

Exercise 1: Bohr's quantization rule

- (2 Points) Let us consider the Rutherford picture of the atom, i.e. we consider an electron moving in circular orbits of radius r around the nucleus. The electron (of charge e) experiences the Coulomb attraction induced by the positive charge of the nucleus (Ze). Applying Bohr's quantization law for the allowed orbits ($pr = n/\hbar$), and imposing that for every possible orbit the centrifugal force compensates the Coulomb force, calculate the possible orbits (r_n) that an electron can follow around the nucleus, and the energies associated with these orbits (Energy=Kinetic Energy + Coulomb Energy)
- (2 Points) Suppose that instead of an electron in the Coulomb potential of a nucleus, you have a particle of mass m describing orbits of radius r under the influence of an harmonic-oscillator potential $V(r) = m\Omega^2 r^2/2$. Applying Bohr's quantization rule, and compensating the harmonic force with the centrifugal force, calculate the possible energies the particle can have.

Exercise 2: Compton scattering (3.0 Points)

A photon with frequency ω collides with an electron which is initially at rest. Calculate the maximal energy gained by the electron after the scattering. Once you have calculated it, consider the case $\hbar\omega \ll mc^2$, where m is the mass of the electron, and express the maximal energy gained at first order in $\hbar\omega/mc^2$.

Exercise 3: Uncertainty principle (3.0 Points)

Remember that for an harmonic oscillator the energy is given by $E = p^2/2m + m\Omega^2 x^2/2$. Classically the minimal energy is $E = 0$ which corresponds to $x = 0$ and $p = 0$. Quantum-mechanically the minimal values of x and p are not exactly zero, but they have an uncertainty Δx and Δp . Since we cannot bring both uncertainties to zero (as you should already know), then the minimal energy is not zero. Estimate up to a constant the minimal energy (also called the ground-state energy).