Klassische Teilchen und Felder

Präsenzübung, Blatt 06 **WS 08/09 18.11.2008**

Vorlesung: Luis Santos – Übungen: Garu Gebreyesus & Tobias Wirth

[P12] Lorentz force – generalized potentials

Later in this course we will see that a particle moving with velocity $\vec{v}$ in an electromagnetic field is acted upon by the so-called Lorentz force

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$

with $\vec{E}$ the electrical field and $\vec{B}$ the magnetic induction.

We will see as well that $\vec{E}$ and $\vec{B}$ can be written as

$$\begin{aligned} \vec{B} &= \vec{\nabla} \times \vec{A} \\ \vec{E} &= -\vec{\nabla}\phi - \frac{\partial}{\partial t}\vec{A} \end{aligned}$$

where $\vec{A}$ is a vector potential and ϕ is a scalar potential.

a) Show that

$$F_{x_j} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial}{\partial \dot{x}_j} U - \frac{\partial}{\partial x_j} U \quad \text{with } x_{j=1,2,3} = x, y, z$$

where $U = Q(\phi - \vec{v} \cdot \vec{A})$.

b) Show that the D'Alembert principle $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial T}{\partial \dot{x}_j} - \frac{\partial T}{\partial x_j} = F_{x_j}$ is again of the form $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}_j} - \frac{\partial L}{\partial x_j} = 0$ (even though $\vec{F} \neq -\vec{\nabla}U$).
U is a so called *generalized potential* and L is now a *generalized Lagrange function.*

[P13] friction

We consider a system with holonomic constraints. Conservative forces (with associated potential V) exist as well as friction (with generalized force $Q_j^{(R))}$). As you know friction is not a conservative force, a typical form of it is $Q_j^{(R)} = -\sum_{j=1}^{s} \beta_{jl}\dot{q}_l$.

a) Show that for such type of systems

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = 0$$

where D is the so called dissipation function which you have to derive.

b) Derive $\frac{\mathrm{d}}{\mathrm{d}t} E = -2D$ where $E = T + V$ is the total energy.

Meldungszeitraum für Bachelorstudiengang beachten: 12.-28. November