* LASER-ASSISTED HOPPING

• Lattice modulations may be employed to induce hopping under conditions in which natural hopping vanishes.

• To understand photon-assisted tunneling in a lattice, it is a good idea to begin with bosonic atoms in a biased (tilted) double well with time-dependent tunneling rate $J(t)$. The energy difference due to the tilting between the 2 sites is E, and the onsite interactions are U:

$$H = -J(t)\left[a_\ell^\dagger a_r + a_r^\dagger a_\ell\right] + \frac{E}{2}\left(a_\ell^\dagger a_\ell - a_r^\dagger a_r\right) + \frac{U}{2}\left(n_\ell(n_\ell-1) + n_r(n_r-1)\right)$$

We consider $E \gg |J(t)|$. In absence of modulation $J(t) = J$, an atom initially localized at one of the wells has basically a vanishing tunneling probability $P(t) \simeq 4\left(J/E\right)^2$.

• Let's consider now a modulation $J(t) = J + \delta J \cos \omega t$. The easiest way to induce such a modulation is by simply modulating the lattice depth.

We will perform a very similar Floquet analysis as that of p. (55).

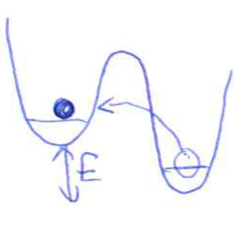
Let's consider the Floquet basis $|n_\ell, n_r; m\rangle \equiv e^{im\omega t}|n_\ell, n_r\rangle$

$$\langle\langle n_\ell', n_r'; m' | H | n_\ell, n_r, m\rangle\rangle = \frac{1}{T}\int_0^T e^{-i(m-m')\omega t}\langle n_\ell' n_r' | H_0 | n_\ell n_r\rangle$$

$$+ \frac{1}{T}\int_0^T e^{-i(m-m')\omega t}\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right)\delta J \langle n_\ell' n_r' | a_\ell^\dagger a_r + a_r^\dagger a_\ell | n_\ell n_r\rangle$$

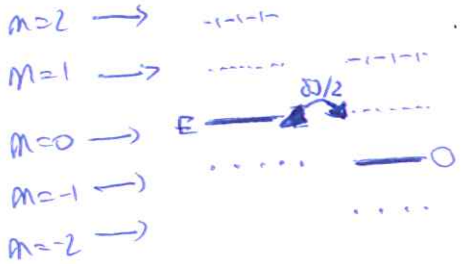
$$= \delta_{m',m}\langle n_\ell' n_r' | H_0 | n_\ell n_r\rangle + \underbrace{\frac{\delta J}{2}\left(\delta_{m',m+1} + \delta_{m',m-1}\right)\langle n_\ell' n_r' | a_\ell^\dagger a_r + a_r^\dagger a_\ell | n_\ell n_r\rangle}_{\text{There's a coupling between neighboring Floquet manifolds}}$$

* Let's consider just one particle:



This process is forbidden for $J(t) = J$

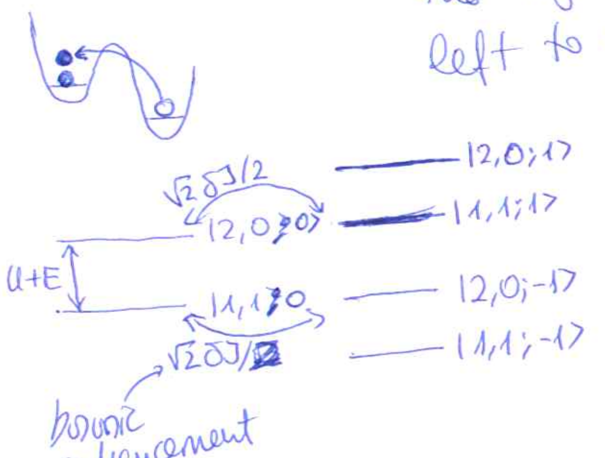
* But now we will have a coupling between Floquet sectors. This coupling is resonant for $\omega = E$:



. An effective hopping rate $\frac{\delta J}{2}$ is hence induced.

. the modulation provides a photon to the atom with the requisite energy to tunnel.

* If instead we begin with 2 strongly-interacting atoms ($U \gg |J(t)|$) one on each site, the energy cost to tunnel now becomes $U+E$ for the right-to-left tunneling and $U-E$ for the left to right tunneling



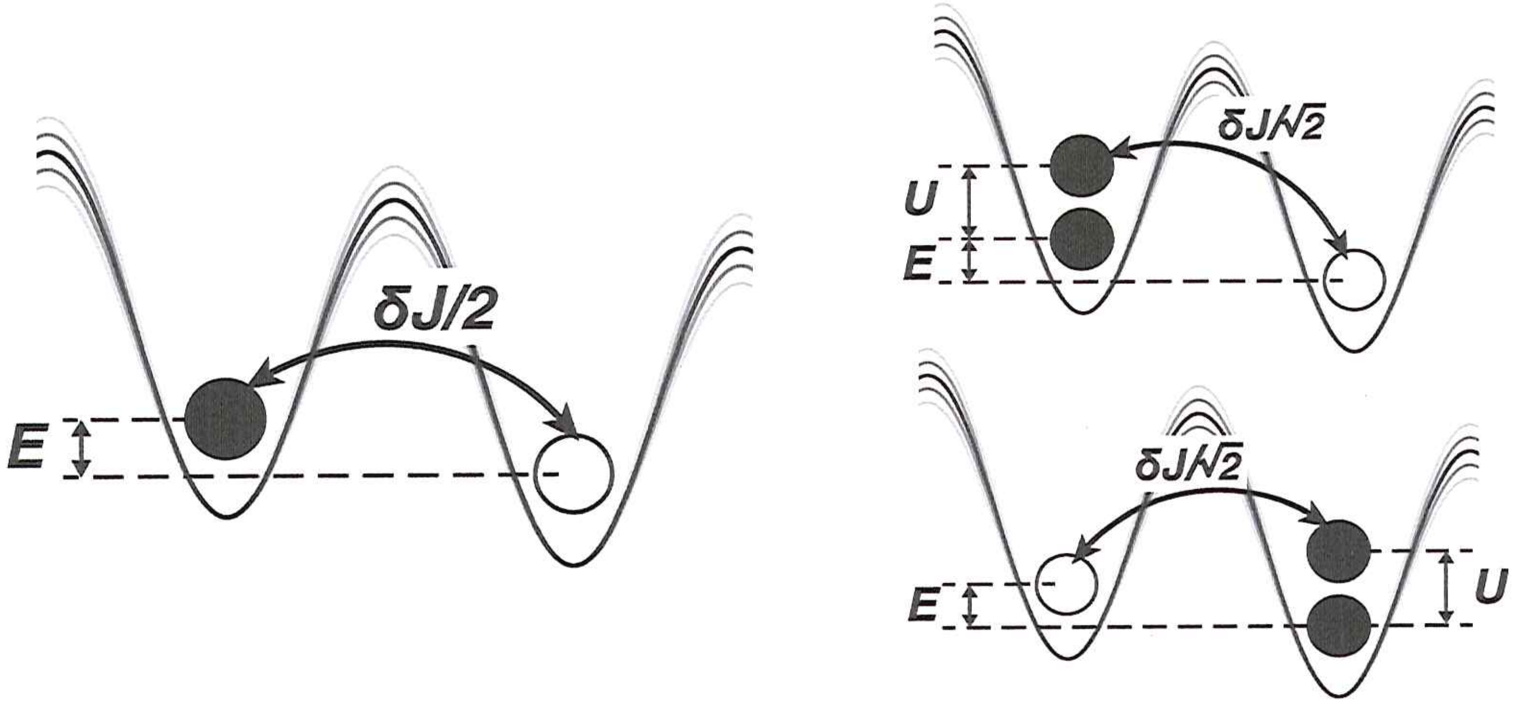
* The double-well picture can be directly extended to 1D lattices. Tunneling is strongly suppressed in a Mott insulator due to the repulsive interactions (U)

. For a tilting $E \neq U$ produces a tunneling barrier that depends on the tunneling direction. Modulating the lattice depth at a frequency equal to that barrier enables <u>resonant tunneling</u>.

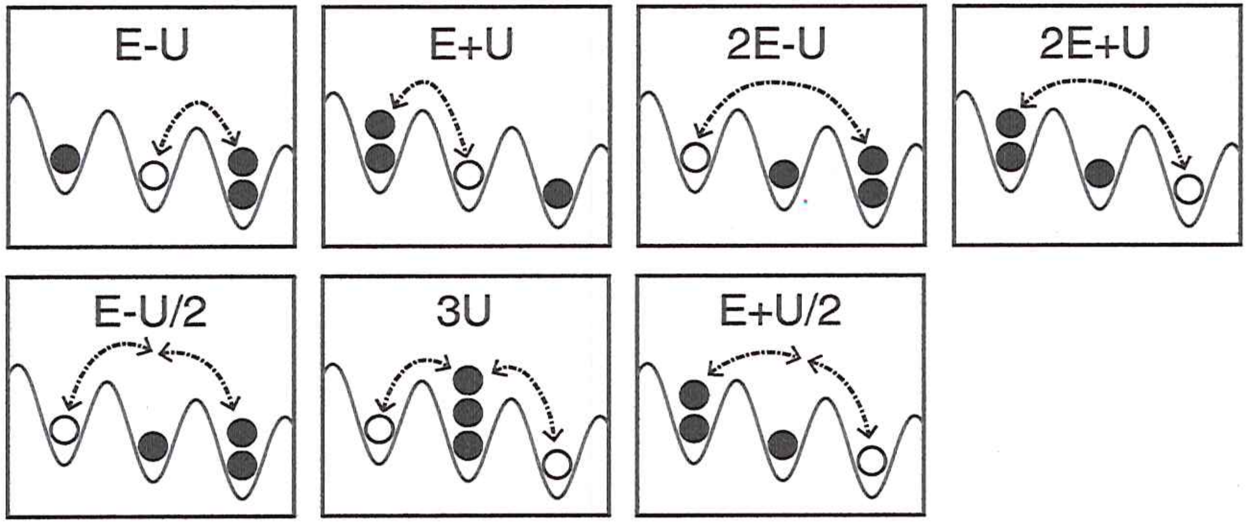
* This effect was measured in 2011 in M. Greiner's group at Harvard [Ma et al., PRL 107, 095301 (2011)]. In that experiment, they create 1D lattice gases and modulate the lattice depth to produce photon-assisted hopping, which they detect using in-situ fluorescence imaging with single-site resolution (p. 83). This technique is sensitive to parity. They start very deep inside the Mott (1111...). Laser-assisted hopping changes the parity of 2 neighbors, which is what they detect. The probability of odd occupation parity shows clear dips that signal the tunneling resonances (see p. 84).

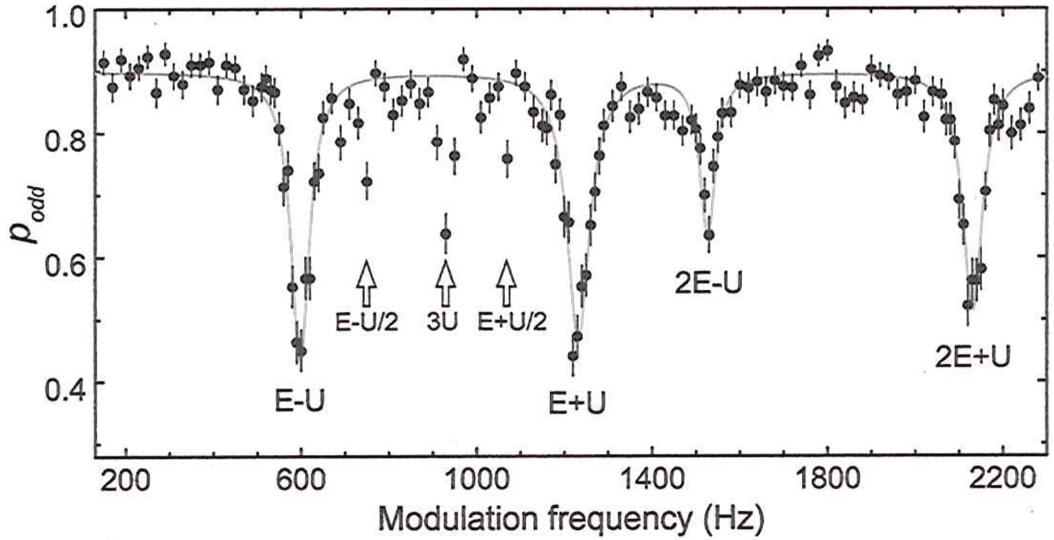
Laser-assissted hopping [Ma et al., PRL **107**, 095301 (2011)]

$$J(t) = J + \delta J \cos E t$$



Laser-assisted hopping [Ma et al., PRL **107**, 095301 (2011)]





* GENERATION OF SYNTHETIC MAGNETISM IN OPTICAL LATTICES

· Up to now, we just consider the case in which J may change the sign (as in the lattice shaking of p. (54)), but it remains real.

