ULTRACOLD ATOMIC GASES

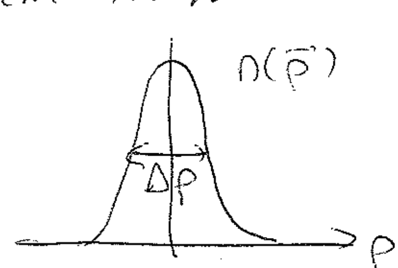
· In the final lectures of this course, we are going to have a very brief look on one of the major topics or research on modern quantum optics. Actually this field has just partially to do with standard quantum optics, having exciting resemblances to other field as condensed-matter and non-linear physics.

· This field is the field of ultra cold atomic gases. The temperatures demanded in typical experiments in this field are extremely low, of the order of μK or even nK. Hence, the first major question we have to face is how can we cool the atoms to such incredibly low temperatures. Here is where the ideas of laser cooling play a key role. In the following I will introduce to you, necessarily in a very brief way, some key ideas on laser cooling. You will quickly recognize some of the concepts we have introduced during the course, in particular in what concerns the interaction of atoms with light.

· LASER COOLING

· We know from statistical mechanics that a dilute gas in thermal equilibrium presents a Maxwell-Boltzmann momentum distribution of the form

$$n(\vec{p}) \propto e^{-p^2/2mk_BT}$$



where T is the temperature of the sample

· Note that the temperature can be ~~[illegible]~~ defined from the width of the momentum distribution $\Delta p \propto \sqrt{T}$. In this sense, the narrower the momentum distribution the colder the atomic sample.

· Therefore a cooling technique can be understood as a method to narrow the momentum distribution (and in general the energy distribution if not in free space).

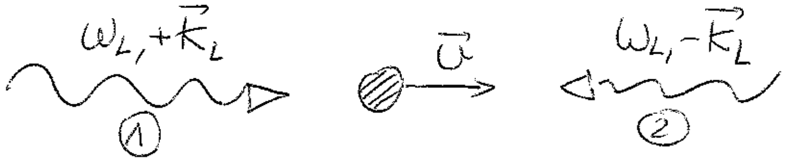
(Note: of course a thermal distribution is not possible for a single atom, but it's common the use of the term cold atom when dealing with an atom possessing a narrow momentum distribution)

· Note that the narrower is the momentum distribution of an atom, the broader it's its spatial wavepacket, and as a consequence ultracold atoms will behave in many senses as waves (matter waves) as we will see later.

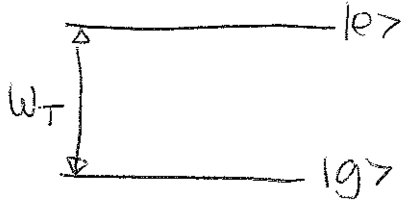
· In the following we will review three different laser cooling techniques, namely Doppler cooling, Sysiphus cooling and VCSPT cooling. You will see that in these methods (which we will not discuss in full detail) some already known concepts play a key role, as detuning and resonance (p. 32), dressed states (e.g. p 112), and dark states (remember the exercise sheet 2).

* Doppler cooling

· This is probably the simplest laser cooling technique.
Let's consider an atom moving with an initial velocity $\vec{v}$.
Let's consider also two counterpropagating laser beams of frequency ω_L and momentum $\pm\hbar\vec{k}$:



· Let's assume that the electronic structure of the atom can be reduced to two states (remember the two-level atom approximation that we introduced in p. 28):



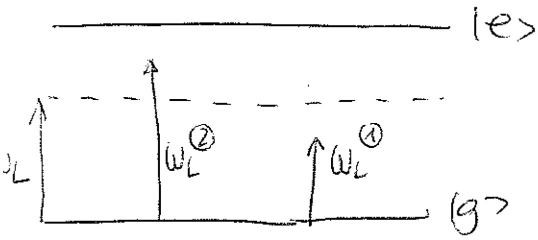
* If the atom is not moving ($v=0$) it sees both lasers ① and ② with the same frequency ω_L. But if $v \neq 0$, then the atom experiences the Doppler effect, and hence it feels an effective frequency

$$\omega_L' = \omega_L - \underbrace{\vec{k}\cdot\vec{v}}_{\text{Doppler effect}}$$

Hence
$$\left.\begin{aligned} \omega_L^{①} &= \omega_L - k_L v \\ \omega_L^{②} &= \omega_L + k_L v \end{aligned}\right\} \text{ they get shifted in an opposite way}$$

· Let's assume red detuning (remember p. 32)

$$\omega_L < \omega_T$$

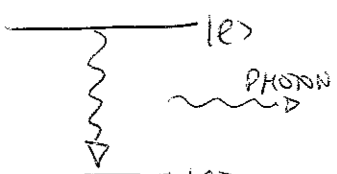


Clearly, the photon ② which opposes to the motion of the atom is closer to resonance with the atomic transition.

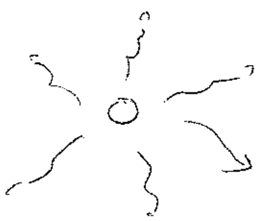
Remember from our discussion of the two-level atom (e.g. p. 33) that the closer is the laser to be resonant the larger is the probability that the photon is absorbed. Hence the counter-propagating photon is absorbed with a larger probability than the other photon.

· Remember that an excited-state atom can decay in one of two ways. Either via stimulated emission (remember the Rabi oscillations of population (p. 34)) or via spontaneous emission (p. 91). An absorption followed by an ~~spont~~ stimulated emission produces no net effect.

· However, if an absorption is followed by an spontaneous emission then some net effect occurs. Let's see this.



The spontaneously-emitted photon is emitted in a random direction (remember our discussion on fluorescence (p. 100))



The emission of the photon is accompanied by the corresponding recoil (similar to a canyon firing a canonball).

* So in a cycle of absorption plus spontaneous emission the atom absorbs a photon with momentum $-\vec{k}_L$ and emits a randomly-oriented photon getting a recoil $\vec{k}_{RECOIL}$.

So the net effect of the cycle is

$$\Delta \vec{k} = -\vec{k}_L + \vec{k}_{RECOIL}$$

- After the spontaneous emission the atom is back in the ground state, and hence the cycle starts again.

- Hence we have a combination of
  - Selective absorption in the direction against the motion
  - Randomly-oriented emission accompanied by a corresponding random recoil.

- Note that this means that if $v_x > 0$, then the atom experiences a net force in the $-x$ direction, and if $v_x < 0$ a net force in the $+x$ direction.

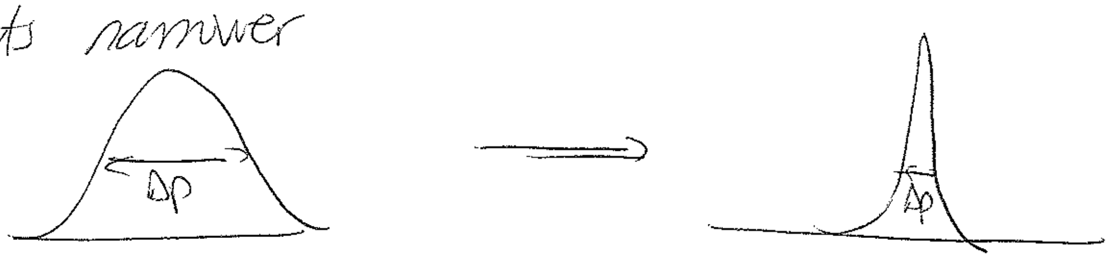
In fact, an expansion around $\vec{v} = 0$ leads to a simple friction equation for the atomic velocity

$$\frac{d\vec{v}}{dt} = -\vec{v}/\tau_D$$

where the damping time is $\tau_D \sim \frac{m}{\hbar k_L^2}$ (typically $\sim$ tens of ms).

The counterpropagating lasers act as a viscous medium (optical molasses). By using three intersecting orthogonal pairs of counterpropagating beams we can get a 3D friction force.

· As a consequence of the net friction the momentum distribution gets narrower



and from our previous discussion this means that the distribution has been cooled down.

· In addition to the deterministic viscous equation, there's an stochastic contribution due to the momentum fluctuations produced by the recoil of the spontaneously-emitted photons. This introduces a heating mechanism which competes to the Doppler cooling leading to a minimal temperature achievable with this technique $T_D \simeq \hbar\gamma/2k_B$ (Doppler temperature) where γ is the line width.
(Note: for alkali atoms $T_D \sim 100\,\mu K$)

· <u>Sisyphus cooling</u>

· As pointed above Doppler cooling works for a ~~negative~~ red detuning ~~[illegible]~~). However this is only true for sufficiently weak lasers. If the laser standing wave (created by the two counterpropagating beams) is strong the situation changes, and cooling is achieved for blue-detuned lasers. Let's see this.

* Let's recall the idea of dressed states (p. 42), i.e. the eigenstates of the atom plus laser.

