $T(\vec{0}, t) = T_0 e^{tD} 4\kappa^2 \stackrel{!}{=} 2T_0$, $t_1 = \frac{\ln(2)}{D 4\kappa^2}$.
- 10) $\rho = \varepsilon_0 \nabla \vec{E} = \varepsilon_0 2\alpha z$, $\nabla \times \vec{E} = 3\alpha(-y, x, 0) \stackrel{!}{=} -\partial_t \vec{B}$, $\vec{B} = -3\alpha t(-y, x, 0)$. Daß $\nabla \vec{B} = 0$ ist, sieht man im Kopf. Bleibt $\vec{j} = \varepsilon_0 c^2 \nabla \times \vec{B} = -\varepsilon_0 c^2 3\alpha t(0, 0, 2) = -6\varepsilon_0 c^2 \alpha t \vec{e}_3$.
- 11) $\nabla \cos(\vec{k} \vec{r} - ct) \times \vec{E}_0 = -\vec{k} \sin(\dots) \times \vec{E}_0 \stackrel{?}{=} -\partial_t \frac{1}{c} \frac{\vec{k}}{k} \times \vec{E}_0 \cos(\dots) = -\vec{k} \times \vec{E}_0 \sin(\dots)$, ja.
- 12) $c_n = \frac{1}{L} \int_0^L dx \alpha \delta\left(x - \frac{L}{2}\right) e^{in\frac{2\pi}{L}x} = \frac{\alpha}{L} (-1)^n$, $T(x, t) = \frac{\alpha}{L} \sum_n (-1)^n e^{-tD(n\frac{2\pi}{L})^2} e^{in\frac{2\pi}{L}x}$
 $T\left(\frac{L}{2}, t\right) = \frac{\alpha}{L} \sum_n e^{-tD(\frac{2\pi}{L})^2 n^2} \rightarrow \frac{\alpha}{L} \int dn e^{-tD(\frac{2\pi}{L})^2 n^2} = \frac{\alpha}{L} \sqrt{\frac{L^2}{(2\pi)^2 t D}} \int dn e^{-n^2} = \frac{\alpha}{\sqrt{4\pi t D}}$.
- 13) $\tilde{T}(\vec{k}) = \int d^3 r e^{-i\vec{k} \vec{r}} T(r) = 2\pi \int_R^\infty dr r^2 \frac{T_0}{\alpha r} e^{-\alpha r} \int_{-1}^1 du e^{-ikru}$, u -Integral = $\frac{2 \sin(kr)}{kr}$,
 $\tilde{T}(\vec{k}) = \frac{4\pi T_0}{\alpha k} \int_R^\infty dr \sin(kr) e^{-\alpha r} = \frac{2\pi T_0}{\alpha k i} \int_R^\infty dr e^{ikr - \alpha r} + \text{dito}_{-k} = \frac{2\pi T_0 e^{-\alpha R}}{\alpha k i} \left(\frac{e^{ikR}}{\alpha - ik} - \frac{e^{-ikR}}{\alpha + ik} \right)$
 $() = \frac{(c + is)(\alpha + ik) - (c - is)(\alpha - ik)}{\alpha^2 + k^2} = 2i(\dots)$, $\tilde{T}(\vec{k}) = \frac{4\pi T_0 e^{-\alpha R}}{\alpha k} \frac{\alpha \sin(kR) + k \cos(kR)}{\alpha^2 + k^2}$.