· We are going to see now how to create a complex hopping of the form $J = |J| e^{i\phi}$. [The phase ϕ receives the name Peierls phase]

· The first question we should ask ourselves is why having a complex phase is relevant. We will see that having a phase is directly related with having an effective vector potential.

· Let's consider the minimal-coupling Hamiltonian:

$$H = \frac{(\hat{p} - A)^2}{2m} + V_{lattice}(x)$$

("vector potential" labels A; "lattice" labels $V_{lattice}(x)$)

Let's evaluate J:

$$J = \int dx\, W_{j+1}^*(x) \left[\frac{(-i\partial_x - A)^2}{2m} + V_{latt}(x) \right] W_j(x)$$

$$\Rightarrow (-i\partial_x - A)\, W(x) \xrightarrow[\text{transf.}]{\text{gauge}} (-i\partial_x - A)\left[\tilde{W}_j(x) e^{i\Phi x}\right] \quad \Phi = A$$

$$= e^{i\phi(x)}\left[(\Phi - A)\tilde{W} - i\partial_x \tilde{W}\right] = e^{i\phi x}(-i\partial_x \tilde{W})$$

Then $(-i\partial_x - A)^2 W(x) = e^{i\Phi x}(-i\partial_x)^2 \tilde{W}$

$$\Rightarrow J = \int dx\, e^{-i\Phi[x-(j+1)D]}\, \tilde{W}_{j+1}(x) \, e^{i\Phi(x-jD)} \left[\frac{(-i\partial_x)^2}{2m} + V_{latt}(x)\right] \tilde{W}_j(x)$$

$$= e^{i\Phi D} \int dx\, \tilde{W}_{j+1}(x) \left(\frac{\hat{p}^2}{2m} + V_{latt}(x)\right) \tilde{W}_j(x) = e^{i\Phi D} J_0$$

Then having a phase $e^{i\phi} b_{j+1}^\dagger b_j$ is equivalent to having a vector potential $A = \phi/D$. You can then easily understand that if ϕ is spatially dependent one may have $\nabla \times A = B \neq 0$. One may create an effective magnetic field!

* So the question is: how can one create $e^{i\Phi[J]}$? let's see this first, and then let's move to the creation of $\phi(\vec{r})$ and hence of a magnetic field.

* let's discuss first the <u>generation of a Peierls phase using lattice shaking</u>, as performed by K. Sengstock's group in Hamburg [Struck et al., PRL 108, 225304 (2012)].

* We can generalize the formalism of page (51) to a general shaking:
$$\hat{H}(t) = \hat{H}_0 + K f(t) \sum_j j \hat{n}_j$$

We introduce the Floquet basis:
$$|\{n_j\}, m\rangle = |\{n_j\}\rangle \exp\left\{+\frac{i}{\hbar}\chi(t)\sum_j j\hat{n}_j + im\omega t\right\}$$

such that $\dot{\chi}(t) = K f(t)$

The right choice (for the particular case of lattice shaking) is:
$$\chi(t) = -K\int_0^t dt' f(t') + \frac{1}{T}\int_0^T dt \int_0^t dt' f(t')$$

we do not enter in detail in the reasons of why it is so.

[• <u>Note</u>: But this deserves some consideration. Note that $\chi(t) = -K\int^t dt' f(t') + \chi_0$ leads to $\dot{\chi}(t) = Kf(t)$ for any χ_0 → gauge-freedom. However for the lattice shaking this is fixed. The main reason is that the shaking determines $x_0(t)$ [illegible], and $F = -m\ddot{x}_0 \rightarrow V_j = -Fj$ [illegible] One fulfills not only that $x_0(t) = x_0(t+T)$ but also that $\frac{1}{T}\int_0^T x_0(t)\,dt = 0$, and $\frac{1}{T}\int_0^T \dot{x}_0(t)\,dt = 0$; Note that $\chi(t)$ is directly related to $\dot{x}_0$; hence we know that $\frac{1}{T}\int_0^T \chi(t)\,dt = 0$. Hence the gauge chosen above.]

* Once we have chosen that, let's consider

$$f(t) = \begin{cases} f_0 \sin \frac{2\pi}{T_1} t & 0 \le t \le T_1 \\ 0 & T_1 \le t \le T = T_1 + T_2 \end{cases}$$ (see p. (88))

hence: $$\int_0^t dt' f(t') = \begin{cases} -f_0 \frac{T_1}{2\pi} \left[\cos \frac{2\pi t}{T_1} - 1 \right] & 0 \le t \le T_1 \\ 0 & T_1 \le t \le T \end{cases}$$

$$\frac{1}{T}\int_0^T dt \int_0^t dt' f(t') = -\frac{f_0 T_1}{2\pi} \frac{1}{T} \int_0^{T_1} dt \left[\cos\left(\frac{2\pi t}{T_1}\right) - 1 \right] = \frac{f_0 T_1}{2\pi} \frac{T_1}{T}$$

• Then $$x(t) = \begin{cases} \left(\frac{f_0 T_1}{2\pi}\right) \left[\left(-1 + \cos\left(\frac{2\pi t}{T_1}\right)\right) + \frac{T_1}{T} \right] \\ \left(\frac{f_0 T_1}{2\pi}\right) \frac{T_1}{T} \end{cases}$$

• Recall from p. (55) that (let $K = f_0 T_1 / 2\pi$)

$$J_{eff} = J \frac{1}{T} \int_0^T dt\, e^{i x(t)} = J \frac{1}{T} \int_0^{T_1} dt\, e^{iK\left[\frac{-T_2}{T}\right]} e^{iK\cos\left(\frac{2\pi t}{T_1}\right)} + J \frac{1}{T} \int_{T_1}^{T} dt\, e^{iK T_1/T}$$

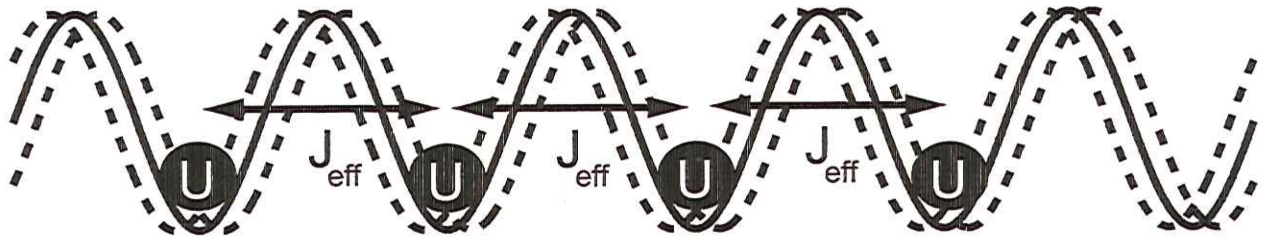
$$\Rightarrow J_{eff} = J \left\{ \frac{T_1}{T} e^{-iK\frac{T_2}{T}} J_0(K) + \frac{T_2}{T} e^{iK T_1/T} \right\} = |J_{eff}| e^{i\phi}$$

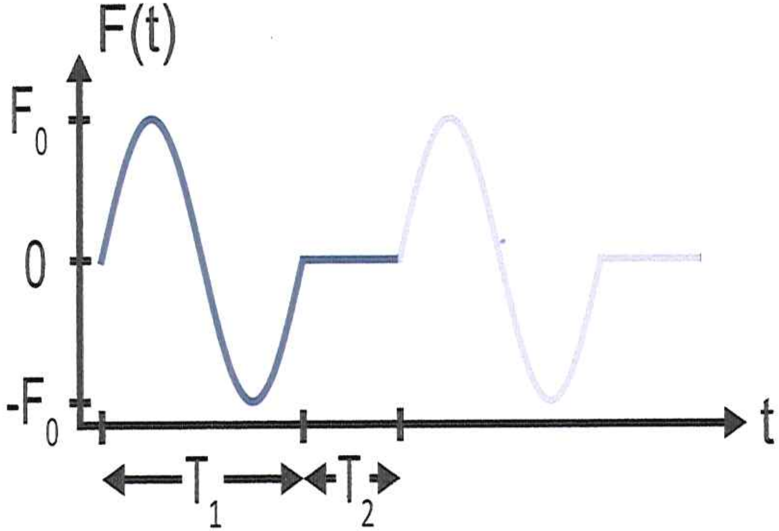
• Note that the Peierls phase depends on K, i.e. on the shaking amplitude. As we discussed before ϕ leads to an effective vector potential and then to a displaced $p \to p - \phi/D$. We hence expect a shift of the momentum distribution as a function of K. This is precisely what was observed on Sengstock's experiments. (see p. (89))

[<u>Note</u>: in Sengstock's experiments the quasimomentum distribution remains actually fixed, whereas what is shifted is actually the overall envelope given by the Fourier transform of the Wannier function. This is due to the fact that one changes frame twice, once to jump into the non-inertial system and then a jump into a rotating frame in the Floquet analysis.]

Synthetic vector potential using lattice shaking

[Struck et al., PRL **108**, 225304 (2012)]

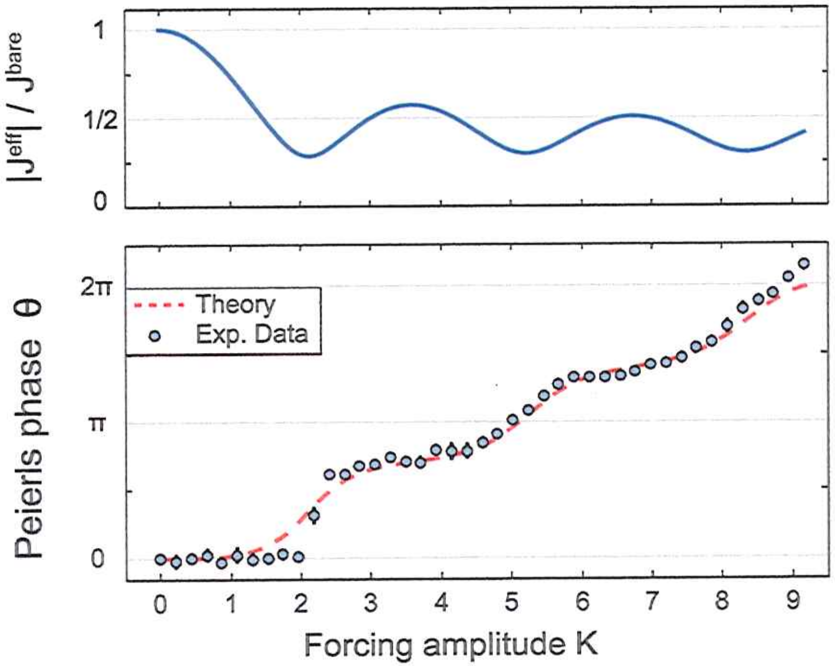




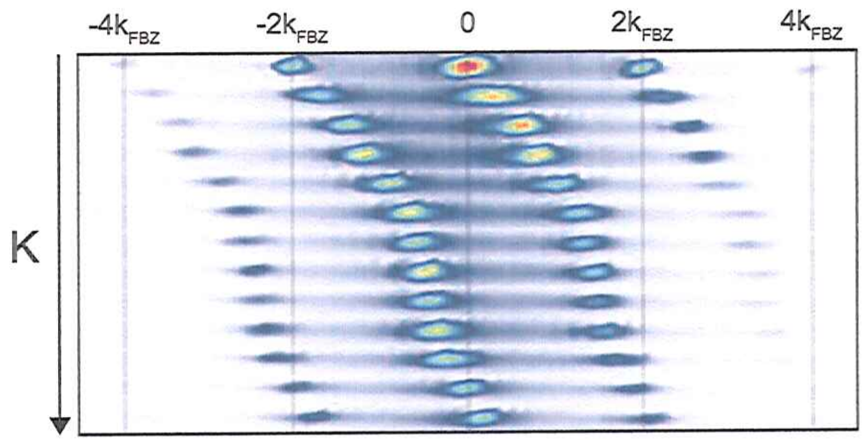
$$J_{eff} = \frac{T_2}{T} e^{-i\kappa\frac{T_1}{T}} + \frac{T_1}{T} J_0(\kappa) e^{i\kappa\frac{T_2}{T}} = \left|J_{eff}\right| e^{i\phi}$$

Synthetic vector potential using lattice shaking

[Struck et al., PRL **108**, 225304 (2012)]

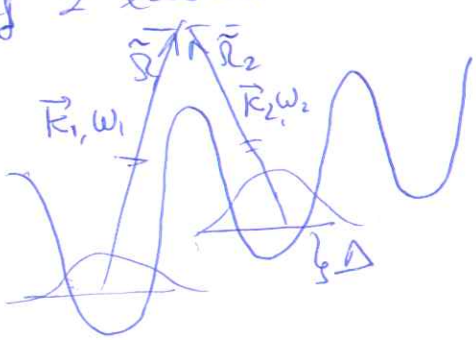


The Peierls phase leads to a K-dependent shift og the momentum distribution obtained in time-of-flight



* Let's have a look now to the generation of Peierls phases (and actually synthetic magnetic fields) in optical lattices using Raman-assisted hopping. Two simultaneous experiments achieved that in 2013, at W. Ketterle's lab at MIT and at I. Bloch's experiments in Munich.

