Sheet 8 (to be returned on 22.06.07 before 12:00)

Exercise 1: Three-dimensional potential well (3 Points)

Consider a potential $V(r) = -V_0$ for $r < a$ and $V(r) = 0$ otherwise. Show for $l = 1$ that the condition for the allowed energies $E < 0$ is

$$\frac{2 - (\kappa a)^2 - 2\kappa a \cot \kappa a}{1 - \kappa a \cot \kappa a} = 2 + \frac{(\alpha a)^2}{1 + \alpha a},$$

with $\kappa = \sqrt{\frac{2m(E+V0)}{\hbar^2}}$, and $\alpha = \sqrt{\frac{2m|E|}{\hbar^2}}$.

Exercise 2: Three-dimensional potential well II (3.5 Points)

Consider the same potential as in the first exercise, but now with $E > 0$.

- Show for that for $r \gg a$, one can express for all l the radial wavefunction in the form:
$$R_l(r) \simeq \frac{B}{kr}\left[\sin(kr - l\pi/2) - C\cos(kr - l\pi/2)\right]$$
where B and C are constants, and $k = \sqrt{\frac{2mE}{\hbar^2}}$.
- Show that for $l = 0$,
$$C = \frac{q \cot qa - k \cot ka}{k + q \cot qa \cot ka},$$
where $q = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$.

Exercise 3: Hydrogen-like atom (2.0 Points)

Consider a Hydrogen-like atom with charge Ze in the nucleus. Calculate for the 2S state the value of $\langle r \rangle$, and $\langle r^2 \rangle$. What is the value of $\langle z \rangle$? (Hint: $\int_0^\infty d\rho \rho^s e^{-\rho} = s!$).

Exercise 4: Positronium (1.5 Points)

Positronium is formed by a positron (i.e. an antielectron, with mass as that of the electron but opposite charge) and an electron. Obtain the eigenenergies of positronium and compare them with that of Hydrogen.