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*Exercise 1: Properties of the $\gamma$-matrices*

(a) Show that $\Delta\tau = \frac{-i}{4}\sigma_{\mu\nu}\Delta\omega^{\mu\nu}$ with $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ fulfills

$$[\gamma^\mu, \Delta\tau] = \Delta\omega^\mu_\nu\gamma^\nu, \quad (1)$$
$$\mathrm{Tr}\Delta\tau = 0 \quad (2)$$

where $\Delta\omega_{\mu\nu} = -\Delta\omega_{\nu\mu}$.

(b) Show that $(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$.

*Exercise 2: Plane wave solutions*

(a) Show $\not{p} \cdot \not{p} = m^2c^2$.

(b) Let $\begin{cases} \Psi_r^+(x) = u_r(p)e^{-ipx/\hbar} \\ \Psi_r^-(x) = v_r(p)e^{ipx/\hbar} \end{cases}$ show that

$$(\not{p} - mc)u_r(p) = 0 \quad (3)$$
$$(\not{p} + mc)v_r(p) = 0 \quad (4)$$

(c) Show that

$$u_r(p) = N(\not{p} + mc)u_r(0), \quad (5)$$
$$v_r(p) = N(\not{p} - mc)v_r(0), \quad (6)$$

where $N$ is a normalization constant, $u_r(0) = \begin{pmatrix} \chi_r \\ 0 \end{pmatrix}$, $v_r(0) = \begin{pmatrix} 0 \\ \chi_r \end{pmatrix}$, and $\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(d) Calculate $(\not{p} + mc)u_r(0)$ and $(\not{p} - mc)v_r(0)$.

(e) Impose $\bar{u}_r(p)u_r(p) = 1$; $\bar{v}_r(p)v_r(p) = 1$, and find the normalization constant $N$.

*Exercise 3: Density*

Calculate the density $\rho = j^0/c$ for the solutions of the Dirac equation in free space. Why is it not Lorentz invariant? Relate it to the idea of Lorentz contraction.

*Exercise 4: Equivalent representations of the $\gamma$-matrices*

Let $M$ be an arbitrary non-singular matrix. Show that $M\gamma^\mu M^{-1}$ is also a possible representation of the $\gamma$-matrices, i.e. it fulfills $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}$.