* Let's consider a tilted 2D lattice along the x direction. The tilting $\Delta \gg J \Leftarrow$ hopping along x. Such that along x there's no natural hopping. Hopping may be however reduced using 2 lasers in a Raman configuration:



- We consider 2 lasers with wave vectors $\vec{k}_{1,2}$, frequencies $\omega_{1,2}$ and Rabi frequency $\tilde{\Omega}_{1,2}$ far from resonance from excited states (let $\Omega \equiv \frac{\tilde{\Omega}_1 \tilde{\Omega}_2}{\delta}$ ← detuning)

- Absorbing one laser and emitting the other one may get resonant tunneling if $\delta\omega = \omega_1 - \omega_2 = \Delta/\hbar$. The atom experiences a kick $\vec{\delta k}$. The effective tunneling may be easily evaluated for $J \ll \Delta$.

- It's particularly convenient to work with the so-called Wannier-Stark states, instead of the usual Wannier states. The Wannier-Stark states are the right basis when one has tunneling (they are e.g. employed in the theory of Bloch oscillations). For $J \ll \Delta$, one may easily obtain an approximation of the Wannier-Stark states:

$$\psi_m(x) \cong W_m(x) + \frac{J}{\Delta}\left[W_{m+1}(x) - W_{m-1}(x) \right]$$

WS-state centered at site m ← $\psi_m(x)$; Wannier states ← $W_m(x)$, $W_{m+1}(x)$, $W_{m-1}(x)$

· The effective tunneling between a site (m, n) [$x \nearrow$ $\uparrow y$] and a site $(m+1, n)$ is then:

$$J_{eff} = \frac{\Omega}{2} \int d^2r \, \psi^*_{m+1}(x) W^*_n(y) \, e^{+i\delta\vec{k}\cdot\vec{r}} \, \psi_m(x) W_n(y)$$

$$\simeq \frac{\Omega}{2} \int dx \left[W_{m+1}(x) + \frac{J}{\Delta}\left[W_{m+2}(x) - W_m(x)\right]\right] e^{i\delta k_x x} \left[W_m(x) + \frac{J}{\Delta}\left[W_{m+1} - W_{m-1}\right]\right]$$

$$\times \int dy \, |W_n(y)|^2 e^{i\delta k_y y}$$

Note that using the harmonic approximation $W(x - x_j) \simeq \frac{e^{-(x-x_j)^2/2\ell^2}}{\sqrt{\sqrt{\pi}\,\ell}}$ with $\ell \simeq \frac{1}{\pi s^{1/4}} D$ with $s = V_0/E_{rec} \to$ lattice depth.

If the lattice is deep $\ell \ll D$. We will employ this in a second.

$$J_{eff} \simeq \frac{\Omega}{2} \int dx \left[W_{m+1}(x) W_m(x) + \frac{J}{\Delta}\left[W_{m+1}(x)^2 - W_m(x)^2\right] + \dots\right] e^{i\delta k_x x}$$

$$* \int dy \, |W_n(y)|^2 e^{i\delta k_y y}$$

($\uparrow$ very small terms since $J/\Delta \ll 1$ and $\ell \ll D$.)

· Note something quite crucial: δk_x must be non-zero. Otherwise we have zero because the Wannier function are orthogonal (Note: this may be sometimes forgotten if one abuses of the harmonic approximation $\to$ be careful!).

Using $\ell \ll D \to$

$$\int dy \, |W_n(y)|^2 e^{i\delta k_y y} \simeq e^{i\delta k_y n D}$$

$$\int dx \, W_{m+1}(x) W_m(x) e^{i\delta k_x x} \simeq 0$$

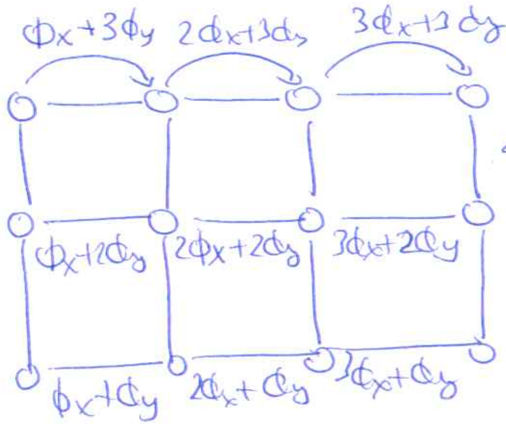
$$J_{eff} \simeq \frac{\Omega}{2} \frac{J}{\Delta} \left[e^{i\delta k_x (m+1)D} - e^{i\delta k_x m D}\right] e^{i\delta k_y n D}$$

$$= \frac{\Omega J}{2\Delta} e^{i\delta k_x m D} e^{i\delta k_y n D} e^{i\delta k_x \frac{D}{2}} \left(e^{i\delta k_x D/2} - e^{-i\delta k_x D/2}\right)$$

$$= i \frac{\Omega J}{\Delta} \sin\left(\frac{\delta k_x D}{2}\right) e^{i\delta k_x (m+\frac{1}{2})D} e^{i\delta k_y n D}$$

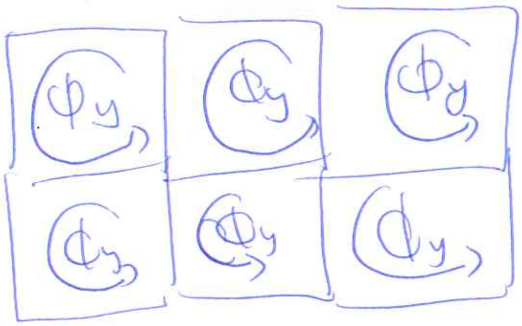
$$\equiv J_{eff}^{(0)} e^{i\Phi_x m} e^{i\Phi_y n}$$

• Then:



← along y we consider natural hopping and hence there's no extra phase

Hence the phase accumulated when going around a plaquette is



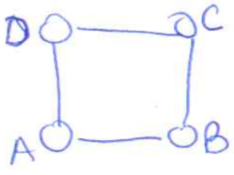
← Uniform flux per plaquette.

One generates in this way an homogeneous magnetic field in the z-direction. This is clear from the discussion of p. (85), since now $\vec{A} \propto (\phi_x x + \phi_y y)\vec{e}_x \rightarrow \nabla \times \vec{A} \propto (\partial_y A_x)\vec{e}_z = \phi_y$

Hence only ϕ_y is relevant for the magnetic field created.

• At I. Bloch's group they study the created B field by isolating individual plaquettes. In Bloch's experiment they have actually 2 states $|\uparrow\rangle \equiv |F=1, m_F=-1\rangle$ and $|\downarrow\rangle \equiv |F=2, m_F=-1\rangle$ which have opposite magnetic moments. A magnetic field gradient hence leads to opposite tilting Δ for $\uparrow$ and $-\Delta$ for $\downarrow$, and hence since $\phi_y \propto \Delta^{-1} \rightarrow \phi_y$ for $\uparrow$ and $-\phi_y$ for $\downarrow$.

× In a plaquette: By doing even-odd resolution (i.e. resolving between the populations in even/odd sites) they can determine $N_{LEFT} = N_A + N_D$, $N_{RIGHT} = N_C + N_B$, $N_{UP} = N_D + N_C$, $N_{DOWN} = N_A + N_B$

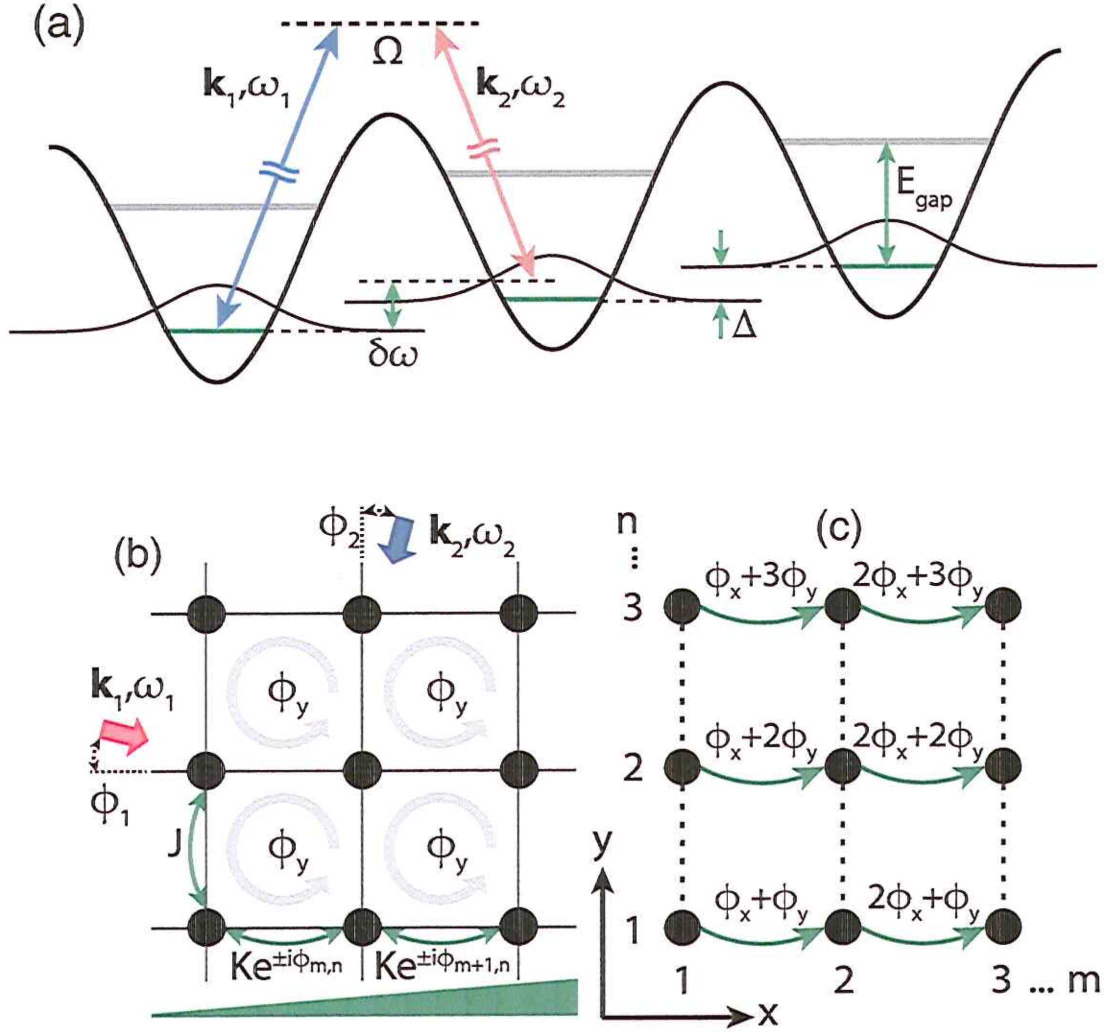


and hence analyze the mean position

$\langle X \rangle = (N_{RIGHT} - N_{LEFT}) D / 2N$; $\langle Y \rangle = (N_{UP} - N_{DOWN}) D / 2N$

