Exercise 1: Free electron in a magnetic field (4.5 Points)

Consider a free electron (so not bound to an atom) in a magnetic field $\vec{B} = B\vec{u}_z$. The free electron is on the xy-plane. Do not consider the z-dimension (i.e. consider a 2D motion of the electron).

- Use the Hamiltonian that we obtain in the theory class, but now do not neglect the term dependent on B^2 ("third term" in the theory class). Change from (x, y) coordinates to polar coordinates (r, ϕ), and write the time-independent Schrödinger equation for the eigenfunction $\psi(r, \phi)$ with eigenenergy E.
- Use the method of separation of variables $\psi(r, \phi) = R(r)e^{im\phi}$ (as you did for a similar 2D problem in Sheet 7). After making a convenient change of variables $r \to \eta$, obtain that the radial equation for $R(\eta)$ can be written in the form:

$$\frac{d^2}{d\eta^2}R + \frac{1}{\eta}\frac{d}{d\eta}R - \frac{m^2}{\eta^2}R - \eta^2 R + \lambda R,$$

  where

$$\lambda = \frac{4\mu c E}{eB\hbar} - 2m$$

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- Use the expression $R(\eta) = \eta^{|m|}e^{-\eta^2/2}G(\eta)$, and show that $G(\eta) = L_{n_r}^{|m|}(\eta^2)$, where $n_r = 0, 1, 2, \ldots$. Find n_r as a function of λ and m, and from that obtain that the eigenenergies are of the form

$$E = \frac{eB\hbar}{2\mu c}(2n_r + 1 + |m| + m)$$

Exercise 2: Spin-1/2 particles (2.0 Points)

Consider a spin-1/2 system (e.g. an electron, like in the theory class).

- Calculate the eigenstates and eigenvalues of $\hat{S}_x + \hat{S}_y$.
- Suppose that you measure $\hat{S}_x + \hat{S}_y$ and you obtain that the system is in the eigenstate with maximal eigenvalue. Suppose that after that you measure $\hat{S}_z$. What is the probability to obtain as a result of the measurement $\hbar/2$?

Exercise 3: Spin dynamics in the Heisenberg picture. (3.5 Points)

A spin 1/2 particle (e.g. an electron) is under the influence of a magnetic field $\vec{B}$. Consider that the electron cannot move. Then, as we show in the theory class, the Hamiltonian describing the electron is of the form:

$$\hat{H} = \frac{eg}{2mc}\hat{\vec{S}} \cdot \vec{B},$$

where $\hat{\vec{S}} = \{\hat{S}_x, \hat{S}_y, \hat{S}_z\}$, and $g \simeq 2$.

- Obtain the Heisenberg equation for the dynamics of the operator $\hat{\vec{S}}(t)$.
- Assuming $\vec{B} = (0, 0, B)$ calculate $\hat{S}_x(t)$ and $\hat{S}_y(t)$, as a function of $\hat{S}_x(0)$ and $\hat{S}_y(0)$.