Sheet 2 (to be returned on 04.05.07 before 12:00)

Exercise 1: Potential box (3 Points)

Consider a particle initially in the ground state of a box potential with walls at $x = \pm a$. Suddenly (at $t = 0$) the walls of the box are displaced to $x = \pm b$ where $b > a$.

- What is the probability to find the particle in an odd state of the new box? Can you discuss why?
- What is the probability to find the atom in the even state $u_n^{(+)}(x)$ of the new box?
- Using the expression of the wavefunction in terms of the new eigenfunctions, write down the wavefunction after a given time t.

Exercise 2: Potential box II (2 Points)

Suppose again a particle in the ground state of a box with walls in $x = -a$ and $x = a$. Suppose that we move the walls like in the first exercise but now to $\pm\infty$. What are the eigenfunctions of the Hamiltonian after moving the walls to $\pm\infty$? What is the probability to find a particle with momentum in the range from p to $p + dp$? What happens with this probability at a later time $t > 0$?

Exercise 3: Uncertainty principle in the box potential (2 Points)

Consider a box potential with walls at $x = \pm a$. Calculate for the state the n-th even state $u_n^{(+)}$, $\langle \hat{p} \rangle$, $\langle \hat{p}^2 \rangle$, $\langle \hat{x} \rangle$, $\langle \hat{x}^2 \rangle$, Calculate $\Delta x \Delta p$ and check the uncertainty principle $\Delta x \Delta p > \hbar/2$.

Exercise 4: Box potential with periodic boundary conditions (3 Points)

Consider again a box potential with walls at $x = 0$ and $x = L$. Consider the following, so-called periodic, boundary conditions: $\psi(0) = \psi(L)$ and $\psi'(0) = \psi'(L)$, where ψ' denotes $d\psi/dx$. Calculate the eigen-functions and the eigen-energies, and compare them to the case we have seen during the theory class. Find the right combinations of the wavefunctions which are symmetric/antisymmetric with respect to the transformation $\psi(x - L/2) \leftrightarrow \psi(L/2 - x)$.