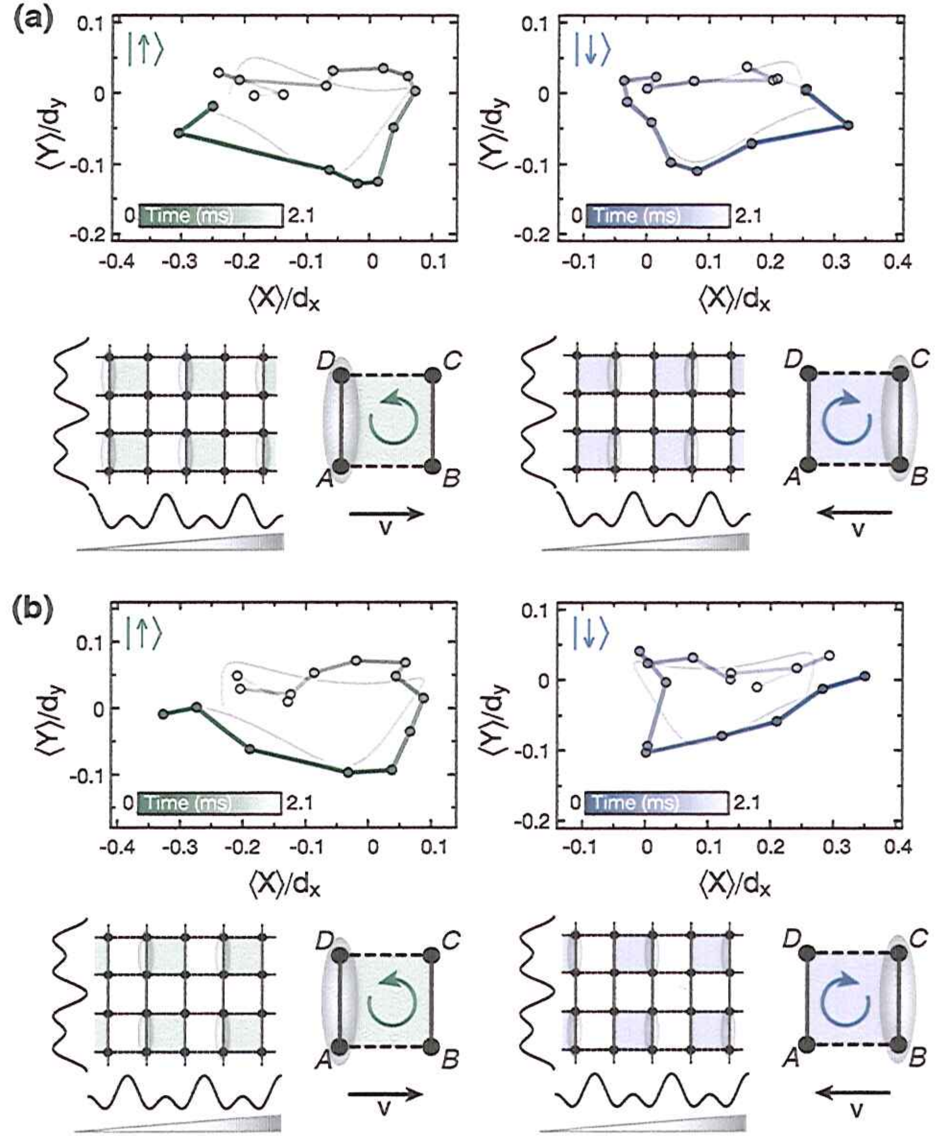
Synthetic magnetism using Raman-assisted hopping

[Miyake et al., PRL **111**, 185302 (2013)]



Synthetic magnetism using Raman-assisted hopping

[Aidelsburger et al., PRL **111**, 185301 (2013)]



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- They observe the corresponding "quantum ~~cyclotron orbits~~" induced by the effective Lorentz force, showing that the chirality of the cyclotron orbit is reversed depending on ↑ or ↓.

- They checked the uniformity of the Lorentz force by having a look to the cyclotron orbits when shifting by one (to the right) the plaquettes measured.

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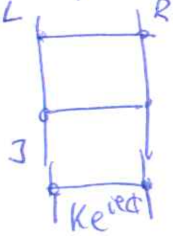
* Let's consider at this point other recent experiment at Bloch's lab on synthetic magnetism in ladders.

By means of a superlattice they create an array of isolated tilted double-well potentials. (p. 97) Each double-well corresponds to a single ladder realization. Using the Raman techniques we just discussed they create a Peierls phase $e^{il\phi}$ with $\phi = \pi/2$. Hence the flux per plaquette is $\pi/2$.

$$H = -J \sum_l (\hat{a}^+_{l+1;L} \hat{a}_{l;L} + \hat{a}^+_{l+1;R} \hat{a}_{l;R}) - K \sum_l (e^{il\phi} \hat{a}^+_{l;R} \hat{a}_{l;L} + h.c.)$$



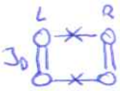
* An important observable is the average current on either side of the ladder $j_{\mu=L,R} = \frac{1}{N_{eag}^{-1}} \sum_l \langle j_{l,l+1;\mu} \rangle$

where $j_{l,l+1;\mu=L,R}$ denotes the current operator for currents flowing from site l to site $l+1$. ($j_{l,l+1;\mu} \propto a^+_{l;\mu} a_{l+1,\mu} - a^+_{l+1;\mu} a_{l;\mu}$)

- One may construct an important quantity, the so-called chiral current $j_c = j_L - j_R$ (see p. 98)

- For fluxes $\phi \le \phi_c$ the ground state of the Hamiltonian is a Meissner phase with maximal and opposite $|j_\mu| = (2J/\hbar) \sin\phi/2$. Increasing ϕ increases $|j_\mu|$ (and hence j_c) to a maximum at ϕ_c critical flux ϕ_c. Beyond that, for $\phi > \phi_c$, the current abruptly

decreases. At Φ_c the system enters a vortex phase with decreasing $|j_\mu|$. This resembles the Meissner phase in type-II superconductors and the corresponding transition to an Abrikosov vortex lattice.
[Note: The resemblance is however imperfect because here the currents do not affect the B field because the particles are at the end of the day neutral!]

- In the experiments they extract j_μ by suddenly projecting the wavefunction into isolated double wells along y' (the leg axis) and holding for a certain time t. During the projection the legs of the ladder are decoupled as well.  (see p. 99)

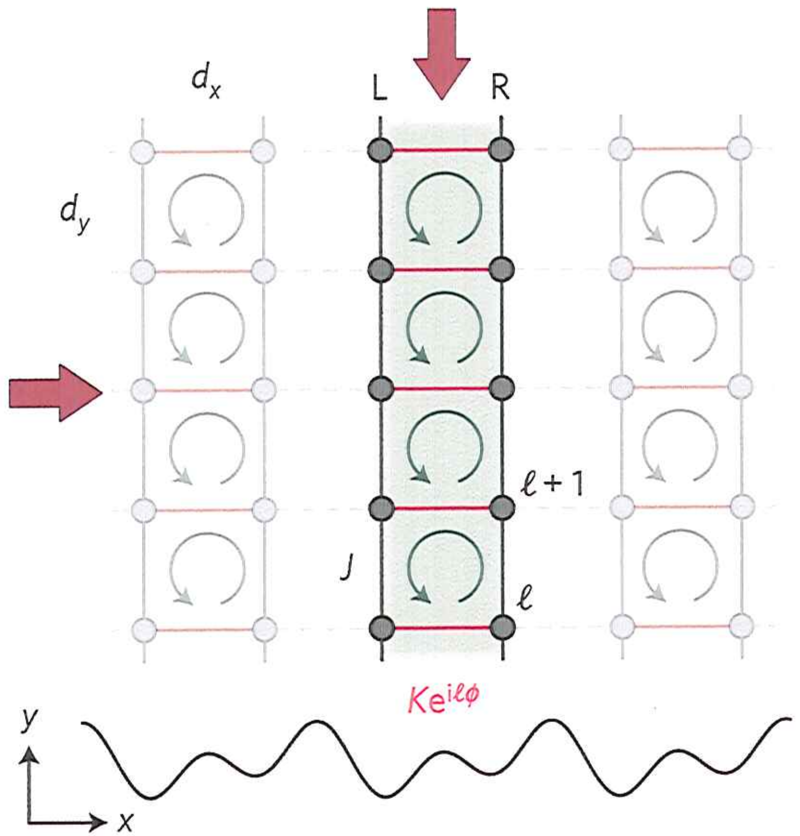
If J_D is the tunnel coupling inside each double well, then the average even-odd atom fraction oscillates as

$$n_{even;\mu}(t) - n_{odd;\mu}(t) \simeq \frac{-j_\mu}{J/\hbar}\sin(2J_D t/\hbar)$$

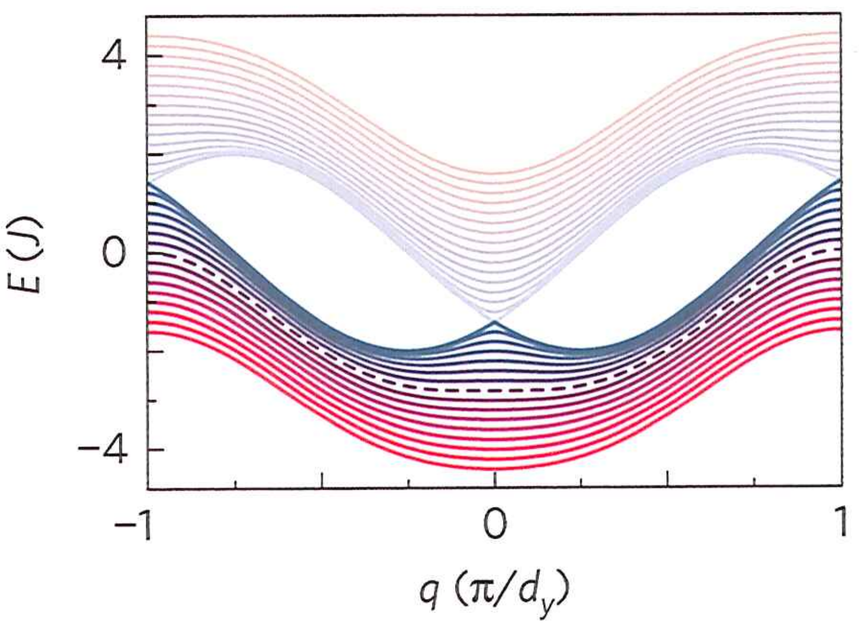
- Measuring for $\phi = \pi/2$ the $j_c = j_L - j_R$ for various K/J they are able to map the Meissner to vortex transition. (p. 100)

Synthetic magnetism in ladder-like lattices

[Atala et al., Nature Physics **10**, 588 (2014)]