Remember that the states $\{|n+1, g\rangle, |n, e\rangle\}$ form a closed set of states, and that the eigenstates are the dressed states

$$|\phi_1\rangle = \cos\phi\, |n+1, g\rangle + \sin\phi\, |n, e\rangle$$
$$|\phi_2\rangle = -\sin\phi\, |n+1, g\rangle + \cos\phi\, |n, e\rangle$$

with

$$\omega_{1,2} = \omega(n+\tfrac{1}{2}) \pm \sqrt{g^2(n+1) + \frac{\Delta^2}{4}} \overset{n \gg 1}{\simeq} \omega n \pm \underbrace{\sqrt{\Omega^2 + \Delta^2/4}}_{\Omega_G \,\equiv\, \text{generalized Rabi freq.}}$$

with $\tan\phi = \dfrac{\Delta/2 + \sqrt{\Omega^2 + \Delta^2/2}}{\Omega}$

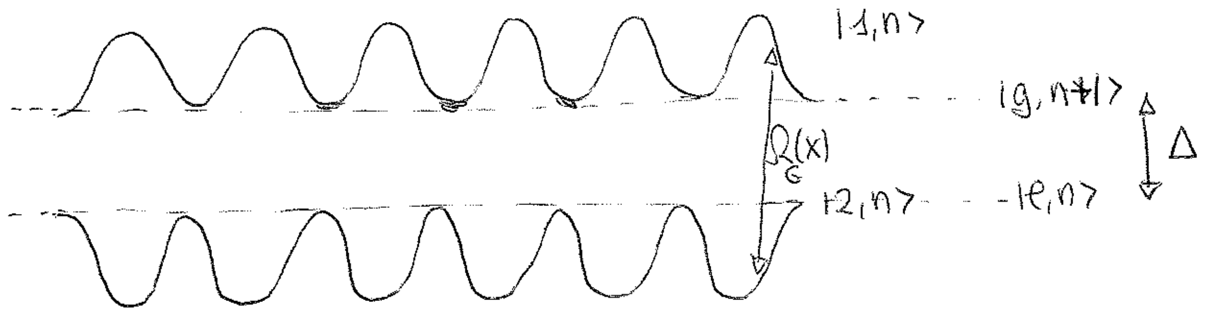
~~where~~ where $\Delta = \omega_T - \omega_L$

For blue detuning $\rightarrow \Delta < 0$ and hence for $\Omega \rightarrow 0$

$$|\phi_1\rangle \longrightarrow |n, e\rangle$$
$$|\phi_2\rangle \longrightarrow |n+1, g\rangle$$

* In a laser standing wave $\Omega_G = \Omega_G(x) \leftarrow$ sinusoidal function.

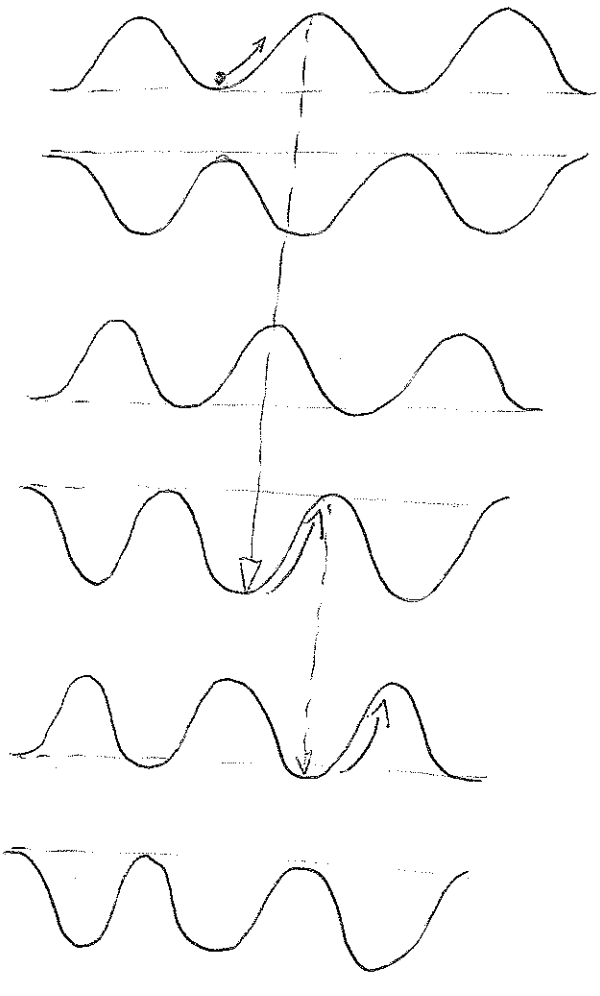
Hence the dressed-state energies also perform an oscillatory behaviour:



At a node of the standing wave the dressed state $|1,n\rangle$ coincides with $|n+1, g\rangle$ and $|2,n\rangle$ with $|n, e\rangle$. Out of a node the dressed states are linear combinations of ground and excited state, and hence both dressed states may decay spontaneously (remember our discussion of the resonance fluorescence in p. 112)

· The key point is that the spontaneous emission rates for the different dressed states are spatially dependent. For the state $|1,n\rangle$ it is minimal at the nodes (since there $|1,n\rangle \cong |g,n\rangle$) and maximal at the antinodes, whereas for $|2,n\rangle$ it's the opposite.

· Let's suppose an atom initially in a node in the level $|1,n\rangle$, and moving to the right with some velocity



· Starting from the valley the atom climbs the potential hill until it approaches the top (antinode) where the spontaneous emission is most probable.

· If it jumps from $|1,n\rangle \to |1,n-1\rangle$ then it jumps from hill to hill and nothing happens for the atomic motion.

· But if it jumps from $|1,n\rangle \to |2,n-1\rangle$ it jumps from hill to valley. Then the atom must climb again the hill when moving to the right

· Once more when reaching the top of the hill (now at a node) the probability of spontaneous emission is maximal. $|2,n-1\rangle \to |2,n-2\rangle$ does nothing to the atomic motion, but $|2,n-1\rangle \to |1,n-2\rangle$ jumps again from hill to valley, and again the atom must climb again. And so on.

* As a consequence, the atomic sample gets progressively colder. This cooling mechanism is called Sisyphus cooling (in the greek mithology Sisyphus rolls uphill a ball for ever).

. Note that for weak laser beams, the dressed states are not resolved, and we recover our picture of the Doppler cooling (remember our discussion of p. 113 when talking about Mollow's triplet).

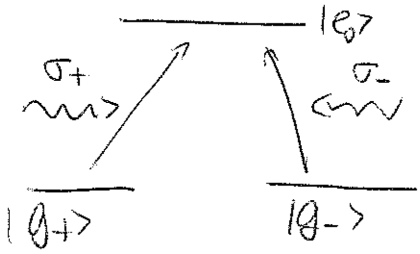
. This an related techniques may beat the Doppler temperature pushing the cooling towards the so-called recoil limit. The recoil temperature $K_B T_{rec} = \frac{\hbar^2}{2M} K_L^2$ is a limit due to the fact that the spontaneous emission is present, and necessarily leads to the recoil of the atom.

. <u>VELOCITY-SELECTIVE COHERENT POPULATION TRAPPING</u> (VCSPT)

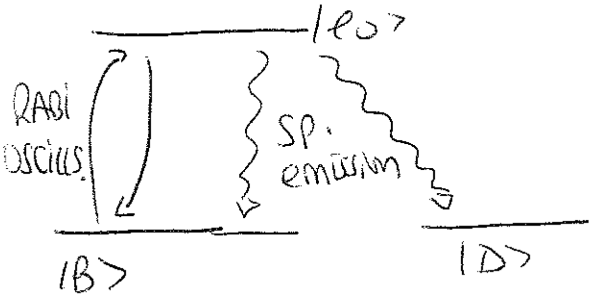
. There are however ways to cool down further below the recoil limit. This is possible by allowing the damping of the atomic motion via spontaneous emission, but "switching-off" the spontaneous emission at some point to avoid the recoil heating.

. This is achieved by means of the so-called dark states. Remember that in the exercise sheet 2 you have already had a look on the idea of dark state, i.e. a state which by some reason becomes uncoupled to the laser

- Let's consider a three-level lambda configuration in which two degenerate ground states $|g_\pm\rangle$ are coupled to an excited state $|e_0\rangle$ by two counterpropagating lasers of freq. ω_L and polarization $\sigma_\pm$.

