Exercise 1: Composition of 2 spins (2 Points)

The Hamiltonian for a system of two spins is given by

$$\hat{H} = A + \frac{B}{\hbar^2}\hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2 + \frac{C}{\hbar}\left(\hat{S}_{1z} + \hat{S}_{2z}\right),$$

where A, B and C are constants. Calculate the eigenvalues and eigenstates of the Hamiltonian assuming that:

- The two particles have spin 1/2.
- One particle has spin 1/2 and the other spin 1.

Exercise 2: Time evolution of spins (3.5 Points)

Consider two particles. Particle 1 has spin 1, whereas particle 2 has spin 1/2. At time $t = 0$, the state of the two-particle system is

$$\Psi(0) = a|1, -1/2\rangle + b|0, 1/2\rangle,$$

where a and b are constants (real numbers). We use the notation $|m_1, m_2\rangle \equiv |s_1 = 1, m_1; s_2 = 1/2; m_2\rangle$. We assume that the dynamics of the two-particle system is provided by the Hamiltonian:

$$\hat{H} = \frac{\alpha}{\hbar}\hat{\vec{S}}_1 \cdot \hat{\vec{S}}_2,$$

where α is a constant. Calculate after a given time t the state $|\Psi(t)\rangle$, and the expected value $\langle \hat{S}_{1z}\rangle$.

Exercise 3: Composition of three spins (4.5 Points)

Consider three particles of spin 1/2. Let $\hat{\vec{J}} = \hat{\vec{S}}_1 + \hat{\vec{S}}_2$, and $\hat{\vec{S}} = \hat{\vec{J}} + \hat{\vec{S}}_3$. obtain the states which are simultaneously eigenstates of $\{\hat{S}^2, \hat{S}_z, \hat{J}^2, \hat{S}_3^2, \hat{S}_2^2, \hat{S}_1^2\}$, and the corresponding eigenvalues. Express the eigenstates as a linear combination of the eigenstates of the basis $\{\hat{S}_1^2, \hat{S}_{1z}, \hat{S}_2^2, \hat{S}_{2z}, \hat{S}_3^2, \hat{S}_{3z}\}$. (Hint: First compose the first two spins, and then compose with the third one.)

Hint: In these exercises you will need at some point the Clebsch-Gordan coefficients for the composition of angular momenta. You can find tables of Clebsch-Gordan coefficients in internet. E.g. *http://en.wikipedia.org/wiki/Table_of_Clebsch-Gordan_coefficients*. You can find a calculator (java aplet) of Clebsch-Gordan coefficients e.g. in *http://www.gleet.org.uk/cleb/cgjava.html*.