$$\phi = \frac{\pi}{2}$$

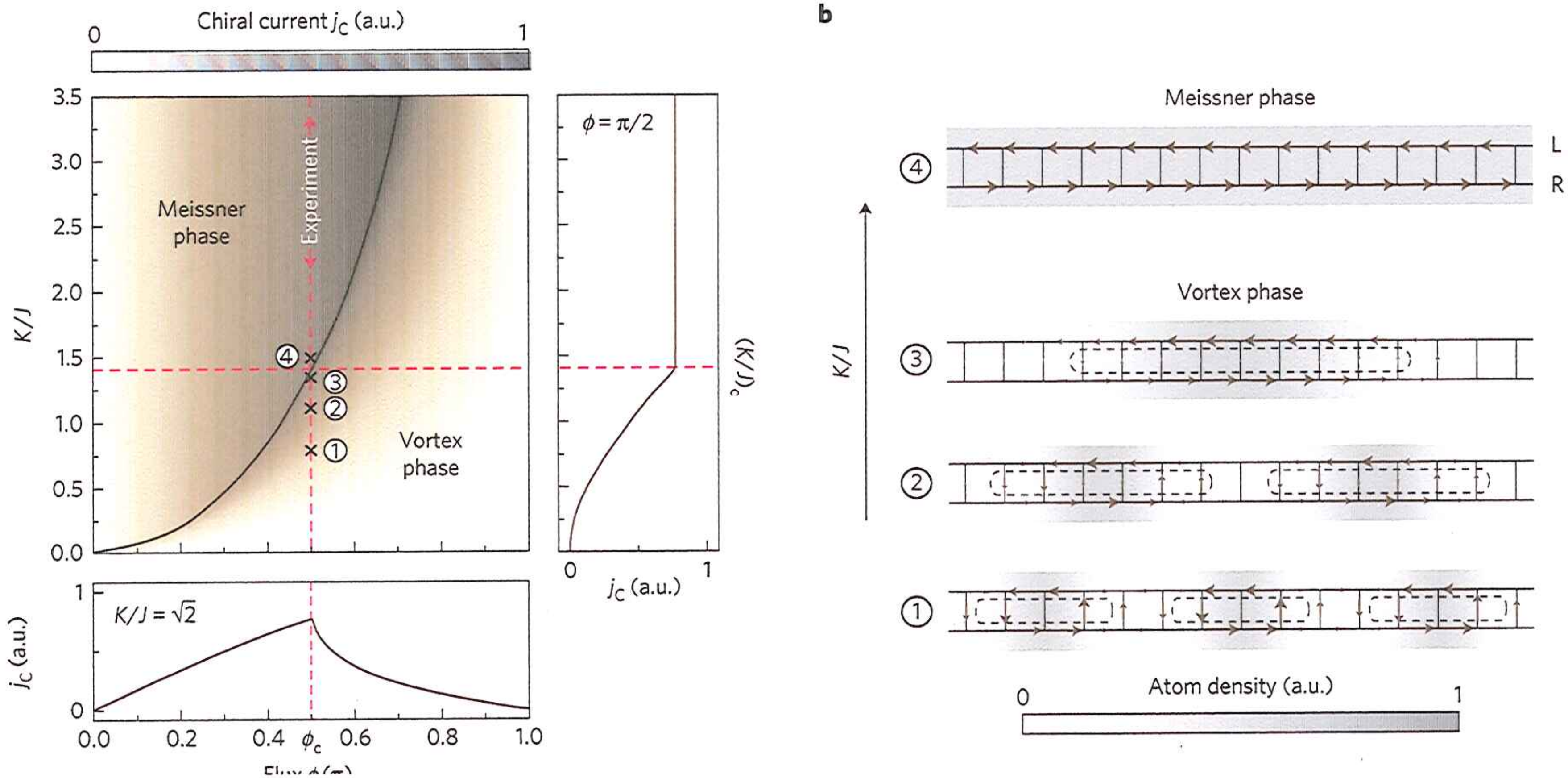


$$\frac{K}{J} < \left(\frac{K}{J}\right)_c = \sqrt{2}$$ Two minima: Vortex phase

$$\frac{K}{J} > \left(\frac{K}{J}\right)_c$$ One minimum: Meissner phase

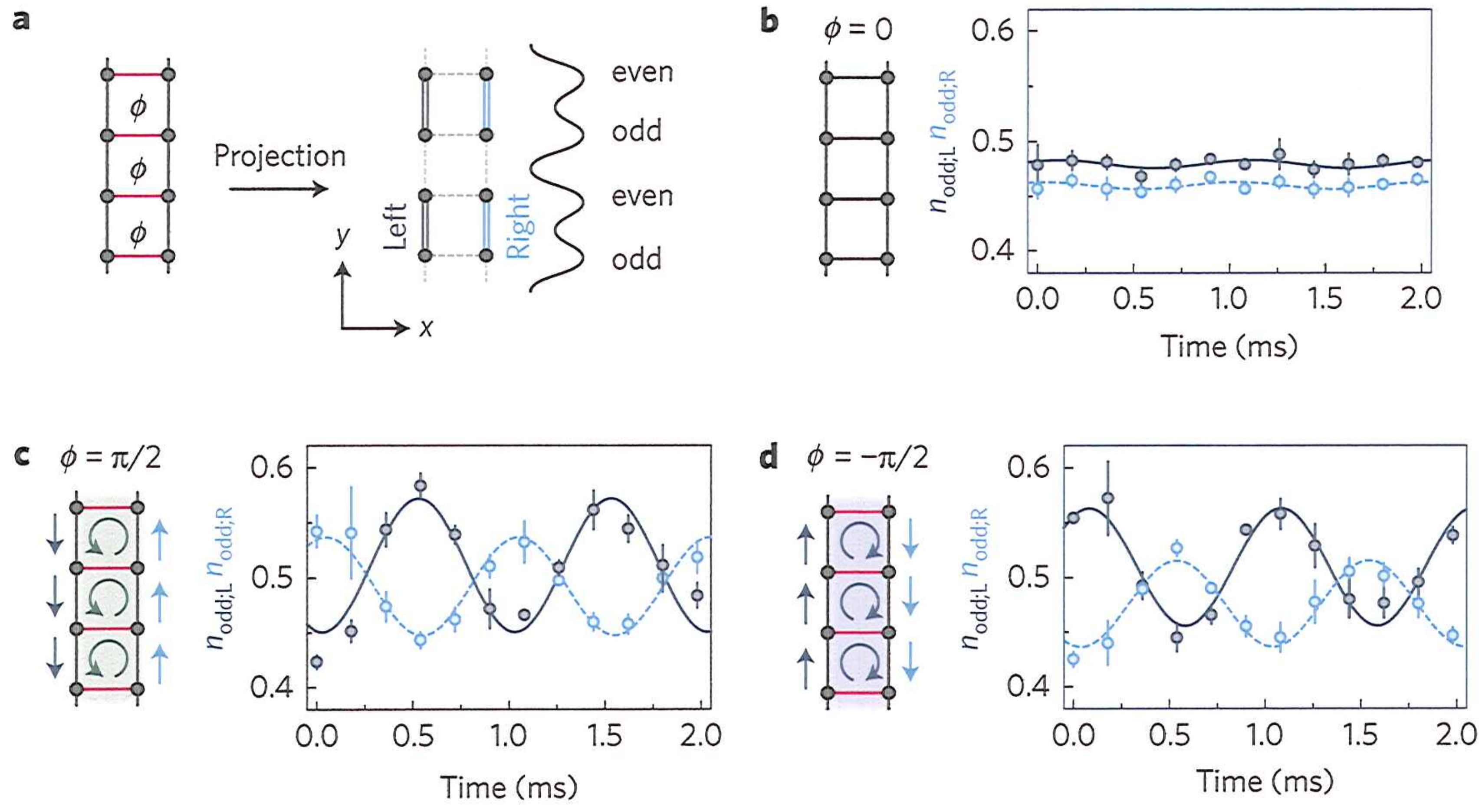
Synthetic magnetism in ladder-like lattices

[Atala et al., Nature Physics **10**, 588 (2014)]



Synthetic magnetism in ladder-like lattices

[Atala et al., Nature Physics **10**, 588 (2014)]



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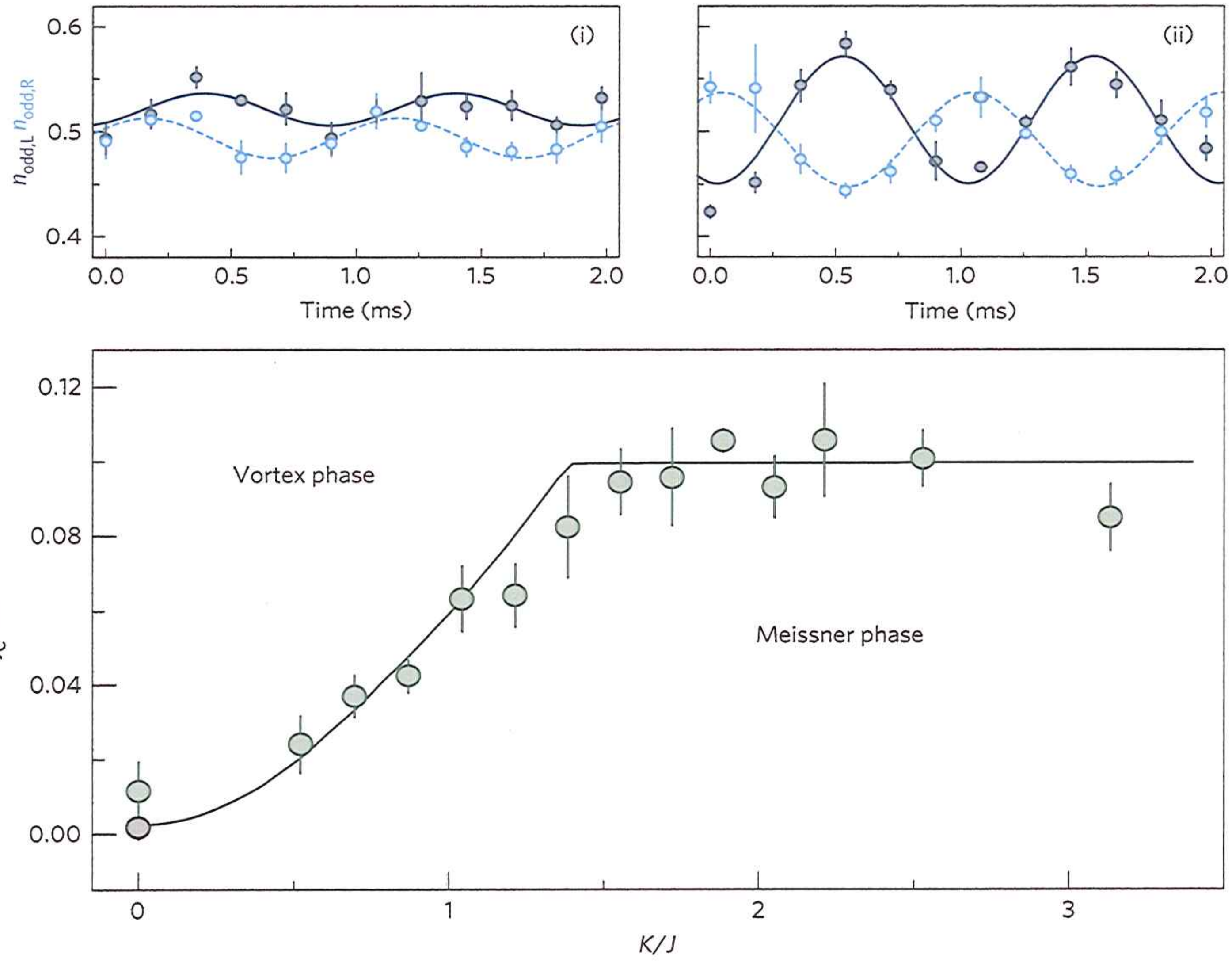
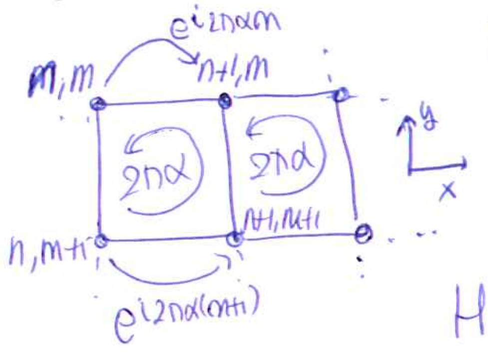


* Let us discuss at this point the band structure expected for a lattice gas in the presence of a strong magnetic field. This problem was studied by D. Hofstadter in a classic paper of 1976.

• Let's consider a lattice with an induced gauge as that induced in the experiments of Ketterle or Bloch. Let's consider only the non-interacting part:



$$H = -J \sum_{n,m} \psi^\dagger_{n,m} \left[\psi_{n,m+1} + \psi_{n,m-1} + e^{-i2\pi\alpha m} \psi_{n+1,m} + e^{i2\pi\alpha m} \psi_{n-1,m} \right]$$

* In solid state systems in the presence of a (real) magnetic field, the parameter α is of the form: $\alpha = \frac{D^2 B}{2\pi(\hbar c/e)}$ with $\begin{cases} D \equiv \text{lattice spacing} \\ B \equiv \text{magnetic field} \\ c \equiv \text{speed of light} \\ e \equiv \text{electron charge} \end{cases}$

* Typical lattice spacings in real crystals are of the order of $D \sim 2Å$. Hence $\alpha = 1$ requires brutally large magnetic field of the order of 10^9 gauss. It's important to point this fact because the physics we will discuss right now will be around this regime of large α.

• This must be compared with synthetic gauge field where $\alpha \sim 1$ is rather easy to achieve! (see p. 104)

[Note: recent experiments (see Dean et al., Nature 497, 598 (2013); and Ponomarenko et al., Nature 497, 594 (2013)) have been able to create larger D values in graphene crystals, by using the Moiré pattern when superimposing 2 lattices; they have observe traces of the Hofstadter butterfly, discussed below]

* Since the phase only depends on the y-direction, we will assume a plane wave behavior along x:

$$\psi_{nm} = e^{i\nu n} g_m$$

* We are interested in the eigenfunctions:

$$\epsilon \psi_{nm} = \psi_{n,m+1} + \psi_{n,m-1} + e^{-i2\pi\alpha m} \psi_{n+1,m} + e^{+i2\pi\alpha m} \psi_{n-1,m}$$

(with $\epsilon \equiv E/(-J)$)

Then:

$$\epsilon e^{i\nu n} g_m = e^{i\nu n} (g_{m+1} + g_{m-1}) + (e^{-i2\pi\alpha m} e^{i\nu(n+1)} + e^{i2\pi\alpha m} e^{i\nu(n-1)}) g_m$$

$$\Rightarrow \boxed{\epsilon g_m = g_{m+1} + g_{m-1} + 2\cos(2\pi\alpha m - \nu) g_m}$$

This equation is the so-called <u>Harper's equation</u>. It appears also in the discussion of lattice gases in the presence of quasidisorder (e.g. given by bychromatic lattices), but we will not discuss that at this point.

* It can be shown that the actual value of ν doesn't change the spectrum, which on the contrary crucially depends on α (i.e. on the flux).

Strikingly when $\alpha = p/q$ (i.e. a rational number) the Bloch band always breaks up into precisely q distinct energy bands. The resulting spectrum presents a fine-grained fractal structure, in which bands cluster into group, which themselves may cluster into larger groups, and so on.

This structure (shown in p. (105)) receives the name of <u>Hofstadter butterfly</u>.

* of course, in reality, fluctuations of the flux (however small) will partially blurred the fractal structure. But still, a large part of the structure survives. (see p. (106)).

* Neither the MIT experiments nor the Munich ones could directly measure the fractal butterfly spectrum. In the future one way to look for the Hofstadter butterfly may be to load a degenerate <u>Fermi gas</u> instead of bosons. One would expect that the <u>density profile</u> could reveal then the fractal spectrum.