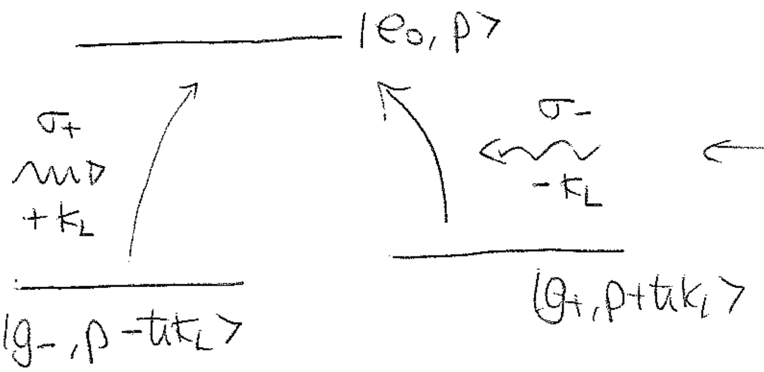


- Let's consider first the case of the atom at rest. In that case we can find a linear superposition of $|g_\pm\rangle$ which forms a dark state [if the 2 lasers have the same intensity $|D\rangle = \frac{1}{\sqrt{2}}(|g_+\rangle - |g_-\rangle)$].
Hence, in the new basis of dark $|D\rangle$ and bright state $|B\rangle$ (the bright state remains connected by the lasers to $|e_0\rangle$):



Note that $|D\rangle$ isn't connected by the laser to $|e_0\rangle$, but the spontaneous emission may introduce a decay from $|e_0\rangle$ into $|D\rangle$. When this occurs the atom can't go out any more from the state $|D\rangle$ (this is what is called a population trapping).

- Let's introduce now the atomic motion. Now we have to take into account the center-of-mass motion.



One can easily see that this family of three states remains closed with respect to absorption and stimulated emission.

* Note that for $p=0$ both ground states of the closed family have the same energy. However for $p \neq 0$ the ground states don't possess the same kinetic energy. Therefore any linear combination of them (in particular the dark state) can't be stationary. Actually there will be a transference of population between $|D\rangle$ and $|B\rangle$ and hence no true population trapping occurs. This is only exactly true for $p=0$.

· The spontaneous emission provides the way to jump (randomly) from one family of closed states to any other.
In this way the atoms perform a random path in momentum space until reaching the family with $p=0$, where they remain "trapped".
The larger the time of interaction between atoms and lasers the larger the population trapped at zero momentum ($p=0$).
Note that in principle the recoil isn't a limit now any more, since for $p \to 0$ the sp. emission is suppressed due to the dark state.

In this way one can get $T < T_{recoil}$.

· There are other laser-cooling techniques, but we have no time to review them all in detail. We have seen with these 3 techniques, how concepts that we already know may be employed to cool atoms (detuning + Doppler shift, dressed states + resonance fluorescence, dark states). These techniques are crucial for the study of cold atomic gases (Nobel award, 1997).

- Radiation pressure force versus dipole force

· In the following we will see that the interaction of light with atoms may lead to two fundamentally different types of forces, the radiation pressure and the dipole force.

· In order to see this, let's recall that the force is defined as

$$F = \langle F \rangle = \frac{d}{dt}\langle p \rangle \underset{\substack{\uparrow \\ \text{Heisenberg} \\ \text{eq.}}}{=} +\frac{i}{\hbar}\langle [H, p] \rangle \underset{\substack{\uparrow \\ p = -i\hbar\partial_z}}{=} -\frac{\partial}{\partial z}\langle H \rangle$$

$$= -\mathrm{Tr}\left\{ \rho \frac{\partial H}{\partial z} \right\}$$

Remember (p. 37) that the Hamiltonian (for intense lasers) may be written as $\hat{H} = \hat{H}_A + \hat{H}_{INT}$. $\hat{H}_A$ is the atomic Hamiltonian and it's space independent. $\hat{H}_{INT}$ describes the laser-light interaction:

$$\hat{H}_{INT} = \frac{1}{2}\hbar\Omega(z)\,|e\rangle\langle g| + \frac{\hbar}{2}\Omega^*(z)\,|g\rangle\langle e|$$

where $\Omega(z) = \dfrac{E(z)(e\vec{d})}{\hbar}$ is the Rabi frequency.

Since the electric field may be z-dependent (space dependent in general), then $\hat{H}_{INT} = \hat{H}_{INT}(z)$.

Hence

$$F = \frac{\hbar}{2}\frac{\partial \Omega}{\partial z}\rho_{eg}^* + \frac{\hbar}{2}\frac{\partial \Omega^*}{\partial z}\rho_{eg} \qquad \text{where } \rho_{eg} = \langle e|\rho|g\rangle = \rho_{ge}^*$$

Let $q_r + i q_i = \dfrac{1}{\Omega}\dfrac{\partial \Omega}{\partial z}$ (logarithmic derivative)

Hence $F = \dfrac{\hbar}{2} q_r \left[\Omega \rho_{eg}^* + \Omega^* \rho_{eg}\right] + i\dfrac{\hbar}{2} q_i \left(\Omega \rho_{eg}^* - \Omega^* \rho_{eg}\right]$

* Note that for a travelling wave

$$E(z) = \frac{E_0}{2}\left(e^{ikz}e^{-i\omega t} + c.c\right) \longrightarrow \begin{array}{l} q_r = 0 \\ q_i = k \end{array}$$

(Note: only the negative freq. part survives the RWA)

whereas for a stationary wave

$$E(z) = E_0 \cos kz \cos\omega t \longrightarrow \begin{array}{l} q_r = -k\tan qz \\ q_i = 0 \end{array}$$

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* Let's calculate ρ_{eg}, and in this way F:
* Remember the Master equation for an atom in the presence of a radiation and spontaneous emission (fluorescence problem) (p. 101) (but now we don't assume resonance). In the rotating frame with ω_L:

$$\dot{\hat\rho} = i\frac{\Delta}{2}\left(\hat\sigma_z\hat\rho - \hat\rho\hat\sigma_z\right) + i\frac{1}{2}\left\{(\Omega\hat\sigma_+ + \Omega^*\hat\sigma_-)\hat\rho - \hat\rho(\Omega\hat\sigma_+ + \Omega^*\hat\sigma_-)\right\}$$

$$+ \frac{\gamma}{2}\left(2\hat\sigma_-\hat\rho\hat\sigma_+ - \hat\sigma_+\hat\sigma_-\hat\rho - \hat\rho\hat\sigma_+\hat\sigma_-\right)$$

where $\Omega = dE/\hbar$ is the Rabi frequency and $\Delta = \omega_L - \omega_A$ is the detuning.

* Let's calculate $\dot\rho_{ee}$, $\dot\rho_{gg}$ and $\dot\rho_{eg}$, where $\rho_{ij} = \langle i|\hat\rho|j\rangle$, $i,j = e,g$

$$\frac{d\rho_{gg}}{dt} = \frac{d}{dt}\langle g|\hat\rho|g\rangle = \langle g|\frac{d\hat\rho}{dt}|g\rangle$$

$$= i\frac{1}{2}\left[\Omega^*\rho_{eg} - \Omega\rho_{ge}\right] + \frac{\gamma}{2}\left[+2\rho_{ee}\right] = i\frac{1}{2}(\Omega^*\rho_{eg} - \Omega\rho_{eg}^*) + \gamma\rho_{ee}$$

$$\frac{d\rho_{ee}}{dt} = -i\frac{1}{2}(\Omega^*\rho_{eg} - \Omega\rho_{eg}^*) - \gamma\rho_{ee}$$

$$\frac{d\rho_{eg}}{dt} = -\left(\frac{\gamma}{2} - i\Delta\right)\rho_{eg} + i\frac{\Omega}{2}(\rho_{gg} - \rho_{ee})$$

• Let $W = \rho_{gg} - \rho_{ee}$ (so-called population inversion)

then: $$\frac{d\rho_{eg}}{dt} = -\left(\frac{\gamma}{2} - i\Delta\right)\rho_{eg} + \frac{i\Omega}{2}W$$

$$\frac{dW}{dt} = i\left(\Omega^*\rho_{eg} - \Omega\rho_{eg}^*\right) + \underbrace{2\gamma\rho_{ee}}$$

$\longrightarrow \left.\begin{matrix}\rho_{gg}+\rho_{ee}=1\\ \rho_{gg}-\rho_{ee}=W\end{matrix}\right\} 2\rho_{ee} = 1-W$

$$= i\left[\Omega^*\rho_{eg} - \Omega\rho_{eg}^*\right] + \gamma - \gamma W$$

* Let's look for the stationary solution

$$\frac{d\rho_{eg}}{dt} = \frac{dW}{dt} = 0$$

$$\Rightarrow \left(\frac{\gamma}{2} - i\Delta\right)\rho_{eg} = \frac{i\Omega}{2}W$$

$$\Rightarrow i\left(\Omega^*\rho_{eg} - \Omega\rho_{eg}^*\right) + \gamma - \gamma W = 0$$

$$\rho_{eg} = \frac{i\frac{\Omega}{2}}{\left(\frac{\gamma}{2} - i\Delta\right)}W = \frac{i\frac{\Omega}{2}\left(\gamma/2 + i\Delta\right)}{\left(\frac{\gamma}{2}\right)^2 + \Delta^2}W$$

$$\Omega^*\rho_{eg} = \frac{i\frac{|\Omega|^2}{2}\left(\frac{\gamma}{2} + i\Delta\right)}{\left(\frac{\gamma}{2}\right)^2 + \Delta^2}W$$

$$\Omega^*\rho_{eg} - \Omega\rho_{eg}^* = 2i\,\mathrm{Im}(\Omega^*\rho_{eg}) = \frac{2i\frac{|\Omega|^2}{4}\gamma W}{\left(\frac{\gamma}{2}\right)^2 + \Delta^2} = \frac{i\frac{|\Omega|^2}{2}\gamma}{\left(\frac{\gamma}{2}\right)^2 + \Delta^2}W$$

