Exercise 1: Born approximation (6p)

Consider particles of incoming energy E that scatter from a spherically symmetric potential of the form:

$$V(r) = \begin{cases} V_0, & \text{for } r < r_0 \\ 0, & \text{for } r \geq r_0. \end{cases} \tag{1}$$

This problem is a reasonably good approximation for the scattering of microscopic particles from an atomic nucleus, where the radius r_0 is called *nuclear radius*.

(a) Calculate the scattering amplitude $f(\theta)$ to the first order in the Born approximation.(4p)

(b) Obtain from (a) the differential cross section and discuss it for the case of small energies. What happens when $V_0 \to \infty$? Is it correct? (2p)

Exercise 2: Partial wave expansion (6p)

Consider now the s-wave scattering ($l = 0$ in the partial wave expansion) in the previous problem, for the case where the incoming energy satisfies $E < V_0$.

(a) Obtain the equation that determines the scattering phase δ_0. (Hint: solve for $r < r_0$ and for $r > r_0$, and then match both relations.) (4p)

(b) Calculate the phase shift δ_0 and the scattering cross section σ_0 in the limit $V_0 \to \infty$, the so called *Hard Sphere* limit. (Hint: you should recover the results dicussed in the theory lecture.) (2p)

Exercise 3: Indistinguishable and distinguishable particles (2p)

Consider the case of spin-1/2 fermions with random orientation interacting via a Yukawa potential (recall the Präsenzübungen). Calculate in the 1st Born approximation the back-scattering ($\theta = \pi$) differential cross section, and compare it to the case of distinguishable particles. (2p)