Sheet 11 (to be returned on 16.07.07 before 12:00)

Exercise 1: Perturbed Hydrogen atom (3 Points)

Consider the Hydrogen atom ($Z = 1$), where we only consider the Coulomb interaction between proton and electron. If instead of assuming the proton as a point charge, one assumes it as an homogeneously charged sphere of radius $R \ll a_0$ (where a_0 is the Bohr radius), then instead of a Coulomb potential $V(r) = -e^2/r$ for all values of r, one has $V(r) = -e^2/r$ only for $r > R$, whereas for $r < R$ one has:

$$V(r) = -\frac{3e^2}{2R^3}\left[R^2 - \frac{r^2}{3}\right].$$

- What is the perturbation Hamiltonian in this problem? (The umperturbed Hamiltonian is that with the usual Coulomb interaction $-e^2/r$ for all r.)
- Calculate the energy shift ΔE (in first order) for the ground state of the Hydrogen atom.
- Show that the ratio between ΔE and the umperturbed ground state energy $\bar{E}$ is

$$\Delta E/\bar{E} = -\frac{4}{5}\left(\frac{R}{a_0}\right)^2.$$

Hint: At some point the fact that $R \ll a_0$ will simplify very much your calculations.

Exercise 2: Box potential with a delta perturbation (3 Points)

Consider a box potential $V(x) = 0$ for $|x| < a$, and $V(x) = \infty$ for $|x| > a$. Consider a perturbation of the form $U(x) = \epsilon a \delta(x)$, where ϵ is very small. Calculate the energy correction of the ground state of the box potential up to second order in ϵ.

Hint: The fact that many matrix elements are zero will help you a lot. You will need $\sum_{n=2}^{\infty} \frac{1}{n(n-1)} = 1$.

Exercise 3: Box potential with a small sinusoidal perturbation (4.0 Points)

Consider again the same box potential $V(x)$ of the previous exercise. Assume now a perturbation $U(x) = \epsilon \sin(\pi x/2a)$. Calculate the first non-zero correction of the energy for the even states of the box.

Hint: Use again that many matrix elements are zero due to parity reasons.