Then

$$\frac{-|\Omega|^2/2}{\left(\frac{\gamma}{2}\right)^2 + \Delta^2}W + 1 - W = 0 \longrightarrow W = \frac{1}{1 + \frac{|\Omega|^2/2}{\left(\frac{\gamma}{2}\right)^2 + \Delta^2}} = \frac{1}{1+S}$$

where $S \equiv \dfrac{|\Omega|^2/2}{\left(\frac{\gamma}{2}\right)^2+\Delta^2} \equiv$ saturation parameter

Then $\rho_{eg} = \dfrac{i\frac{\Omega}{2}}{\left(\frac{\gamma}{2}-i\Delta\right)} \dfrac{1}{1+S}$

Note also that $\rho_{ee} = \dfrac{1-w}{2} = \dfrac{S}{2(1+S)}$

Hence

$$F = \frac{\hbar q_r}{2}\left\{\Omega \frac{\left(-i\frac{\Omega^*}{2}\right)}{\left(\frac{\gamma}{2}+i\Delta\right)} \frac{1}{1+S} + \Omega^* \frac{\left(i\frac{\Omega}{2}\right)}{\left(\frac{\gamma}{2}-i\Delta\right)} \frac{1}{1+S}\right\}$$

$$+ \frac{i\hbar q_i}{2}\left\{\Omega \frac{\left(-\frac{i\Omega^*}{2}\right)}{\left(\frac{\gamma}{2}+i\Delta\right)} \frac{1}{1+S} - \Omega^* \frac{\left(\frac{i\Omega}{2}\right)}{\left(\frac{\gamma}{2}-i\Delta\right)} \frac{1}{1+S}\right\}$$

$$= \frac{\hbar|\Omega|^2}{4(1+S)} q_r \left[\frac{-i\left(\cancel{\frac{\gamma}{2}}-i\Delta\right)+i\left(\cancel{\frac{\gamma}{2}}+i\Delta\right)}{\left(\frac{\gamma}{2}\right)^2+\Delta^2}\right] + \frac{i\hbar q_i|\Omega|^2}{4(1+S)}\left[\frac{-i\left(\frac{\gamma}{2}\cancel{-i\Delta}\right)-i\left(\frac{\gamma}{2}\cancel{+i\Delta}\right)}{\left(\frac{\gamma}{2}\right)^2+\Delta^2}\right]$$

$$= +\frac{\hbar S}{2(1+S)}\left[-2\Delta q_r + \gamma q_i\right] = -2\hbar q_r \Delta \rho_{ee} + \hbar q_i \gamma \rho_{ee}$$

· Radiation pressure force

· The absorption of light leads to the transfer of the photon momentum to the atoms. If the atom decays by spontaneous emission, the associated recoil gives a kick in a random direction (so its averaged net effect is zero). Hence the force from absorption followed by spontaneous emission is

$$F_{sp} = \underbrace{\hbar k}_{\text{momentum transfer per absorbed photon}} \; \underbrace{\gamma \, \rho_{ee}}_{\text{rate of the process times probability of being in the excited state}}$$

(remember our previous discussion of the Doppler cooling)

Remember that ρ_{ee} saturates to $1/2$ for $\gamma \gg \Omega$ (p. 104), and hence the force saturates at a maximum value $\hbar k \gamma/2$

· Note that F_{sp} clearly corresponds to the second term of the force we found before. This term is called radiation pressure force. It clearly vanishes for an atom at rest in an standing wave, since there $q_i = 0$. This can be easily understood, because atoms can absorb light from either the two counterpropagating beams that make up the standing wave, and the average momentum transfer there vanishes

· The radiation pressure force is clearly dissipative because the reverse of spontaneous emission is not possible, and hence the action of the force can't be reversed.

- Dipole force

* Let's return to the Hamiltonian for the 2-level atom interacting with the light field. Moving into the rotating frame (with ω_L) the Hamiltonian is simply:

$$\hat{H} = \hbar\Delta |e\rangle\langle e| + \frac{\hbar\Omega}{2} |e\rangle\langle g| + \frac{\hbar\Omega^*}{2} |g\rangle\langle e|$$

* Let's consider the case $|\Omega| \ll |\Delta|$, then we can calculate the energies of $|g\rangle$ and $|e\rangle$ up to second order in perturbation theory to get

$$\Delta E_g = \frac{|\langle g|H|e\rangle|^2}{E_g - E_e} = \frac{\hbar |\Omega|^2/\cancel{2}4}{\Delta} = \hbar \frac{|\Omega|^2}{4\Delta}$$

whereas $\Delta E_e = -\hbar \frac{|\Omega|^2}{4\Delta}$.

* The ground state and the excited state suffer a shift due to the applied light, which depends on the intensity of the laser employed ($\propto |\Omega|^2$) and on the inverse of the detuning Δ.

If $\Omega = \Omega(z)$ is spatially modulated, then $\Delta E_{g,e}$ are also z dependent.

We can then calculate the force on ground-state atoms for $|\Omega| \ll |\Delta|$ (low-intensity light). Let Ω real (like in a standing wave)

$$F_{dip} = -\frac{\partial}{\partial z}(\Delta E_g) = -\frac{\hbar\Omega}{2\Delta}\frac{\partial\Omega}{\partial z} = -\frac{\hbar}{2\Delta} q_r \Omega^2$$

When $|\Omega| \ll |\Delta| \longrightarrow S \ll 1 \longrightarrow$ and $\frac{\Omega^2}{2\Delta^2} \simeq S \simeq 2\rho_{ee}$
and $\gamma \simeq 0$

Then $F_{dip} \simeq -2\hbar q_r \Delta \rho_{ee}$

i.e. the 1st term in p. 175

- Note that for

  $\Delta < 0$ (red detuning): the force drives the atoms towards the regions of larger intensities

  $\Delta > 0$ (blue detuning): the force drives the atoms towards the intensity minima.

This force is called the dipolar force. Contrary to the radiation-pressure this force is conservative.

- So, due to the radiation the atoms feel two different types of forces:
  - Radiation-pressure force → dissipative → can be used to cool (see the discussion on laser cooling)
  - Dipole force → conservative → can't be used to cool but can be used to trap as we will see in a moment.

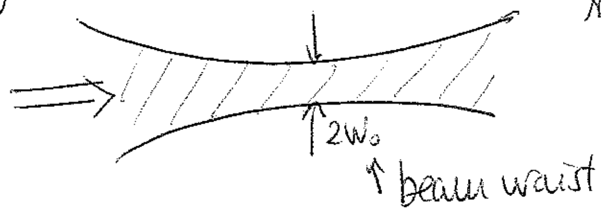
• TRAPPING OF NEUTRAL ATOMS

• Of course the experiments on cold atomic gases demand the confinement of the atoms. However, since typically one must deal with neutral particles, this has to be done in a slightly subtle way. Let's see briefly how we can use different mechanisms that we already know to trap neutral atoms.

• DIPOLE TRAPS

* Let's see how can we use the dipole force we just discussed to trap atoms.

The simplest imaginable trap consists of a single, strongly focused gaussian laser beam, whose intensity at the focus varies transversaly with r as

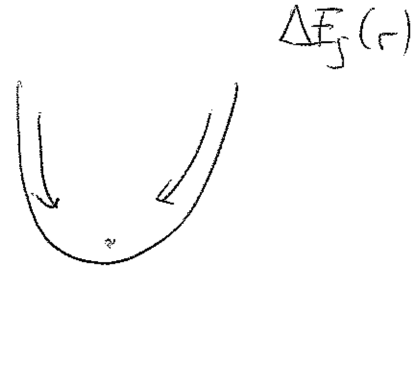


$$I(r) = I_0 \, e^{-r^2/w_0^2}$$

The laser light is tuned below resonance ($\Delta<0$).

The light shift for ground-state atoms is

$$\Delta E_g = \frac{-\hbar}{4\Delta}|\Omega(r)|^2 \propto -\frac{1}{\Delta} I(r)$$



Then, the associated dipole force is:

$$F_{dip} \propto \frac{1}{\Delta}\,\frac{r}{w_0^2}\, e^{-r^2/w_0^2}$$

Since $\Delta<0 \longrightarrow F_{dip} \propto -\text{CONSTANT} \cdot r\, e^{-r^2/w_0^2}$

Clearly the atoms get trapped by the laser

· Actually if $r \ll w_0 \rightarrow F_{dip} \propto -r$ which is like the restoring force of an harmonic oscillator (remember Hooke's law of a spring).

· Actually the beam profile longitudinally is also such that there's also an attractive force in the longitudinal direction, and hence this simple set-up allows for 3D trapping.