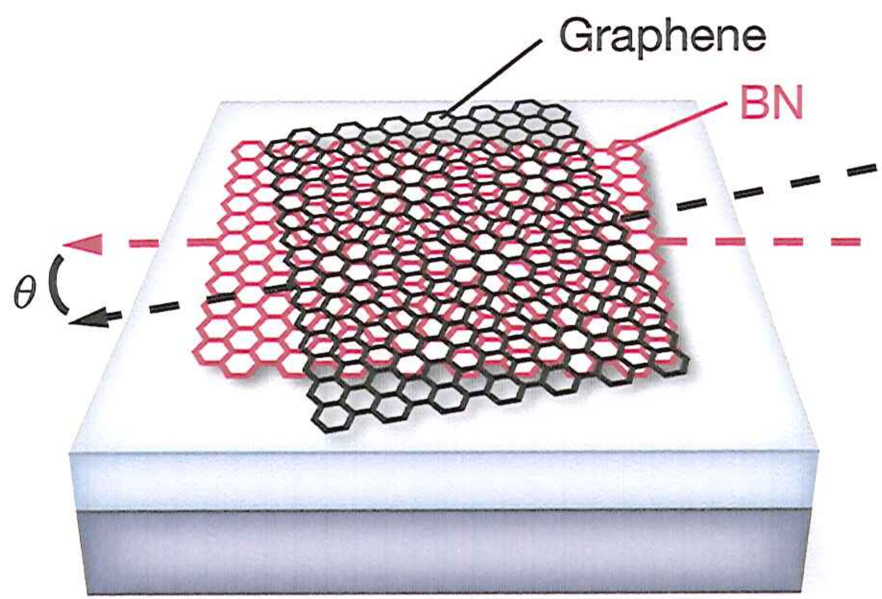
Other possibility (even for bosons) is to use band mapping techniques.

As a caveat: in order to observe the butterfly one would need temperatures smaller than the butterfly gaps, which may be problematic due to the heating associated to the Raman-assisted hopping.

[* <u>Note</u>: see the nice viewpoint article of Physics of C. Chin and E. Mueller on this issues (Physics 6, 118 (2013))].

Hofstadter butterly

[Dean et al., Nature **497**, 598 (2013)]



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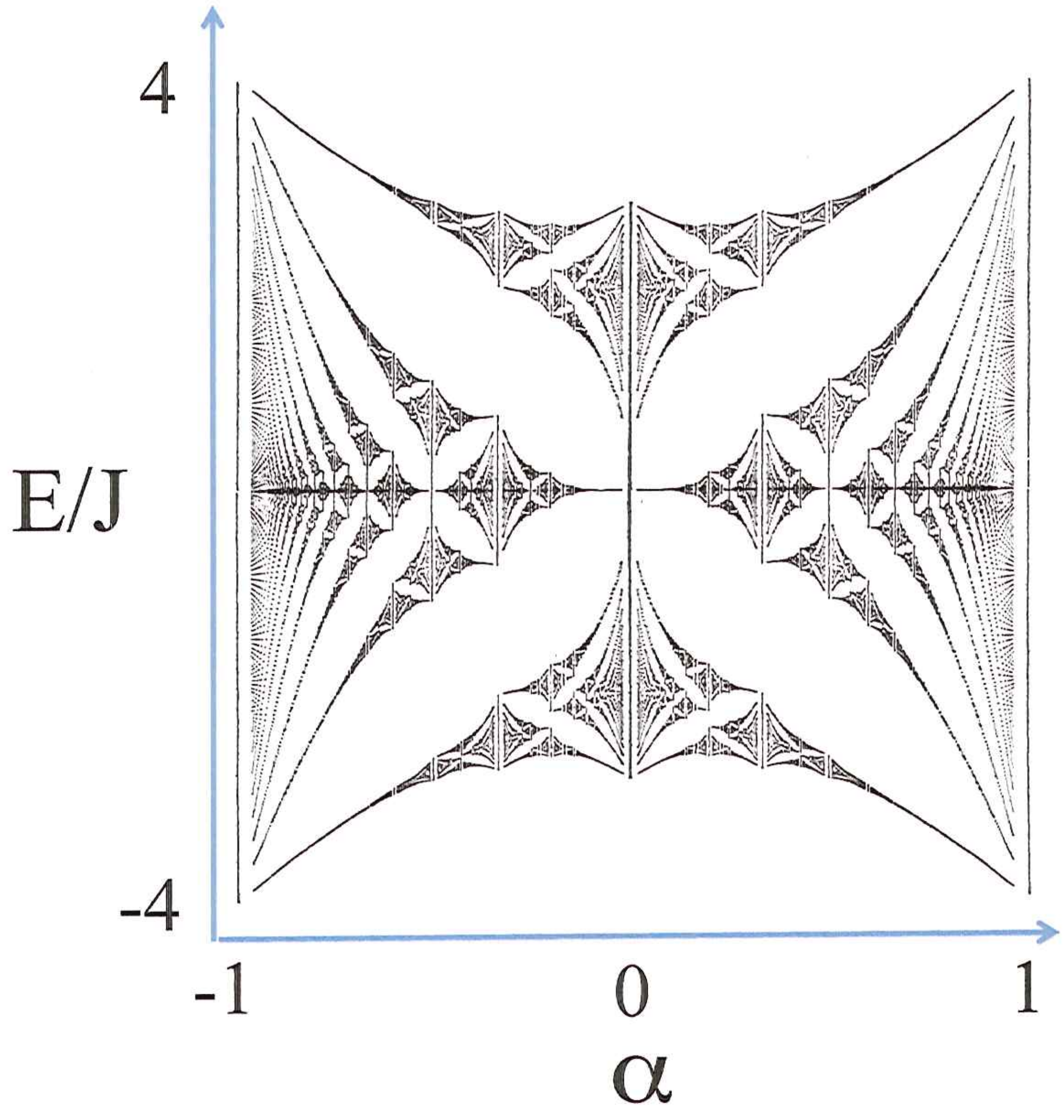
Hofstadter butterly

[Hoftstadter, PRB **14**, 2239 (1976)]



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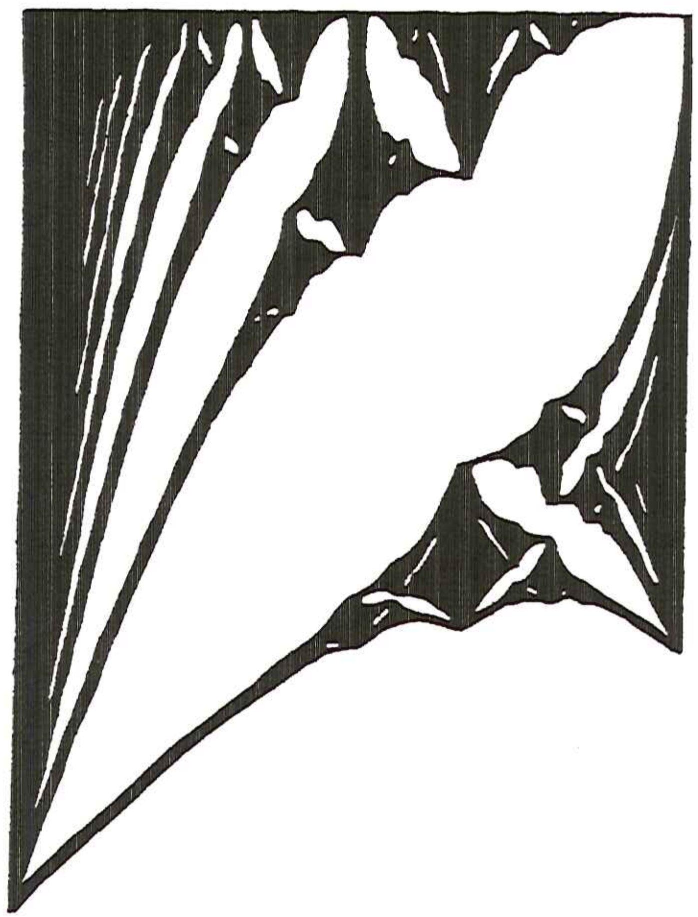
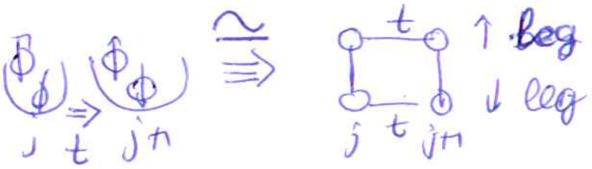
Hofstadter butterly

[Hoftstadter, PRB **14**, 2239 (1976)]

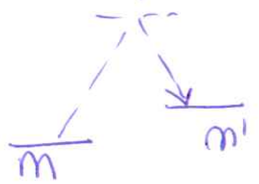


* Interestingly one may engineer ladder-like geometries with just a 1D optical lattice. This is possible because one may use the internal (typically m states) manifold as a separate dimension ⇒ SYNTHETIC DIMENSION



· This interesting analogy was first suggested in this context by the group of M. Lewenstein at ICFO [Celi et al., PRL 112, 043001 (2014)]

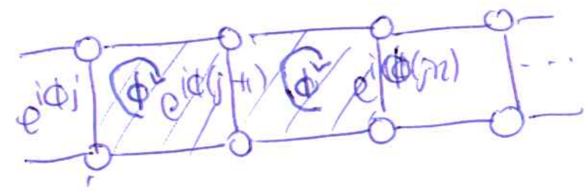
· The tunneling along the rungs of the effective ladder-like lattice can be engineered by establishing a Raman coupling between the internal states ↑, ↓ (note: now we are thinking in on-site Raman coupling between different m states:



· The 2-photon Rabi coupling between m and m' may acquire a phase given by the applied lasers: $\Omega \equiv \Omega e^{i\phi_j}$

· This Rabi coupling acts like the hopping along the rung.

Hence:



One gets hence an "effective plaquette" with a flux ϕ per plaquette.

· This nice idea has been recently realized in 2 experiments, one at L. Fallani's group in LENS (Florence) [Mancini et al., arxiv: 1502.02495] and other in I. Spielman's group at NIST (Maryland) [Stuhl et al. arxiv: 1502:02496].
The experiments at LENS are performed with ^{173}Yb which has available $m: -5/2 \dots 5/2$ (this in nuclear m_I, and as a result the scattering length is the same for any m with any m', and hence there're no spin-changing collisions). The experiment at NIST use ^{87}Rb in $F=1$ (hence $m_F = -1, 0, +1$).

- Let's briefly discuss the experiments at LENS.

They made experiments with both "two-leg" ladders ($m_F = -5/2, -1/2$) and "three-leg" ladders ($m_F = -5/2, -1/2, +3/2$) (see p. (110))

- The two-leg ladder case is basically the fermionic analog of the Bosonic experiment performed at Munich (see above). As for the bosonic case one expects chiral currents with atoms flowing in opposite directions along the legs.

They measure the relative motion of the atoms in the 2 legs by spin-selective imaging of the lattice momentum distribution by spin-selective imaging of the lattice momentum distribution (after switching off the synthetic coupling). (basically doing TOF after switching off the synthetic coupling). The momentum distribution $n(k)$ for the 2 components is anisotropic (with opposite anisotropy for the 2 components).

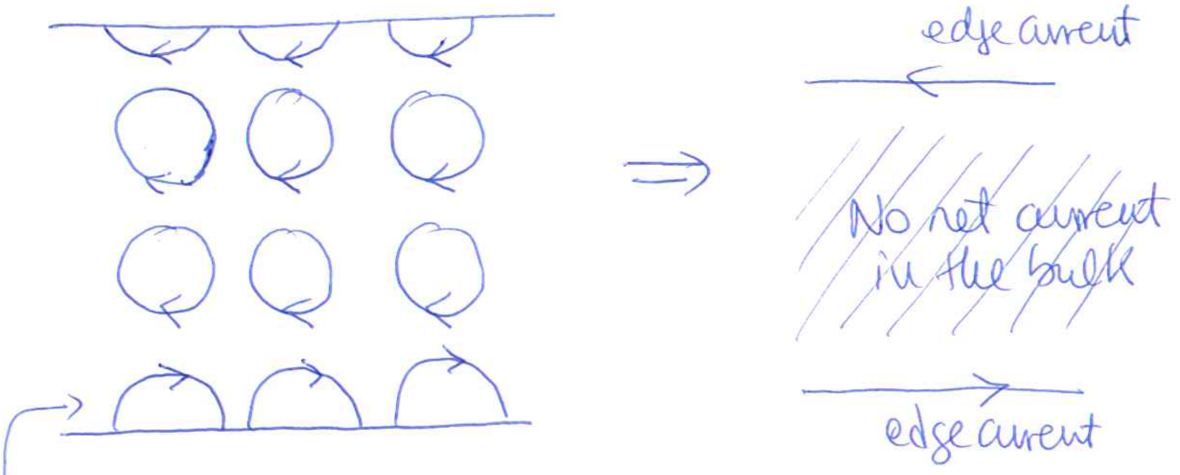
Defining $h(k) = n(k) - n(-k)$ one measures the anisotropy.

