Sheet 3 (to be returned on 11.05.07 before 12:00)

Exercise 1: The potential box once more (2 Points)

Consider the wavefunction of the form $\psi(x) = A(x^2 - a^2)$ in a potential box with walls at $x = \pm a$.

- Find the constant A to normalize the wavefunction.
- Calculate the variances Δx and Δp, and check that $\Delta x \Delta p > \hbar/2$.
- Calculate the total probability to find the particle in any state different than the ground state.

Exercise 2: Parity in the potential box (3 Points)

Consider a particle which is localized in the left half of a box with sides $x = \pm a$, i.e. with a given wavefunction $\psi(x)$ if $x < 0$ and zero otherwise.

- Express the wavefunction as a linear combination of an even function and an odd function.
- Will the particle remain localized in the left half any later time?
- Suppose that the particle can be found with equal probability at any point at the left half of the box. Calculate the probability to obtain in a measurement of the energy the ground state energy of the box.

Exercise 3: The double step potential (5 Points)

Consider the two-step potential given by $V(x) = 0$ for $x < 0$, $V(x) = V_0 > 0$ for $0 < x < a$, and $V(x) = V_1 > V_0$ for $x > a$. Suppose a particle coming from the left (i.e. from $x = -\infty$), with an energy $E > V_1$.

- Write the wavefunction in the three regions of the space (first without solving for the particular coefficients).
- Obtain the form of the probability current in the three different regions and write the conservation relations for the current.
- Calculate the reflection coefficient R
- Calculate the transmission coefficient T, and $|T|^2$.
- For fixed values of V_0 and V_1 find the values of the length a for which you get that $|T|^2$ is minimal.