* MAGNETO-OPTICAL TRAPS (MOT)

The MOT is the most widely used trap for neutral atoms. It employs both optical fields (the radiation pressure actually) and inhomogeneous magnetic fields to trap atoms.

· Let's see how it works.

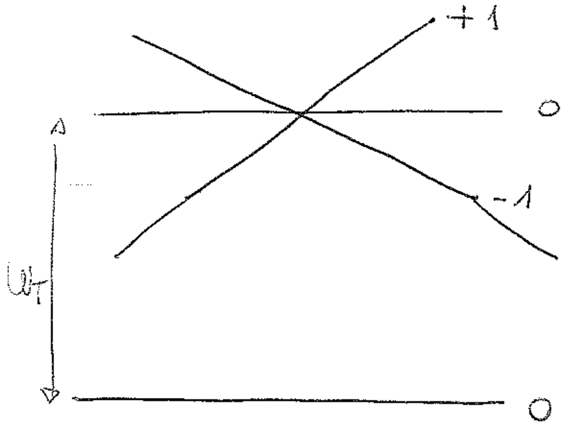
We consider again two counterpropagating lasers, but this time we assume that they have opposite circular polarizations

(I recall you that $\sigma_\pm$ produces a jump of the magnetic quantum number $\Delta m = \pm 1$)

· We will assume that the atom has a ground-state level with spin $J=0$, and an excited state with $J=1$. Hence the upper state has 3 Zeeman sublevels $m = \pm 1, 0$.

* We now assume a spatially inhomogeneous magnetic field $B = \alpha x$

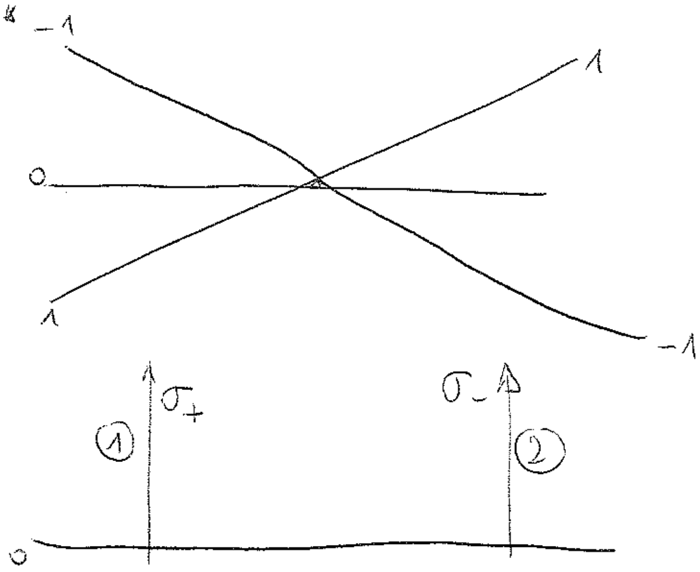
* Due to the inhomogeneous magnetic field the ~~upper~~ Zeeman sublevels present an inhomogeneous Zeeman shift $\Delta E_Z \propto m B \longrightarrow m X$



* We assume both lasers $\omega_L < \omega_T$ (so red detuning.)
* Remember that according to selection rules

$\sigma_+ : m = 0 \longrightarrow m = +1$

$\sigma_- : m = 0 \longrightarrow m = -1$



It's clear that the laser ① linking $m = 0 \rightarrow +1$ is closer to resonance at the left, whereas the laser ② linking $m = 0 \rightarrow -1$ is closer to resonance at the right.

* Hence, when the atoms are at the right they absorb preferentially photons ② which move to the left. Hence an atom at the right gets a kick to the left. A subsequent spontaneous emission will give a random direction. As for the Doppler cooling we get a net force (in the joint absorption + sp. emission cycle) to the right. This is nothing else as the radiation-pressure force we just saw.

. On the contrary, if the atom is at the left it absorbs preferentially ①, and hence the radiation-pressure pushes the atom to the right.

. Hence the radiation-pressure force pushes always towards the center, i.e. it traps.

. Note that the situation is analogous to the velocity damping in an optical molasses, but here the effect operates in position space. Note also that since the light is red detuned in a MOT we get both compression and cooling.

. By using 6 instead of 2 beams one can reach 3D trapping.

. MAGNETIC TRAPS

. There are also purely magnetic traps, which employ the Zeeman shift introduced by inhomogeneous magnetic fields to generate trapping potentials

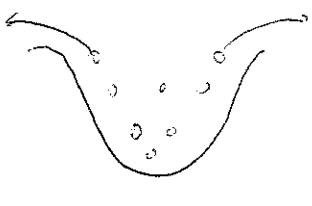
. We will not insist in these traps in detail here.

EVAPORATIVE COOLING

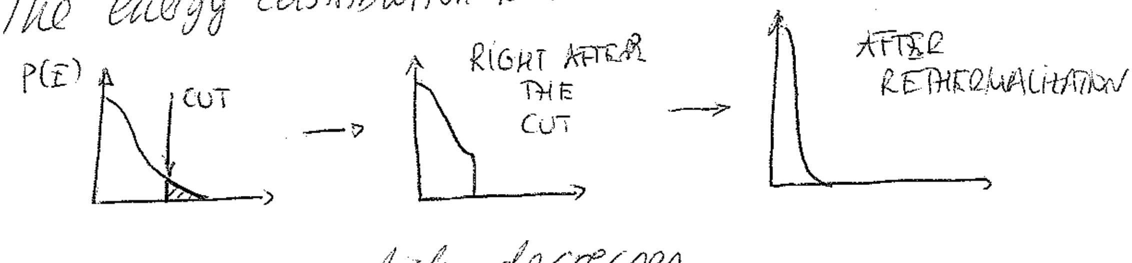
- Laser cooling can be pushed to very low temperatures, as we have seen even below the recoil temperature. However in dense samples a given atom may re-absorb the photon spontaneously scattered by other atom. When this occurs the dark state is not dark for the spontaneously emitted photon and as a consequence of that heating is introduced. This serious problem is known as re-absorption problem, which limits the final temperature reachable by laser cooling.

- To reach the very low temperatures needed for experiments with ultracold gases, other type of cooling must be employed. This method is called evaporative cooling, and as we will see it's very different in nature.

- Contrary to laser cooling (which is basically a one-atom effect) evaporative cooling is based on collisions between atoms. Basically it consists in letting escape the most energetic atoms from the trap.



The energy distribution is the "cut in the wings"



It's clear that the energy per particle decreases. After a rethermalization of the rest of the atoms the sample reaches a lower temperature. This is actually like the cooling of a cup of coffee!

* We will not enter into the technical details here, but they are obviously important. In particular, rethermalization is produced by elastic collisions, so these collisional rates should be large enough (which requires sufficiently dense samples and large collisional cross sections). Also inelastic processes should be "kept under control" (because they produce heating and undesired losses).

· Evaporative cooling loses by definition particles, but those which remain may reach incredibly low temperatures, of the order of $T < 100\,nK$.

· MATTER WAVES: ATOM OPTICS

· As we already commented the width of the momentum distribution is linked to the temperature of the sample

$$\Delta p \sim \sqrt{2mk_BT}$$

· Remember that from the Heisenberg principle

$$\Delta x \, \Delta p \lessapprox \hbar/2$$

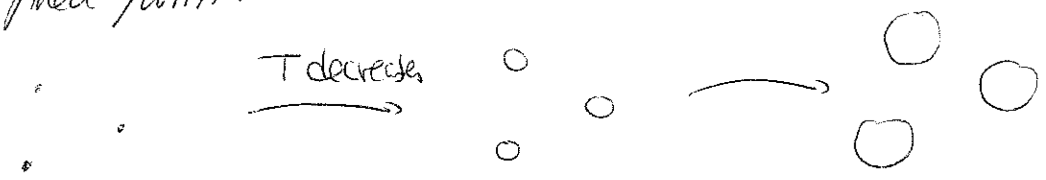
As a consequence of that if $\Delta p \sim \sqrt{T} \implies \Delta x \sim 1/\sqrt{T}$

· This means that the cooler the atoms the more spread they are, i.e. the broader is the corresponding wavepacket.

~~The ...~~ The width of the corresponding wavepacket is characterized by the so-called thermal de-Broglie wavelength:

$$\lambda_T = \left(\frac{2\pi\hbar^2}{mk_BT}\right)^{1/2}$$

· At room temperature λ_T is smaller than the Bohr radius, and hence the atoms can be understood as classical particles with defined position and momentum.



But when T decreases the wavepacket character becomes evident, the momentum distribution gets sharper, and the atom spatial delocalization gets larger.

· Just to know some numbers.

· A temperature of 100 nK (which one can achieve as mentioned previously by combining laser and evaporative cooling) is incredibly low. Just to compare: 100 nK is to 0°C, like 2mm to the Earth radius!

· You can imagine that the delocalization of the atoms may become extreme for these temperatures. λ_T may become larger than 1 μm. Again just to compare: 1 μm is to the Bohr radius, like 10 Km for a usual human being, i.e a person spread in a circle of radius of 10 Km!