Then $J = \int_0^1 h(k)\, dk$ provides a measurement of the lattice momentum imbalance. (Note: basically $J(-5/2) = -J(-1/2)$) (see p. (111))

[Note: Note that J decreases after a maximum as a function of the Raman coupling → due to the formation of "rung" singlets]

- The three-leg ladder case is done similarly. They observe strong chiral currents for $m = -5/2$ and $m = 3/2$, and a much reduced asymmetry in $m = -1/2$ (Note: the nonzero J in $m = -1/2$ in p. (112) is due to technical imperfections)

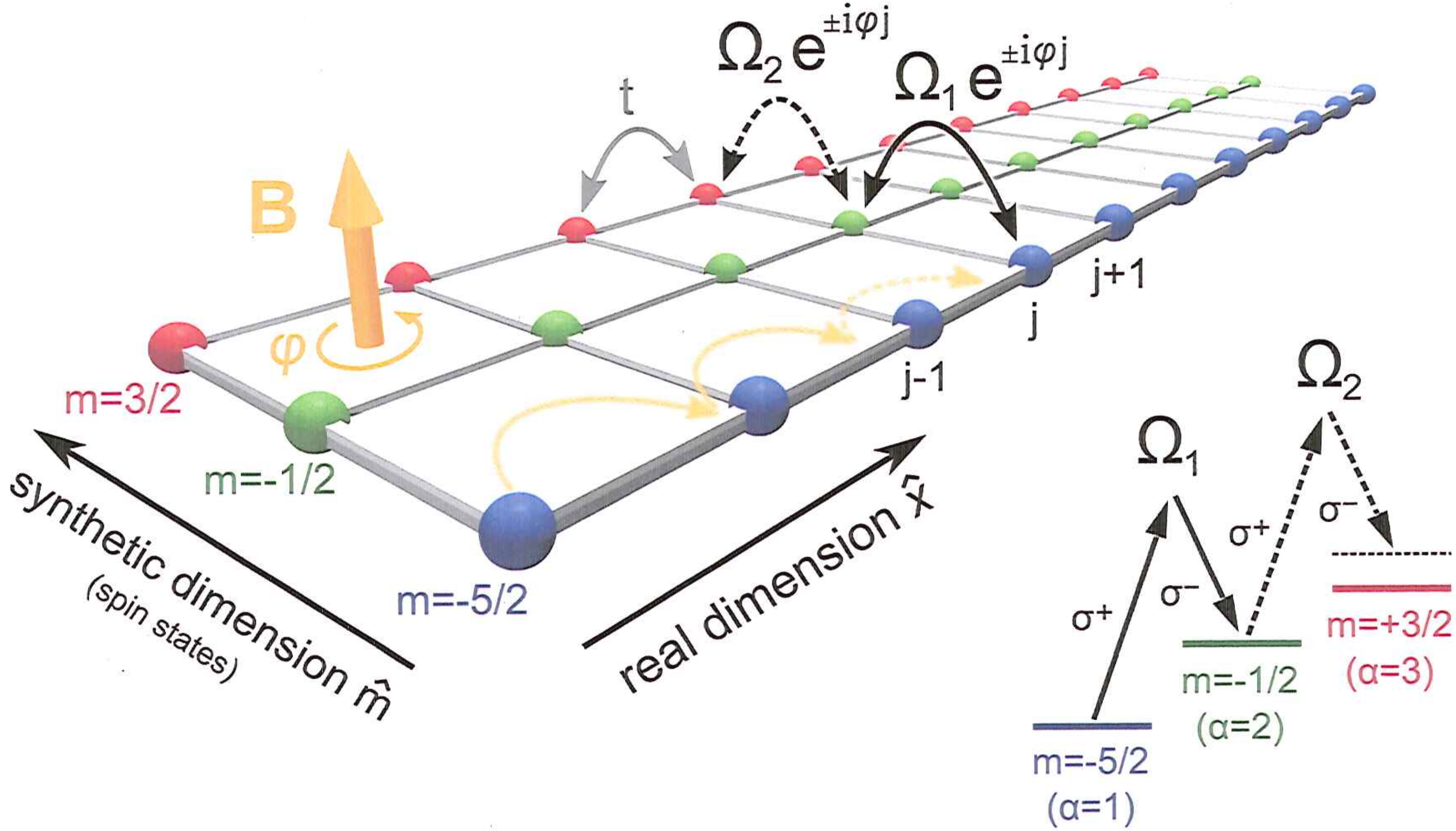
This behavior is basically what one expects for a fermionic system with a magnetic field. Bulk states (the "bulk" is now the middle leg) exhibit only local circulations of current, which average to zero when all states enclosed by the Fermi surface are considered. Only the edges experience a non-zero current.



These, so-called skipping-orbits, have been also observed at LENS by means of a quenching experiment. They start with $m = -5/2$ and switch on the coupling. They are then able to map the trajectory in the synthetic lattice, which shows nicely the skipping orbits. (see (13))

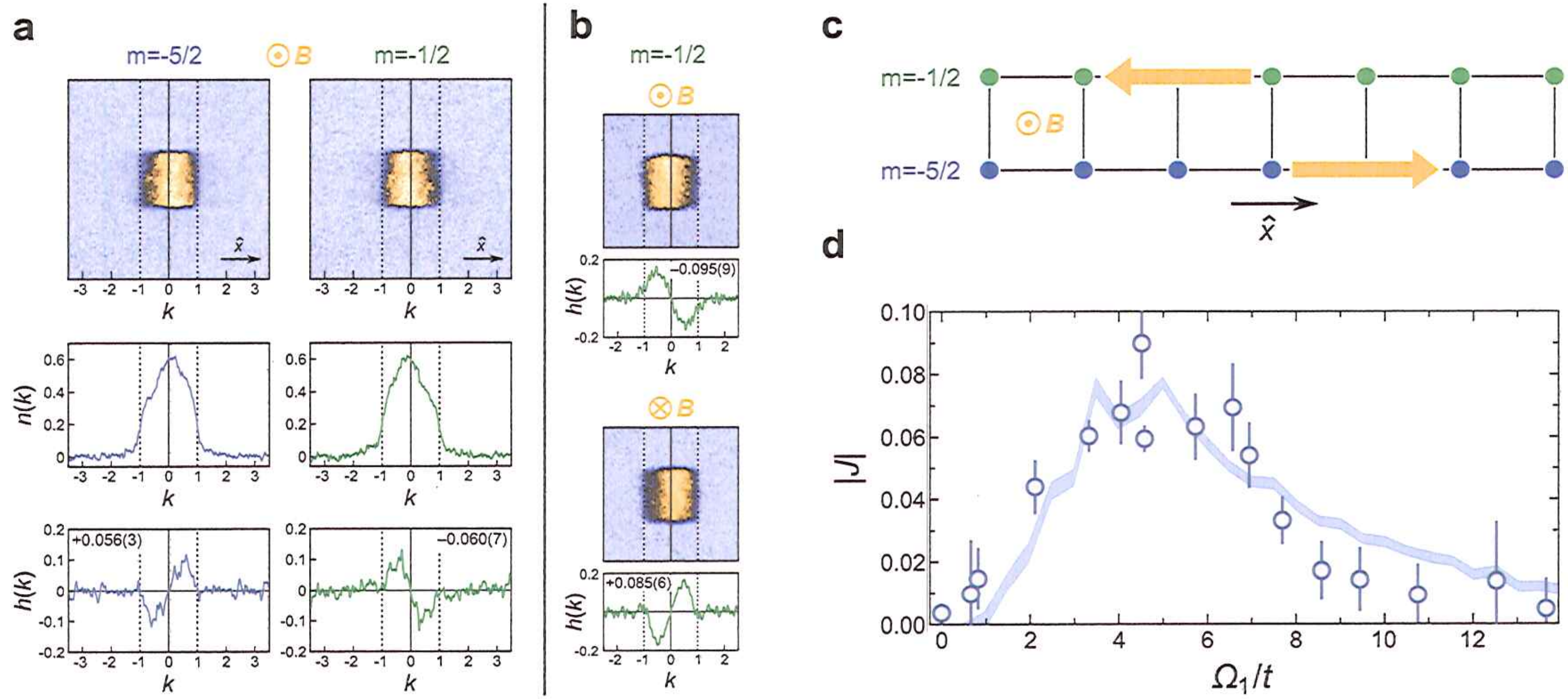
Synthetic magnetism in synthetic dimensions

[Mancini et al., arXiv:1502.02495]



Synthetic magnetism in synthetic dimensions

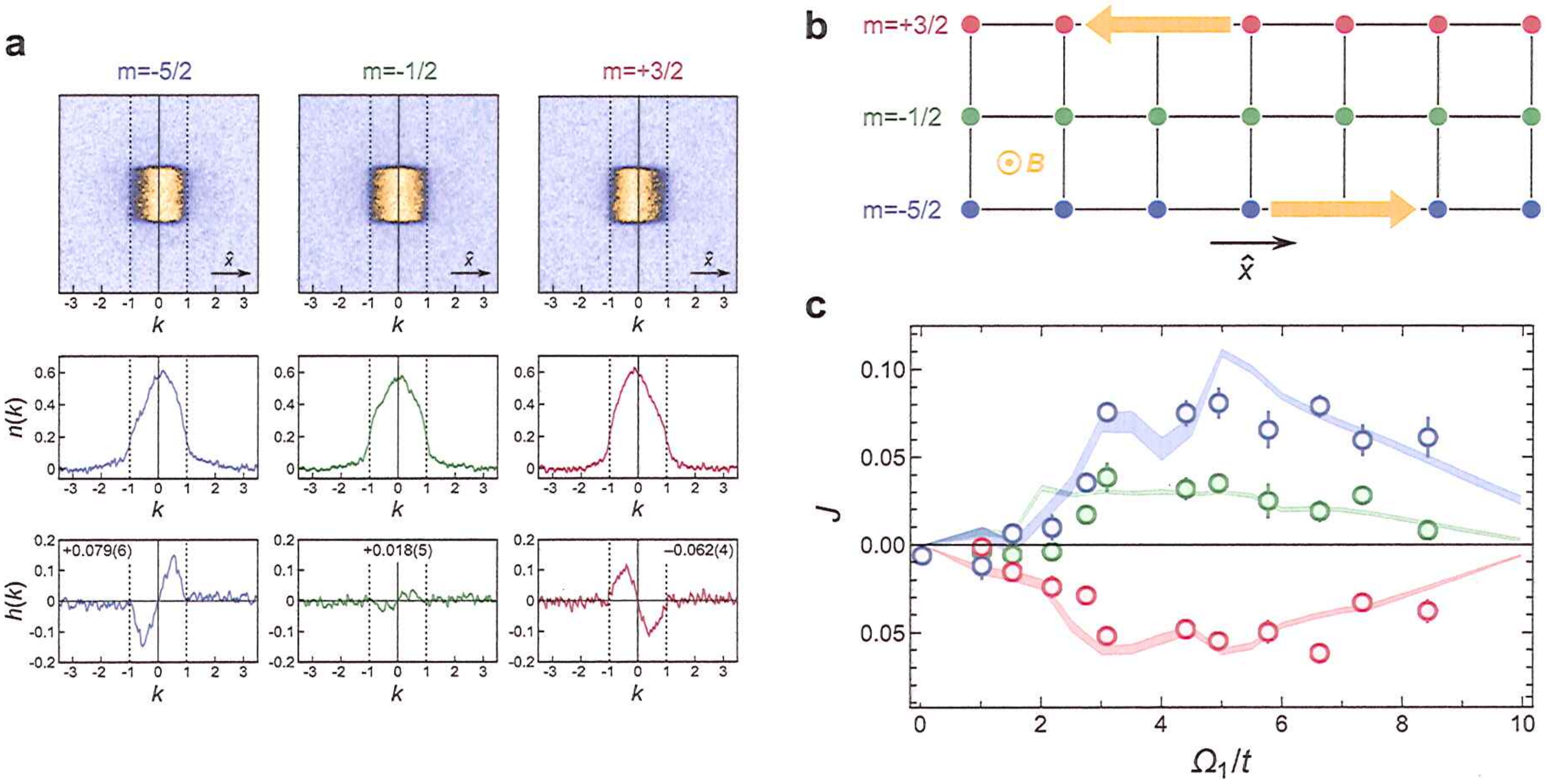
[Mancini et al., arXiv:1502.02495]



$$h(k) = n(k) - n(-k)$$

$$J = \int_0^1 n(k)\,dk$$

Synthetic magnetism in synthetic dimensions [Mancini et al., arXiv:1502.02495]



