Exercise 1: Box-potential (3 points)

In this exercise the following one-dimensional potential is given for $t < 0$: $V(x) = 0$ for $x \in [-a, 0]$ and $V(x) \to \infty$ otherwise. At $t = 0$ the right barrier is shifted instantaneously to $x = a$.

- Write down the eigenfunctions and eigenenergies of the potential for $t < 0$. (0.5 points)
- For $t < 0$ the system is assumed to be prepared in the ground state. Develop the wavefunction for $t \geq 0$ in the basis of the potential with the shifted right barrier. (2.5 points)

Useful identities: $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$, $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \pm (-\sin(\alpha)\sin(\beta))$, $\sin^2(\theta/2) = \frac{1}{2}(1 - \cos(\theta))$.

Exercise 2: Time-evolution in the box-potential (4 points)

Consider again a box-potential as in exercise 1 with barriers at $\pm a$. The system is at $t = 0$ in the state

$$\psi(x, 0) = \frac{1}{\sqrt{2a}}\Big(\sin(\frac{\pi}{a}x) + \cos(\frac{\pi}{2a}x)\Big).$$

- Write down the time-evolved state $\psi(x, t)$. (0.5 points)

  Hint: write down the state in the eigenbases first.
- Calculate the expectation values $\langle \hat{x} \rangle$ and $\langle \hat{p} \rangle$. (1.5 points)

  Hint: They should be time-dependent.
- Calculate $\langle \hat{x}^2 \rangle$ and $\langle \hat{p}^2 \rangle$. (1.5 points)

  Hint: They should not be time-dependent.
- Check (roughly/numerically)that the uncertainty relation $\Delta x \Delta p \geq \hbar/2$ holds for all times. (0.5 points)

Useful identities: $\int_{-\pi/2}^{\pi/2} dy\ y \sin(2y)\cos(y) = 8/9$, $\int_{-\pi}^{\pi} dy\ y^2 \sin(y) = \pi^3/3 - \pi/2$ und $\int_{-\pi/2}^{\pi/2} dy\ y^2 \cos(y) = \pi^3/24 - \pi/4$.

Exercise 3: Box-potential with periodic boundary conditions (3 points)

Here the box-potential has the barriers at $x = 0$ and $x = L$. Given are *periodic boundary conditions*:

$$\psi(0) = \psi(L) \quad \text{and} \quad \psi'(0) = \psi'(L),$$

where ψ' denotes $d\psi/dt$. Calculate the eigenfunctions and eigenenergies, and compare with the results of the lecture without periodic boundary conditions (1.5 points up to here). Find the (normalized) combinations of the eigenfunctions which are symmetric/antisymmetric with respect to the transformation $\psi(x - L/2) \leftrightarrow \psi(x + L/2)$.