· One can even reach situations in which λ_T becomes comparable to (or larger than) the separation between the atoms. When this occurs something remarkable occurs, as we will see in a moment.

· For the moment, we have to understand that $\lambda_T >>$ Bohr radius mean that the atoms behave as matter waves, so it isn't surprising that in some sense atoms start behaving like light beams!

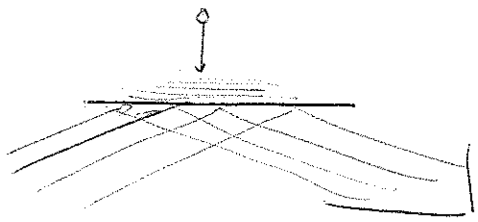
• This brings us to the very successful field of atom optics.

Let's see some examples of atom optics devices

• ATOMIC MIRRORS

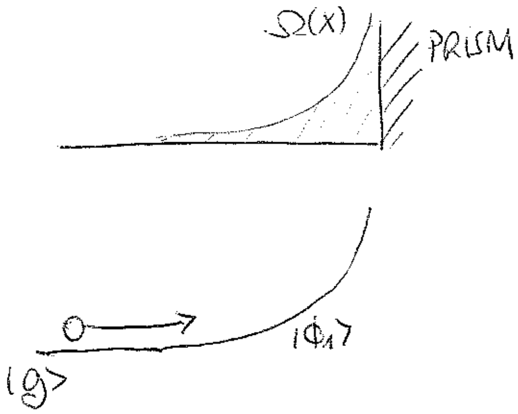
• One may employ the previously discussed dipole force to create a laser mirror for atoms. Let's briefly see how.

• Let's consider a laser beam which is totally reflected inside a prism. An evanescent wave is formed outside the prism.



• Let's assume that the laser is close to resonance to some atomic transition (so we can use a 2-level picture)

• If the laser is blue detuned then (remember our previous discussion about the dressed states) then a ground state atom will feel a repulsive potential when moving towards the prism.



As a consequence the atom is reflected (if it's sufficiently slow)

• There's other form of constructing an atomic mirror, namely the so-called magnetic mirrors. A sheet of spatially alternating currents produces a magnetic field whose magnitude decreases exponentially when going out from the surface. The corresponding Zeeman effect is hence inhomogeneous. In this way an atom with a magnetic moment antiparallel to the magnetic field experiences a repulsive potential. So again a mirror.

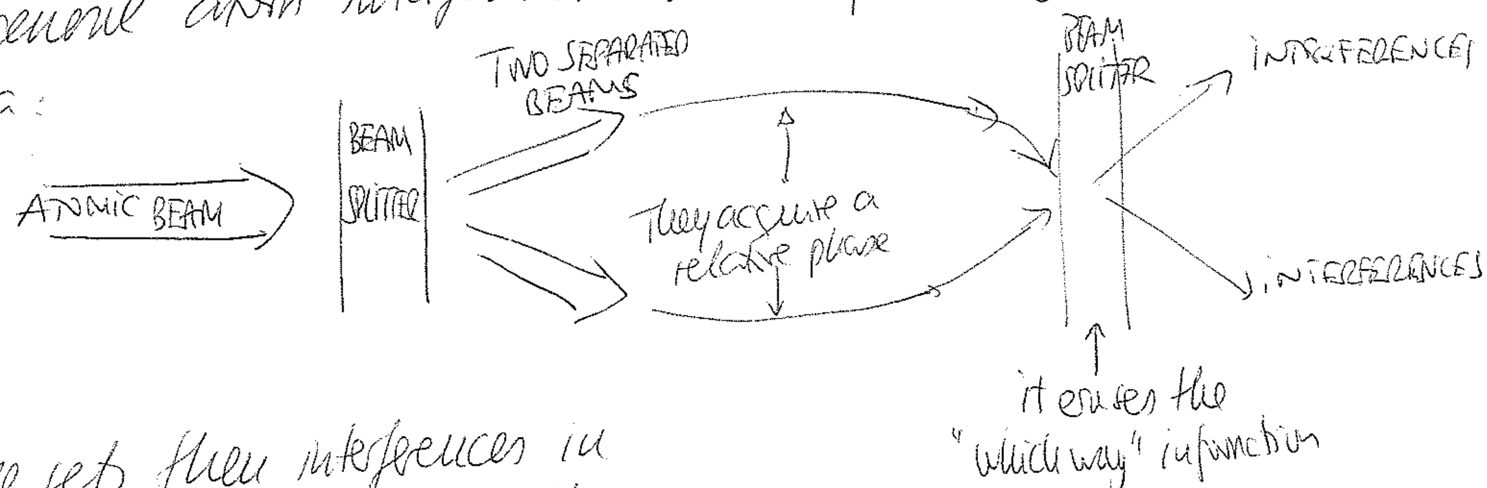
* Note that an atomic mirror demands more a slow atom than a cold one. In this sense it's similar to geometrical (Newtonian) optics, where the wave nature is not relevant (and hence the temperature plays a secondary role).

· We will see now an example of atom optics device where the "de Broglie physics" plays a crucial role:

ATOM INTERFEROMETERS

· Atom interferometers coherently split and recombine matter waves (typically by means of laser beams). This may be done in different ways as we will see in a second.

A general atom interferometer works following a very simple idea:

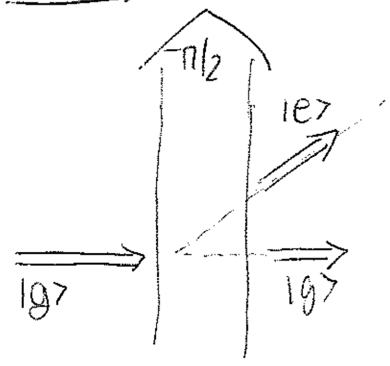


· One gets then interferences in the resulting matter waves!!

· This interferences contain a very precise information about the relative phase accumulated, and this information may be employed for many purposes as

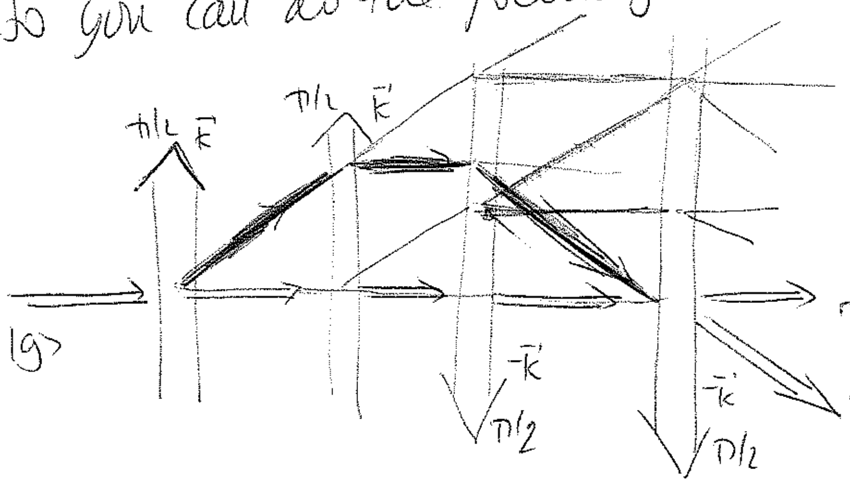
- Atom clocks (frequency standards)
- Precision measurements of e.g. the gravitational constant (G) or the fine-structure constant (α)
- gravimetry, and general relativity test, etc.

* let's see one possible scheme (quite used in experiments, e.g. at Hannover University). This scheme is known as Ramsey-Bordé interferometer. Let's consider a 2-level atom



As a beam splitter we employ a so-called π/2 pulse. Remember our discussion of p. (37) about the Area Law. A π/2 pulse is such that the area is such that starting with |g> one reaches a 50-50 population in |g> and |e>. The excited state absorbs the photon momentum and hence it's deflected.

So you can do the following:

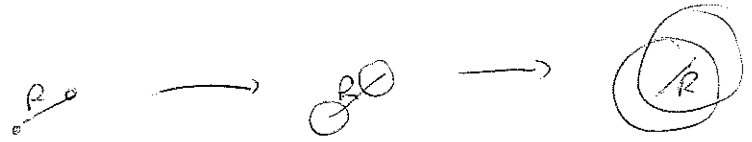


This is the particular Ramsey-Bordé scheme employed at Hannover University.

*QUANTUM DEGENERACY

* We have already seen that when the temperature decreases then $\lambda_T \sim 1/\sqrt{T}$ increases.

. At some critical temperature ~~the~~ the thermal de Broglie wavelength λ_T becomes comparable to the interparticle distance. When this occurs, the particles become undistinguishable and hence Quantum statistics begins to



play a crucial role.

. This is the regime of quantum degeneracy.

. If the particles are bosons they will obey Bose-Einstein statistics, whereas if they are fermions they obey Fermi-Dirac statistics (and hence the Pauli exclusion principle).