Synthetic magnetism in synthetic dimensions

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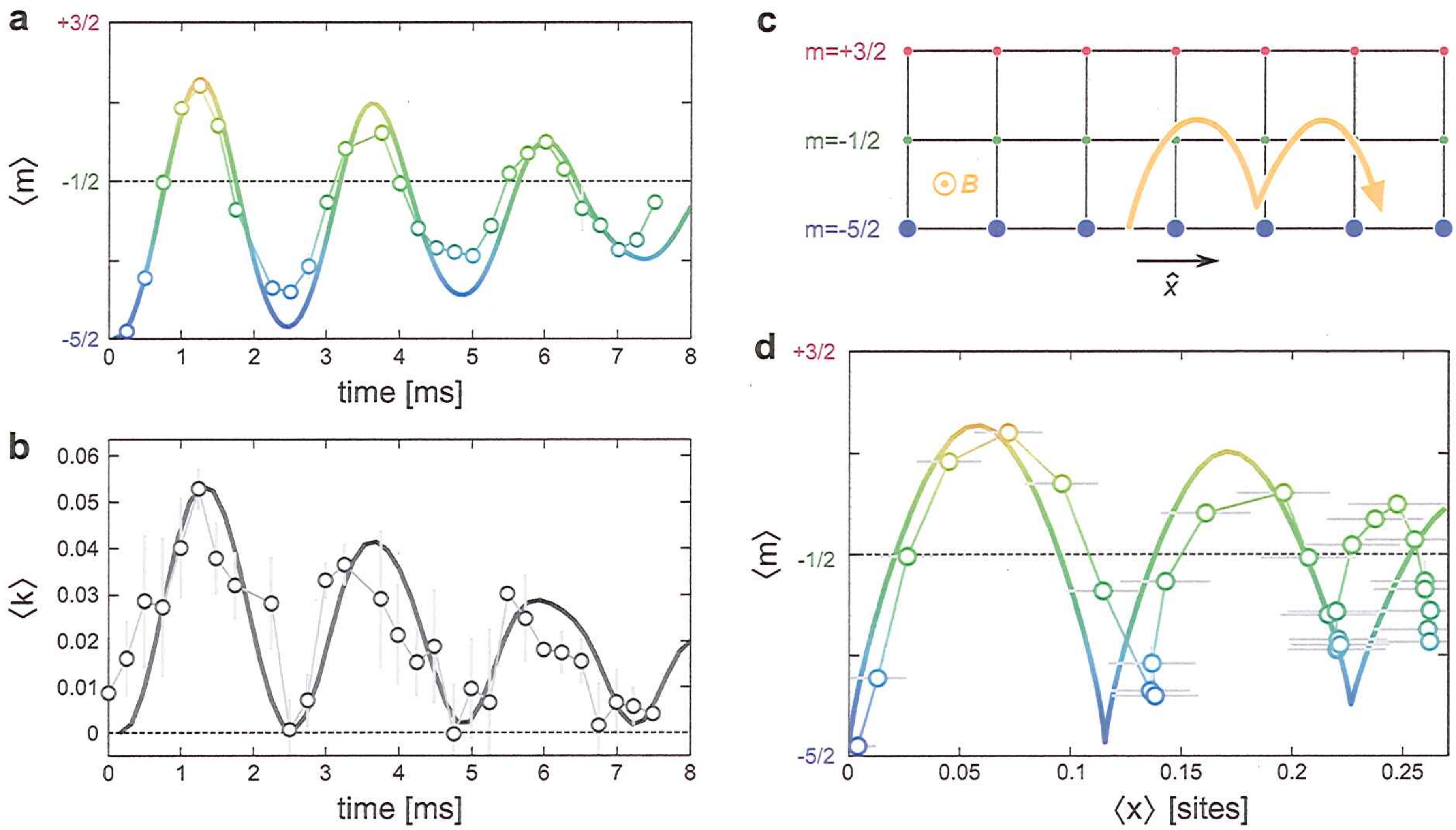
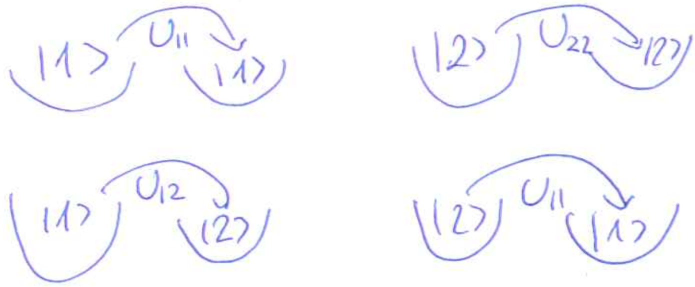


* More "exotic" possibilities

· Let's end our discussion about lattice manipulation and synthetic magnetism with a brief comment about a couple of more "exotic" possibilities.

· Non-Abelian gauge fields [Osterloh et al., PRL 95, 010403 (2005)]

· Let's suppose that at each site we may have 2 possible spin states $|1\rangle$ and $|2\rangle$ (they could be e.g. 2 different hyperfine states). Let's imagine that we engineer e.g. a Raman-assisted hopping which leads to 4 types of processes

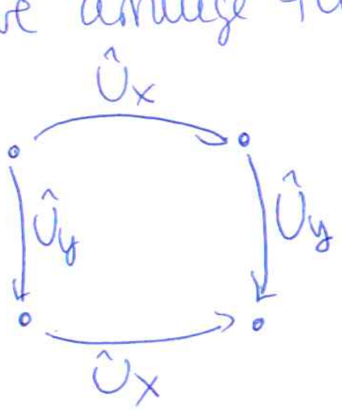


Then the hopping is not any more a c-number but rather a 2×2 matrix:

$$H_{HOPPING} = \hat{\vec{\Psi}}^{\dagger}_{j+1} \hat{U} \hat{\vec{\Psi}}_j \quad \text{with} \quad \hat{\vec{\Psi}}_j \equiv \begin{pmatrix} \hat{\Psi}_{j,1} \\ \hat{\Psi}_{j,2} \end{pmatrix}$$

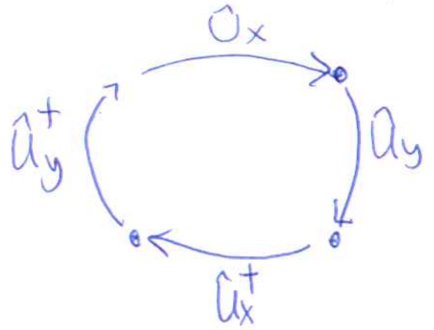
$$\hat{U} \equiv \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix}$$

· If we arrange things such that



such that $[\hat{U}_x, \hat{U}_y] \neq 0$ then we have generated a non-Abelian gauge field.

• Note that something funny occurs. When moving around the plaquette:



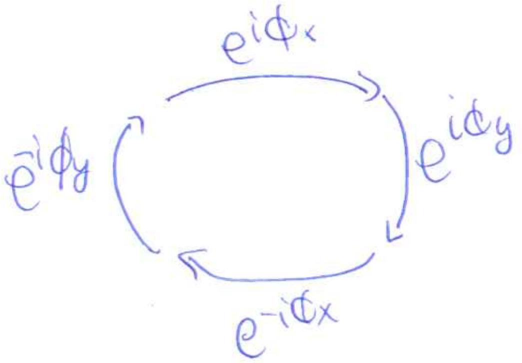
$\Rightarrow \hat{U}_y^+ \hat{U}_x^+ \hat{U}_y \hat{U}_x$

If $[U_x^+, U_y] = 0 \longrightarrow \underbrace{\hat{U}_y^+ \hat{U}_y}_{\mathbb{1}} \underbrace{\hat{U}_x^+ \hat{U}_x}_{\mathbb{1}} = \mathbb{1}$

But if $[\hat{U}_x^+, \hat{U}_y] \neq 0 \rightarrow \hat{U}_y^+ \hat{U}_x^+ \hat{U}_y \hat{U}_x = \mathbb{1} + \hat{U}_y^+ [\hat{U}_x^+, \hat{U}_y] \hat{U}_x \neq \mathbb{1}$

. This is true even if the matrices $\hat{U}_{x,y}$ are spatially homogeneous!

Let's compare with the c-Number case



$\Rightarrow e^{-i\phi_y} e^{-i\phi_x} e^{i\phi_y} e^{i\phi_x} = \mathbb{1}$

If the phases along x,y are homogeneous then there's no net effect per plaquette

(This is why we needed to have an x-dependence to get an effective magnetic field).

* Modulated interactions [Rapp et al., PRL 109, 203005 (2012)]

· Finally, note that up to now we just considered periodic modulations of the lattice potential. However other forms of periodic modulation are possible.

· Let's consider in particular the case of modulated interactions. In the vicinity of a Feshbach resonance one has that the scattering length is of the form: $a = a_{bg}\left(1 - \frac{\Delta B}{B - B_r}\right)$ ← width of the resonance; a_{bg}: background value; B_r: resonant B-field

· If $B = B(t) = B(t+T)$

then $a = a(t) = a(t+T)$

· The on-site interactions in the Hubbard model will hence be $U(t) = U(t+T)$. Let's consider $U = U_0 + U_1 \cos\omega t$

· Then: $H(t) = \underbrace{\left[-J\sum_{\langle ij\rangle} b_i^\dagger b_j + \frac{U_0}{2}\sum_j n_j(n_j-1)\right]}_{H_0} + \underbrace{\frac{U_1}{2}\cos\omega t \sum_j n_j(n_j-1)}_{H_1(t)}$

· Let's define the Floquet basis (the procedure is similar as for the lattice shaking of p. (59)):

$$|\{n_j\},\xi,m\rangle\rangle = e^{im\omega t}\, e^{-i\frac{V(t)}{2}\sum_j n_j(n_j-1)}\, |\{n_j\},\xi\rangle$$

with $\frac{dV(t)}{dt} = U_1\cos\omega t \Rightarrow V(t) = V_0 + \frac{U_1}{\omega}\sin\omega t$ (V_0 may be gauged out, and we will not consider it. We take $V_0=0$)

Then: $\langle\langle\{n_j'\},\xi,m'|\hat{H}_{TUN}|\{n_j\},\xi,m\rangle\rangle = \langle\{n_j'\},\xi|\hat{H}_{TUN}|\{n_j\},\xi\rangle \times$

$$\times \frac{1}{T}\int_0^T dt\, e^{i(m-m')\omega t}\, e^{i\frac{U_1}{\hbar\omega}\sin\omega t\,\frac{1}{2}\sum_j\left[n_j'(n_j'-1) - n_j(n_j-1)\right]}$$

$\frac{1}{2}\sum_j[\ldots] = 2(n_i + n_j - 1)$ ← for $b_i^\dagger b_j$

$$= \langle\{n_j'\},\xi|\left[-J\sum_{\langle ij\rangle} b_i^\dagger\left[\frac{1}{T}\int_0^T e^{i(m-m')\omega t}\, e^{i\frac{U_1}{\hbar\omega}\sin\omega t\,(\hat{n}_i + \hat{n}_j)}\right] b_j\right.$$

· For the same Floquet sector: $m = m' \rightarrow \frac{1}{T}\int_0^T e^{i\frac{U_1}{\hbar\omega}(\hat{n}_i+\hat{n}_j)\sin\omega t} = J_0\left[\frac{U_1}{\hbar\omega}(\hat{n}_i - \hat{n}_j)\right]$

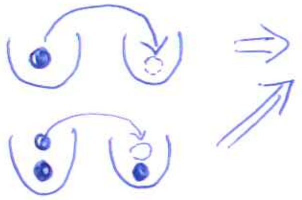
$\Rightarrow$ If $\hbar\omega \gg J, U_0 \Rightarrow$

$$\boxed{\hat{H}_{eff} = -J\sum_{\langle ij\rangle}\hat{b}_i^\dagger J_0\left[\frac{U_1}{\hbar\omega}(\hat{n}_i - \hat{n}_j)\right]\hat{b}_j + \frac{U_0}{2}\sum_j \hat{n}_j(\hat{n}_j - 1)}$$

• Hence modulating the interaction results in a dressed hopping but now the dressing depends on the population difference between the neighbors!

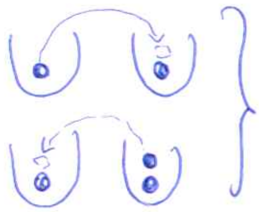
• This has funny consequences, in particular when $J_0\left(\frac{U_1}{\hbar\omega}\right)=0$.

In that case:



⇒ It goes with J

but



⇒ They are forbidden!

• this leads to several consequences. For example, recall our discussion of p. (19) concerning fluctuations in the Mott insulator. If you recall it, quantum fluctuations reduce the parity order O_P^2, which for the perfect Mott (1111..) is $O_P^2 = 1$. Quantum fluctuations are produced by hopping processes  or , which are precisely those processes that we kill! Hence a perfect defect free Mott is expected (as long as there's a Mott at all).