
Exercise 1: Q-Representation of a squeezed state

Show that the Q-representation of a squeezed state $|\beta, r\rangle$ is

$$Q(x_1, x_2) = \frac{1}{4\pi^2 \cosh r} e^{-\frac{1}{2}\left[\frac{(x_1 - X_1)^2}{(1+e^{-2r})} + \frac{(x_2 - X_2)^2}{(1+e^{2r})}\right]}$$

where $\beta = (X_1 + iX_2)/2$.

Hint: Use the antinormally ordered characteristic function $\chi_A(\eta)$. You know from the previous exercise sheet which is the symmetrically-ordered characteristic function $\chi(\eta)$. You have to remember the relation between $\chi(\eta)$ and $\chi_a(\eta)$.

Exercise 2: Degenerate parametric amplifier

Suppose that we employ a degenerate parametric amplifier. In the theory class we have seen the time evolution of the operator $\hat{a}(t)$.

- Consider an initial vacuum state. Show that the equal-time second-order correlation function
$$g^{(2)}(0) = \frac{\langle \hat{a}^\dagger(t)\hat{a}^\dagger(t)\hat{a}(t)\hat{a}(t)\rangle}{\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle^2}$$
is of the form:
$$g^{(2)}(0) = 1 + \frac{\cosh 2\chi t}{\sinh^2(\chi t)}$$
where we employ the same notation as in the theory class.

- Now consider an initial coherent state $|\alpha\rangle$, where $\alpha = |\alpha|e^{i\theta}$. Show that
$$g^{(2)}(0) = 1 + \frac{|\alpha|^2[\cosh 4\chi t + \cos\theta \sinh 4\chi t - \cosh 2\chi t - \cos\theta \sinh 2\chi t] + \frac{1}{2}\sinh^2 2\chi t - \sinh^2 \chi t}{(|\alpha|^2[\cosh 2\chi t + \sinh 2\chi t \cos\theta] + \sinh^2 \chi t)^2}$$

- Consider in the previous point that $|\alpha|^2 \gg \sinh^2 \chi t, \sinh \chi t \cosh \chi t$. Show that for $\theta = 0$ you get photon bunching whereas for $\theta = \pi/2$ you get photon antibunching.

Exercise 3: Damped harmonic oscillator

Show that the thermal density operator:

$$\hat{\rho}_T = e^{-\hbar\omega_0 \hat{a}^\dagger \hat{a}/k_B T}\left(1 - e^{-\hbar\omega_0/k_B T}\right)$$

is a stationary solution (i.e. $d\hat{\rho}_T/dt = 0$) of the master equation of the damped harmonic oscillator of frequency ω_0.

Show also that at any time it is fulfilled that $\frac{d}{dt}\mathrm{Tr}\{\hat{\rho}\} = 0$