. In the following we will have a look to a very remarkable consequence of the Bose-Einstein statistics, namely the Bose-Einstein condensation. This has been an extraordinarily active field during the last years, and later on we will briefly have a look why.

* BOSE-EINSTEIN CONDENSATION

- First of all let's recall the idea of Bose-Einstein condensation.
- For simplicity of the discussion let's consider an ideal Bose gas (I'll later on discuss briefly the role of interactions between the particles of the gas).
- From quantum statistical mechanics we know that the average occupation of a level of energy ϵ_i is provided (we use the grand canonical ensemble) by

$$\bar{n}_i = \frac{1}{e^{\beta(\epsilon_i - \mu)} - 1} \qquad \text{where } \beta = \frac{1}{k_B T}, \text{ and } \mu \equiv \text{chemical potential}$$

The total number of particles is then

$$N = \sum_i \bar{n}_i$$

and the total energy is given by

$$E = \sum_i \epsilon_i \bar{n}_i$$

- Since the occupations $\bar{n}_i \geq 0$, then it's clear that $\mu < \epsilon_0$, where $\epsilon_0 \equiv$ lowest eigenenergy.

Clearly when $\mu \to \epsilon_0$ the occupation number

$$N_0 \equiv \bar{n}_0 = \frac{1}{e^{\beta(\epsilon_0 - \mu)} - 1}$$

becomes increasingly large. This is actually the point behind the Bose-Einstein condensation (BEC)

- let $N = N_0 + N_T$

where $N_T(T,\mu) = \sum_{i\neq 0} \bar{n}_i(T,\mu) \equiv$ Thermal (non-condensed) component

- It's clear that for a fixed value of T,

$N_T(T,\mu)$ reaches it maximum at $N_C(T) = N_T(T, \mu = \epsilon_0)$

(Note that N_C grows with T)

There's a critical temperature T_C, such that

$$N_C(T_C) = N$$

when $T<T_C$ something remarkable occurs, since for $T<T_C$

$N_C < N$. This means that the maximal number of atoms that
we can have in states other that ϵ_0 is smaller than the
total number of atoms N. As a consequence the rest of
the atoms goes (must go) into the groundstate.

If say $N_C = 1000$ and $N = 10^6$, this means that a
single quantum state will be populated by roughly 1 million
atoms. This remarkable phenomenon is called
Bose-Einstein condensation.

- It was discussed for the 1st time by Einstein in 1925,
but (although it played a crucial role in the understanding
of superfluidity and superconductivity) it wasn't observed
in a clean way until the mid 90's (Nobel Prize 2001)
when laser cooling and trapping techniques were finally developed.

* One can easily see that the idea of Bose–Einstein condensation is directly linked to the enormous broadening of the wavepackets. Let's see this briefly.

Let's consider bosons in a volume $V = L^3$

· Eigenenergies $\to$ $\epsilon = p^2/2m$, $p = \frac{2\pi}{L}\hbar \vec{n}$ $\quad \vec{n} = \{n_x, n_y, n_z\}$

$n_j = 0, \pm 1, \pm 2 \dots$

Clearly $\epsilon_0 = 0$, Hence

$$N_T = \sum_{p \neq 0} \frac{1}{e^{\beta\left(\frac{p^2}{2m} - \mu\right)} - 1} \xrightarrow{V \to \infty} \frac{V}{(2\pi\hbar)^3} \int d^3p \, \frac{1}{e^{\beta\left(\frac{p^2}{2m} - \mu\right)}} = \frac{V}{\lambda_T^3} g_{3/2}\left(e^{\beta\mu}\right)$$

$$\left(\underline{\text{note}}:\ g_p(z) = \sum_{\ell=1}^{\infty} \frac{z^\ell}{\ell^p}\right)$$

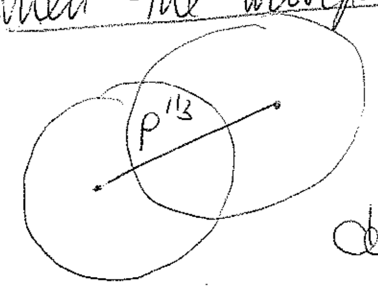
$$N_C(T) = N_T(T, \mu = \epsilon_0 = 0) = \frac{V}{\lambda_T^3} g_{3/2}(1) \simeq 2.612 \frac{V}{\lambda_T^3}$$

$$N_C(T_C) = N \to 2.612 \frac{V}{\lambda_T(T_C)^3} = N \longrightarrow \frac{N}{V} \lambda_T(T_C)^3 = 2.612$$

Hence, let $\rho = \frac{N}{V} \equiv$ density

$$\rho \lambda_T^3 = 2.612 \longrightarrow \lambda_T(T_C) = \frac{2.612^{1/3}}{\rho^{1/3}}$$

Since $\frac{1}{\rho^{1/3}} \equiv$ typical interparticle distance, this means that <u>BEC appears when the wavepackets significantly overlap.</u>



$$\boxed{\rho \lambda_T^3 = 2.612}$$

Now you can understand why we need so low T to get BEC. The typical densities are really very low (otherwise interactions will start to play a dominant role, and the BEC effect becomes "blurred").

Typically $\underline{T \lesssim 100\,nK}$ is needed!

* THE ROLE OF INTERACTIONS

(~ ngrs/liter !)

* As I said right now, the gases are typically extremely dilute. But this doesn't mean that we can treat in general those gases as ideal.

· Interactions between particles play a fundamental role.

· We have here no time to review scattering theory. We will just mention that at very low energies (and you can imagine that with $T \lesssim 100\,nK$ this is the case here!) Only the s-wave scattering is important

(note: remember that one decomposes the scattering problem between 2 particles in partial waves of different angular momentum ℓ. The s-wave means $\ell = 0$, which being the only one without centrifugal barrier contributes most to the scattering)

· The s-wave scattering is characterized by a single parameter, namely the s-wave scattering length a.

One can then safely substitute the real interaction potential by a δ pseudopotential

$$V(\bar{r}) \xrightarrow{\sim} \frac{4\pi\hbar^2}{m} a\, \delta(\bar{r}) \equiv g\, \delta(\bar{r})$$

which has the same s-wave scattering.

· The scattering length can be positive (repulsive gases) or negative (attractive gases). There are experimentally controllable ways of exploiting scattering resonances to change the value and sign of a.

THE GROSS-PITAEVSKII EQUATION. NON-LINEAR ATOM OPTICS

- The interactions may be introduced into the Hamiltonian which in second quantization reads

trapping potential

$$\hat{H} = \int d^3r \, \hat{\psi}^+(\vec{r}) \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{ext}(\vec{r}) \right] \hat{\psi}(\vec{r})$$

$$+ \frac{1}{2} \int d^3r \int d^3r' \, \hat{\psi}(\vec{r})^+ \hat{\psi}(\vec{r}')^+ V(\vec{r}-\vec{r}') \hat{\psi}(\vec{r}') \hat{\psi}(\vec{r})$$

$$\simeq \int d^3r \, \hat{\psi}^+(\vec{r}) \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{ext}(r) + g/2 \, \hat{\psi}^+(\vec{r}) \hat{\psi}(\vec{r}) \right) \hat{\psi}(\vec{r})$$

where $\hat{\psi}(\vec{r}) \equiv$ field operator for the annihilation of a particle in $\vec{r}$.

- Let's consider the Heisenberg equation of motion for $\hat{\psi}(\vec{r})$

$$i\hbar \frac{\partial}{\partial t} \hat{\psi}(\vec{r},t) = [\hat{\psi}(\vec{r},t), \hat{H}] =$$

$$= \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{ext}(\vec{r}) + g \, \hat{\psi}^+(\vec{r}) \hat{\psi}(\vec{r}) \right] \hat{\psi}(\vec{r})$$

- Assuming a macroscopic population of the condensate ~~ground state~~, we may approximate $\hat{\psi}(\vec{r}) \simeq \psi_0(\vec{r}) \equiv$ condensate wavefunction.

(Note: this is similar to what we did in p 37 when considering a field with a lot of photons!)

then we obtain the equation for the condensate wavefunction

$$\boxed{i\hbar \frac{\partial}{\partial t} \psi_0(\vec{r},t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{ext}(\vec{r}) + g |\psi_0(\vec{r},t)|^2 \right] \psi_0(\vec{r},t)}$$

GROSS-PITAEVSKII EQUATION

• This equation is, as you can see, a nonlinear Schrödinger equation (with cubic nonlinearity $\sim g\psi_0^3$).

Hence, the interactions introduce an inherently nonlinear physics in the condensate.

