Exercise 1: Box-potential with a step (4 points)

In this exercise you shall consider a variant of the box-potential. A particle is locked in into the region $[-L, L]$ on the x-axis. The potential is vanishing in the region $[-L, 0]$ and equal to $V_0 > 0$ in the region $[0, L]$.

- Find the condition for the energies from the boundary conditions and the continuity conditions for $E > V_0$ and $E < V_0$. (2 points)
- Is there a solution for $E = V_0$? (1 point)
- Find the energy of the ground state in units of $E_L = \frac{\hbar^2}{2mL^2}$ for $V_0 = 10 \cdot E_L$. You can proceed numerically. (1 point)

Exercise 2: Delta-potential (2 points)

Let the potential be equal to $V(x) = g\delta(x)$. Calculate the transmission-coefficient for a particle coming in from the left and write down the condition from the conservation of the probability current. Under what conditions is there large/small transmission?

Exercise 3: Box-potential with delta-function (3 points)

Here the potential is given by a box potential with barriers at $x = \pm L$, and the potential inside the barriers is given by $g\delta(x)$.

- Write down the equation determining the eigenenergies. (1 point)
- Show that the wavefunctions in the right/left region can be written as

$$A(\cos(kx) \pm \frac{mg}{\hbar^2 k} \sin(kx)))$$

  where $k^2 = \frac{2mE}{\hbar^2}$. (1 point)
- Calculate the norm-factor A. (2 points)