Homeexercises Sheet 11 QUANTENMECHANIK I (SS08) Prof. Dr. Luis Santos
Hand in until 15.07.08 in the lecture. *Please state name and sheet number!*

This is the last sheet of homeexercises. All exercises concern time-independent perturbation theory.

Exercise 11-1: Box-potential with linear perturbation (3 points)

We consider a box-potential with infinitely high walls,

$$\hat{H}_0 = \frac{p^2}{2m} + V_0(x), \quad \text{where} \quad V_0(x) = \begin{cases} 0 \text{ für } |x| < a, \\ \infty \text{ für } |x| \geq a. \end{cases}$$

We add a small perturbation of the form $\hat{H}_1 = Fx$, where $Fa \ll \frac{\pi^2 \hbar^2}{2ma^2}$.

1. Calculate the first non-vanishing correction to the groundstate energy of the Hamilton operator $\hat{H}_0$. Hint: $\sum_{n=1}^{\infty} \frac{n^2}{(4n^2-1)^5} = \frac{\pi^2}{2^{12}}(5 - \frac{\pi^2}{3})$.

Exercise 11-2: Perturbed rigid rotator (3 points)

The Hamilton operator of a rigid rotator in a homogeneous magnetic field $\vec{B} = B\hat{z}$ is given by

$$\hat{H}_0 = \frac{1}{2\theta}\hat{\vec{L}}^2 + gB\hat{L}_z \equiv \frac{\alpha}{\hbar^2}\hat{\vec{L}}^2 + \frac{\beta}{\hbar}\hat{L}_z,$$

where θ is the moment of inertia and g is the gyromagnetic ratio. A second homogeneous magnetic field $\vec{B}' = B'\hat{x}$ is applied, where $\gamma = g\hbar B' \ll \beta$ is assumed to hold. This corresponds to a perturbation $\hat{V} = \frac{\gamma}{\hbar}\hat{L}_x$.

1. Calculate the corrections to the energies of the unperturbed eigenstates $|l, m\rangle$ up to second order in γ. (1 point)
2. Obtain the exact eigenenergies of the perturbed Hamiltonian for $l = 1$. (1.5 points)
3. Develop the exact eigenenergies in $\epsilon \equiv \frac{\gamma}{\sqrt{2}\beta}$ up to the order ϵ^2 and compare the result with the one of the first part of the exercise. (0.5 points)

Exercise 11-3: Anharmonic oscillator (4 points)

The Hamilton operator of the 1d harmonic oscillator is

$$\hat{H}_0 = \frac{p^2}{2m} + \frac{m\omega}{2}x^2.$$

Consider the perturbation $\hat{H}_1 = \mu(x^3 + x^4)$ with small μ.

1. Calculate the corrections of the energy levels up to first order. (1.5 points)
2. Find the expression for $\langle \hat{x} \rangle_n$ to first order, i.e., you have to calculate the expectation values with respect to the perturbed $n-$th eigenstate of the harmonic oscillator. (2.5 points)