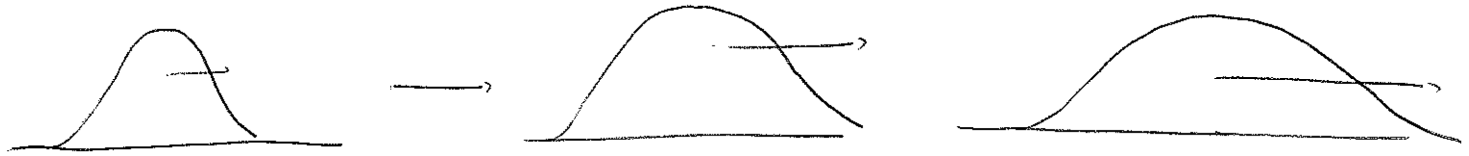
* This constitutes one of the reasons of why condensates are interesting to study.

* This sort of equation is found also in other nonlinear media, and in particular in nonlinear optics in so-called Kerr media (where the refraction index $n = n_0 + n_2 I$, where $I \equiv$ light intensity). Here the density $|\psi|^2$ plays the role of the intensity.

· Hence, similarly to nonlinear optics, one has its counterpart with matter-waves, namely the non-linear Atom optics.

* Many interesting phenomena have been studied in this context, in particular <u>solitons</u>.

Usually a wavepacket spreads when propagating freely



This quantum dispersion is due to the kinetic energy (and the momentum spreading of the wavepacket).

· However, if the scattering length is negative ($g<0$) let's see what may happen.

* When the wavepacket spreads, $|\psi|^2$ is reduced and hence $-|g||\psi|^2$ becomes less negative (the system gains energy). → The interactions hence tend to suppress the spreading.
* The kinetic energy tends to spread.

As a consequence there's a competition.

· If one confines the atoms very strongly in two directions, then in the other direction the gas is basically one-dimensional (this isn't so easy as that, but now we have no time to enter in this details).

Under 1D conditions, the competition between the two tendencies stabilizes the wavepacket, which doesn't spread. This is a <u>soliton</u> (note: more exactly a bright soliton).

· There are other examples of solitons and nonlinear phenomena in BEC, but we will skip them here.

* Other reasons why BEC is interesting

* In addition to the fact that the BEC physics is inherently non-linear, there are at least other 2 major reasons that justify the large interest on BEC in the last years

1) BEC is a very dilute interacting quantum fluid

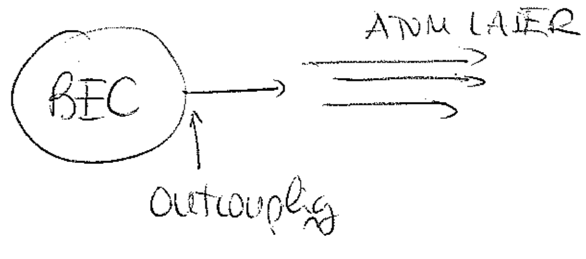
* Since the BEC obtained using cooling and trapping techniques is very dilute, one can then get a very good understanding of fundamental concepts of condensed-matter theory from first principles.

· One of these concepts is the idea of superfluidity, i.e. the flow without viscosity. Unfortunately we cannot enter into details here, but just mention at this point that superfluidity is a consequence of the interaction, and leads to remarkable phenomena as the appearance of quantized vortices.

2) BEC is a coherent matter wave

· The fact that in a condensate one has million(s) of atom living in the same wavefunction is very remarkable.

· One can exploit that to use the BEC as the source of a coherent beam of atoms ⟶ Atom laser



• OPTICAL LATTICES

* Remember our discussion on the dipole force (p. 177).
Remember that if we employ a laser well-detuned from an atomic transition, then the ground-state evolves adiabatically into a potential

$$V(\vec{r}) \propto \frac{I(\vec{r})}{\Delta}$$ where $I(\vec{r})$ is the laser intensity

* If now one employs a standing wave

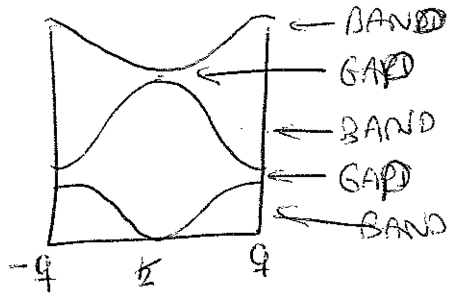
$$I(x) = I_0 \sin^2 qx$$

The atoms are hence feeling a periodic potential.
This has been called an <u>OPTICAL LATTICE</u>.

. This is quite remarkable because this resembles very much the (non-interacting) models of electrons in crystals.
Remember that in solid-state physics you can understand the physics of electrons in crystals as the propagation of an electron in the periodic potential produced by the ions.

· A crucial difference here is that an optical lattice is free from defects and phonons, contrary to (even very pure) crystal samples. This allows for very clean observation of solid-state phenomena using cold gases in optical lattices.

* Remember that a particle living in a periodic potential has a band spectrum of energies, which in the 1st Brillouin zone typically looks like this



· The particles just can live in the allowed bands

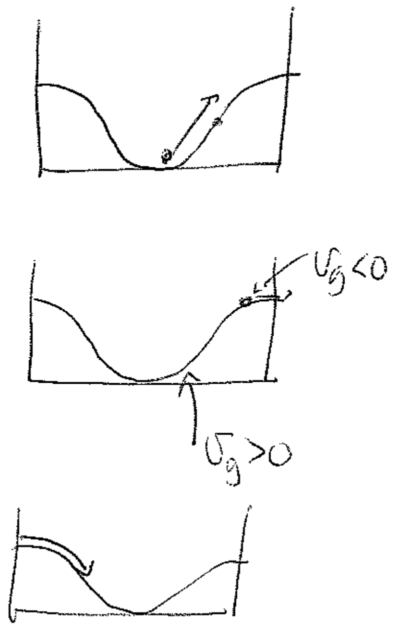
($k \equiv$ Quasimomentum)

* Typical solid-state phenomena have been observed with cold gases ~~......~~ in a very clean way. An example of it is Bloch oscillations. Let's see it very briefly

Let's consider a lattice $V_0 \sin^2 qz$ in the presence of gravity ($+mgz$):

$$V(z) = V_0 \sin^2 qz + mgz$$

Without the lattice the atoms would just accelerate in the gravitational field. In the presence of the lattice it's different



· The acceleration changes the quasimomentum. Suppose that we start at $k=0$.

· Remember that the (group) velocity in the lattice is given by $\partial E/\partial k \equiv v_g$

· As a consequence the atom first moves to the right then decelerates then "appears" at the other side of the Brillouin zone, and hence moves backwards, decelerates, moves forward again and so on.

· This phenomenon is known as Bloch oscillation. It has been observed both with cold (non-condensed) atoms as well as with condensates (recently here at Hannover university).

· As I commented already the dipole potential may be tailored by tailoring the laser intensity. Hence one can play with different types of lattices (also 3D lattices), one can add some defects artificially, one can induce a disordered lattice, etc. The possibilities of control are really remarkable.

· The resemblance between atoms in optical lattices and electrons in solid-state systems has attracted a huge interest recently, and constitutes a major research topic at the very front of physics nowadays.

· Phenomena as the Bloch oscillations are single-particle effects. If the interactions play an important role, then one enters into an exciting field, with resemblances to the theory of strongly-correlated electrons (e.g. high-Tc superconductivity). This is a very active field of physics currently, although this is well beyond the scope of our lecture.

· The properties of cold gases (e.g. their interaction properties), their temperature, the trapping potentials, etc. can be controlled to a very large extent. This has opened many research avenues, which here we just can mention

· One may have atoms in more than one internal state. The internal state play the role of a quasi-spin, and due to that these gases have been called SPINOR GASES. They have a very rich physics.

· When we discussed the interatomic interactions we said that we can reduce the interaction potential to a contact one of the form $g\delta(r)$. This is just true if the particles interact via short-range forces (van-der-Waals like). But if the particles have large (electric or magnetic) dipoles this isn't any more the case. This opens the new field of DIPOLAR GASES

· As already commented one may strongly confine in 2 directions to get an one-dimensional gas. The physics of 1D GASES is very remarkable, and constitutes also a major research topic.

· Although I discussed here only bosons in detail, one may also work with fermions. FERMI GASES in the regime of quantum

degeneracy shows of course the Pauli exclusion principle. At very low T, and if one has a two-component (↑,↓) Fermi gas with attractive ↑↔↓ interactions one may get a form of pairing (in momentum space) known as Cooper-pair. (which is the basic idea behind (usual) superconductivity).
The physics of Fermi gases is also a major research topic.

* There are many other topics, but I think that you can get a feeling already that cold gases is a fascinating topic, linking disparate field of physics, as quantum optics, solid-state physics, non-linear physics, and more.

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* With this we finish our lectures on theoretical Quantum Optics. I hope that with this course (necessarily too compact) you have got some impression of the richness and multi-disciplinarity of the field, which brought us from the "standard" quantum optics realms as (squeezing, bunching and antibunching, coherence and interference, spontaneous emission and fluorescence, ...) to other ~~less~~ "standard" topics as quantum information (q-computing, cryptography, teleportation) and the physics of cold gases (which brought us all the way towards solid-state and non-linear physics). We have also learned some important methodology (about states, stochastic methods, P,Q,W representations, etc) which are very useful for other fields of physics.