
Exercise 1: Three-dimensional potential box (2 Points)

- Calculate the ground state energy and the corresponding normalized eigenfunction of a three-dimensional box potential characterized by $V(x, y, z) = 0$ for $-l_x \leq x \leq l_x$ and $-l_y \leq y \leq l_y$ and $-l_z \leq z \leq l_z$, and ∞ otherwise.
- Suppose that $l_x = l_y = l$, $l_z = \alpha l$. Obtain the first excited state and the corresponding eigenenergy.

Exercise 2: Two-dimensional Schrödinger equation (4 Points)

Consider the two-dimensional time-independent Schrödinger equation in the plane xy. Assume a potential $V(x, y) = V(r)$, where $r^2 = x^2 + y^2$.

- Express the two-dimensional Schrödinger equation in polar coordinates ($x = r \cos \phi$, $y = r \sin \phi$).
- Using the method of separation of variables, show that the eigenfunctions of the Hamiltonian can be written as $\Psi(\vec{r}) = R(r)e^{im\phi}$.
- Obtain the resulting equation for the radial function $R(r)$. What is the form of the centrifugal barrier in the two-dimensional radial equation?

Exercise 3: Spherical Harmonics (4 Points)

A particle in a three-dimensional central field $V(r)$ is described by the wavefunction

$$\Psi(x, y, z) = C(xy + yz + zx)e^{-\alpha r^2},$$

where C and α are constants.

- Express the wavefunction in function of the spherical harmonics.
- Find out which values of the quantum numbers l and m can be obtained in a measurement of $\hat{L}^2$ and $\hat{L}_z$ for this particle, and with which probability.
- Suppose that we have 50% of probability to find the particle in the state $l = 1$, $m = -1$, and 50% probability to find the particle in the state $l = 2$, $m = 2$. Express the wavefunction that represents the particle using Cartesian coordinates.