Exercise 1: Bohr Quantization (4 points)

In the Rutherford model of the atom electrons move on circular orbits of radius r around the nucleus. We assume in this exercise that the potential of the nucleus, in which the electron moves, is given by

$$V(r) = V_0 \cdot \left(\frac{r}{a}\right)^k .$$

- Apply Bohr's quantization rule ($pr = n\hbar$) and calculate the possible radii r_n of the electron-orbits (you should obtain $r_n = \tilde{c} \cdot n^{-2/(2+k)}$ with a constant $\tilde{c}$) and the corresponding energies $E(n)$ (energy=kinetic energy + potential energy) under the assumption that for every possible orbit the force from the potential $V(r)$ compensates the centrifugal force. (3 points)
- Show that for $k \gg 1$ the energies of the orbits are approximately given by $E(n) \approx c \cdot n^2$ and that for the harmonic ($k = 2$) case $E(n) = c' \cdot n$ holds (c and c' are constant).

  (1 points)

Exercise 2: Wavepacket with special dispersion relation (2 points)

We consider here a wavepacket with k-distribution $g(k) = \exp(-\frac{k^2}{2\Delta^2})$ with dispersion law $\omega(k) = -\omega_0 \cos(\chi k)$, which holds in solid state systems in the *tight-binding*-regime.

- Calculate the group velocity of the wavepacket. (0.5 points)
- Assume that $\Delta \ll 1/\chi$ holds, *i.e.* the wavepacket is localized in k-space as compared to $1/\chi$. Paying special attention to the behaviour of $\omega(k)$ in this case calculate the wavefunction $\Psi(x,t)$. You can proceed as shown in the lecture. (1.5 Punkte)

Exercise 3: Compton-scattering (4 points)

In this exercise we ask you to investigate the collision of an electron with momentum $\vec{P}$ with a photon with momentum $\vec{p}$. You can assume that the photon moves along the x-axis in positive direction and that the electron moves in the opposite direction. After the collision the photon has momentum $\vec{p'}$. We call the angle between $\vec{p'}$ and the x-axis θ.

- Calculate the frequency ω' of the photon after the collision as a function of $|\vec{p}|$, $|\vec{P}|$, θ and the frequency ω before the collision from momentum and energy conservation (relativistically!). Check that for $\vec{P} = 0$ your result coincides with the result from the lecture.